Problem Based Learning approach to train pre-service teachers in mathematics education: an experience.

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Abstract

Societies around the world are focusing in motivating scientific vocations in children. STEM education is a priority in many Governments' education policies. Accordingly, actions should begin by improving pre-service teachers training, including the acquisition of skills in teaching methodologies and ICT tools, which will contribute to increase the motivation of students in STEM subjects. Among the most used methodologies for teaching STEM subjects, we consider the Problem-Based-Learning approach, because of its interdisciplinary vision and scientific working method, based on resolution of real situations. Aiming to improve STEM training in pre-service teachers, the authors conducted an experience in an initial-training mathematics teachers master course, where the pre-service teachers were asked to design an activity to be later addressed, with PBL methodology, by real students of a Madrid High-School. Considering the importance of improving teachers' training in ICT tools, it was required, the inclusion of GeoGebra in the design of the proposed activity. The experience is evaluated in terms of analysis of the problems suggested by the pre-service teachers, and of the results of the in-depth interviews conducted with several of the involved master's students, as well as with the teacher responsible for applying the problems in the High School. We appreciate that the inclusion, in the experience, of GeoGebra and other ICT tools within the PBL methodology approach, was a motivating element. Application of learning-by-doing method, led the pre-service teacher to know better about PBL methodology and GeoGebra, increasing their skills and possibilities to apply both tools in their professional future.

1. Introduction

Science, Technology, Engineering and Mathematics (STEM) education is a key approach for an increasingly complex society based on knowledge, that is why it has become a priority for most Governments. STEM education should be addressed from an early age. It is at school that children must be motivated to learn Science, Technology, Engineering and Mathematics to lead them to become responsible, creative, innovative citizens, and to make them aware of the world around them, that they can contribute to improve [15]. In order to assure the best learning process for our students, pre-service teachers must have also the best training. This involves training them in theoretical, as well as in methodology and technology [12].

The philosophy of Problem-Based Learning is to respond to a situation-problem in a real context from an interdisciplinary point of view. This makes it an ideal methodology to address the study of STEM subjects, because it combines this global and practical vision of what scientific work entails [16]. PBL is an excellent methodology to be used in mathematics education, increasing student motivation, giving theoretical mathematics concepts a real context, and the possibility of analysing their applications.

Based on previous studies we present an experience in training pre-service teachers of Secondary Education level in problem-based learning. In a way of *learning by doing*, the pre-service teachers themselves prepare a *Problem* to be applied in a real class, so that they will be capable to repeat it in the future with their own students. As part of their training process, pre-service teachers had to develop not only the *Problem* (including context, mathematical concepts needed as a base, and those to be achieved, a guide to solve it, and a solution), but also how to evaluate the task, hypothetical mistakes made by the students and how to help and guide them.

Information Technology and Communication (ITC) tools and GeoGebra, as well as the interdisciplinarity character of the proposal, were an important part of the learning and teaching process in this experience, both for pre-service teachers and for the students solving the problem.

We will start the article giving a brief context to the experience, and setting some theoretical remarks, both in what Problem Based Learning means as methodology to be used in training teachers, and in educational uses of GeoGebra and ICT in STEAM education. Then, in the next Section, we will explain the experience, the way we developed it, details of the process and ICT tools employed, and an example of a pre-service teachers proposed problem.

We finish, commenting the obtained results, in terms of the designed problems, and in the results of the in-depth interviews with several pre-service teachers, and with the school formal teacher.

2. Conceptual framework

1.1 Training pre-service teachers

In order to assure a good education to our students, the best training must be assured to pre-service teachers [11]. As a matter of fact, teachers are the final responsible of the teaching process, and so, teachers training should be a priority in every country [1]; [4]; [6].

In Spain, the requirement of reviewing the actual model of initial teacher training to make it suitable to the European one, has been established. An appropriate training in didactics and pedagogy, in ICT applied must complement theoretical knowledge of every subject.

In the case of secondary education in Spain (age 12-18), the model of pre-service teacher training is the “consecutive”. It is also applied in the surrounding countries as France, Italy, or Portugal, in it the training is centred in scientific knowledge acquired during the Bachelor degree obtained at the University [14], followed by a Master’s degree in education (in Spain, *Master of Teacher Training in Secondary Education*, is compulsory).

Nevertheless, there exist a gap between the training received at the master’s in education, and the classroom reality. During the master, a practising period of 180 hours is done at a high school, where a formal teacher tutorizes the students. It is useful for pre-service teachers, but insufficient.

In the case of mathematic learning, the problem is aggravated by the abstractness of the subject, which in many children is a lack of motivation and an obstacle to approaching the study of the subject successfully.

For all the above, it is important to give future teachers the opportunity to collaborate in teaching practices that require the use of ICT and active participation in the classroom, and not only attend theoretical classes in which these teaching methodologies are explained without reaching practice. ([2]; [3])

1.2 Problem Based Learning in secondary education students

Problem Based Learning (PBL) has proved to be a methodology specially issued to be used in the frame of STEM subjects [11].

The term PBL has been taking on different meanings and uses, which usually include [2]:

• Education takes place in the context of a situation or problem.

• Key content is learned from context and performance.

• It is part of a challenge or purpose shared by the students.

This is a methodology based in active learning, in which the students play the principal role, and the teachers acts as guide of the teaching and learning process. Knowledge is acquired in the process of solving a problem or project, as a group of scientists in a research context, or engineers working together to develop a project. PBL take the most when is combined with cooperative learning, and is a great way to motivate students, because of the real context of the problem to be solved, the challenge of self-employment in groups [7]

A problem posed in PBL for students from 12 to 17 years old, has the following common characteristics: real context, in which students feel involved; guide questions adapted to the age of the students and the educational stage, which helps students to identify the needs of the problem, the theoretical concepts to be applied, relevant data, and the search for accurate information to acquire them.

The objective of the methodology is not so much to solve the problem posed, as the process carried out for it. In fact, many times problems have open solution, or even several solutions. The acquisition of knowledge occurs when detecting the needs of the problem, performing the search for information to perform the solution process.

According to our experience in the use of this methodology with high school students, a student who approaches mathematical concepts through this system, not only assimilates them in a direct way as well (or better) than he would through a master class, but also develops other equally important skills ([5], [15], [16]):

- The student becomes more autonomous, since he has to face problems very close to real life , and has to "look for life" to be able to solve it using all kinds of "open" tools. Remember that a PBL does not have a single and guided solution but promotes the search for alternative solutions and the use of different tools. In addition, you must interact with real means: face problems that go beyond the merely mathematical, such as the organization of teamwork, decision-making, face technical problems (a file that is lost, a computer that does not work a colleague that does not come ...)
- The student increases his digital competence, especially in relation to the search for information on the web and the use of ICT tools for problem solving.
- The student becomes more collaborative. The problems presented are designed with such complexity that they require teamwork to be solved in a reasonable time.
- During the process of solving the problem, the student observes different paths and different possible solutions to it. He must evaluate and decide the best way to obtain what he considers the most adequate solution, developing the learning-to-learn competency.

- One of the deficits that students present historically in the learning of mathematics is that of the ability to face and solve problems, however simple they maybe. When facing these PBL more complex problems, whose resolution is not immediate, the student is taking an experience and confidence that will serve in the future to face simpler problems (the classic ones). A student who solves PBLs loses the fear of problems of immediate resolution, and even those apparently more complicated.

1.3 GeoGebra and ICT in Problem Based Learning

The PBL methodology supposes, as we have already established, a knowledge and skills in ICT of the students who face the resolution of the problem, and of the teachers who propose it. The search for information through Internet is fundamental in the resolution process; students will carry it out autonomously, guided to a greater or lesser extent according to their age by the questions included in the problem, and by the teacher himself. Looking for the necessary data and discarding the irrelevant ones, extracting the precise information, contrasting the materials obtained, means an important increase in digital competence for students.

In the other side, the best training pre-service teachers received in ICT, the better teaching and learning process of our scholars. That was an important point of including specialized mathematical software in the experience we show here: Master's students learned specialized in mathematical education ICT tools, making them capable to use its as teachers in the future.

Introducing the use of specific software to solve the problem posed is not necessary in PBL methodology, but it is highly recommended, since it expands the range of skills acquired in the learning process, in addition to acting as a motivating element.

There are many technology tools that can be included when considering a PBL to help in the resolution process. From the most basic such as scientific calculators, spreadsheets such as Excel, or the usual Word and Power Point (especially useful the use and learning of the Equation Editor that incorporate both applications), to more specific software such as the dynamic mathematics program GeoGebra, or Google Sketch-Up (free version).

In the case of the experience presented in this article, we have chosen to introduce GeoGebra as a common element in the proposals that the master's students had to present. Based on previous experiences [8],[9],[14], students had to include questions whose resolution went through the use of GeoGebra (functions, geometry, representation).

We consider that the use of freely available software GeoGebra is especially significant due its huge use in high schools and universities, and the millions of users all over the world [10]. On the other hand, it is important for us to transmit to pre-service teachers the spirit inherent in GeoGebra of sharing and contributing to improve for the good of the community, which we consider fundamental to be a good teacher.

It is advisable to distinguish at this point the use of ICT tools from the point of view of the student who solves the problem, and from the teacher who proposes it. In the case of the experience we show, the master's students play both roles, because, as we will see, the task they developed was to propose a problem to use the PBL methodology in the classroom, and to develop a hypothetical solution given by the students who solved the problem.

An important aspect to consider is the current situation arising from the COVID19 pandemic, which demands the use of collaborative remote work environments. In Spain there have been many students

of secondary education who have had part of their teaching remotely during the course 20-21, so that trying to bring active methodologies to distance learning is almost essential [3]. For this purpose, ICT tools for collaborative work such as MS Teams, Google Workspace, Nearpod, or the recent GeoGebra Classroom, are extremely useful.

3. An experience in training pre-service teachers in STEM education

3.1 Experience

The experience we present here was developed during the course 2020-2021 in the frame of the subject Didactic of Mathematics of the Master in Secondary Teachers Training at Rey Juan Carlos University in Madrid. This is an annual subject, with a total of 90 hours in weekly sessions. It is the third course we apply the activity, introducing new elements every year. In the actual course, lessons were conducted in remote, due to pandemic situation. MS Teams platform was used during the sessions to that purpose, but tasks were attached in Rey Juan Carlos University (Moodle) virtual classroom platform.

The number of students who participated in the activity was 33, working in group. There were seven groups of 4 students, one group of 3 students (in this case, there were partial time students who worked together in the activity out of the classroom), and one student with adapted evaluation who participated individually.

The task consisted in proposing a problem within the methodology PBL to be solved by real students aged 15 years (Grade 10) from *Montpellier School* in Madrid. This is a charter school, located in the metropolitan area, with around 1500 students from middle and working social class. This school introduced the project "1x1" (one student, one computer) in 2008 in Grades 7 to 10, and its teaching and learning project is based on cooperative learning, and active learning methodologies.

Formal teacher at Montpellier school sets the curriculum theme the problem must be related to, and the number of hours the students at school are able to dedicate to work in it. The proposed problem should contain the problem statement with the context, questions to guide its resolution, one solution of the problem, and hypothetical errors students form the high school were supposed to commit. This course we focussed our attention in GeoGebra, so that pre-service teachers had to include at least one question within the proposed problem to be performed using GeoGebra. They could ask the students at school to use GeoGebra also to present results (graphics, geometric representations, …), if needed.

PBL methodology has been applied in this high school in mathematics learning for years, combined with cooperative learning. GeoGebra is used frequently in mathematics classes, every student has his own computer, as we mention above. A template is given to each group of children to guide them in the resolution of the problem. This template is provided to master's students to be considered.

The activity took place on November 2020. Pre-service teachers participated in three training session before designing the problem, two in GeoGebra, and one in PBL. During two more classroom session, they work in groups developing the task. As we mentioned above, this master course was in remote learning, and students collaborated doing the task with MS Teams, supervised by the university teacher. After that, they worked autonomous in the activity for two weeks. Feedback was given in a first stage by the formal teacher from the high school, who analysed the proposed problems from a didactic point of view, making recommendations to pre-service teacher, to make them able to be applied with his students at the high school. Students prepared the final version during Christmas holydays, considering the reviews proposed by the teacher.

When the application of PBL in the high school with the proposed problems has concluded, formal teacher provides final feedback to pre-service teachers with the results of the experience. Students at Montpellier school filled a form at the end of the activity, with questions about the level of difficulty of the problem, understanding of the problem statement, questions aimed at reflect on the problem, and a final open question about their personal opinion about the experience.

3.2 Example of a proposed problem

As an example, we consider the following problem proposed by a group of students (we summarize the statement). The topic was trigonometry, and the problem was posed to students ages 15-16:

"Title: Repairing my sailboat. *You have inherited the sailboat from your grandfather, but it needs several major repairs. You must change* the *forestay, mast, and jib of the ship*. *But beware, there are measures that you will not be able to take, you do not have such a high ladder! That is why we provide you with a map of the boat, and we ask you to measure the elements (angles and lengths) that are closest to the deck of the boat*".

Figure 1: Map of the boat

In order to repair the required ship parts, students must find: (1) Length of the forestay, (2) Mast height, (3) Dimensions of the jay (measurement of its sides) and its total surface (with the help of GeoGebra). In addition, students will be required to check the results by creating an scheme of the problem with GeoGebra, using the application's geometric tools.

The most complex part is the measurement of the angles of the jay to obtain the area of fabric necessary to repair it. To solve the problem, once aware that the angle between the mast and the ship deck is 90º, students must take this data into account and apply their knowledge of trigonometry, in addition to a new theorem (sine theorem). In addition, they should use their knowledge of the GeoGebra tool to find the area of the bow sail (jay). Figure 2 shows an example of solving the problem using GeoGebra. You can also see it in https://www.geogebra.org/m/w4cnza5q.

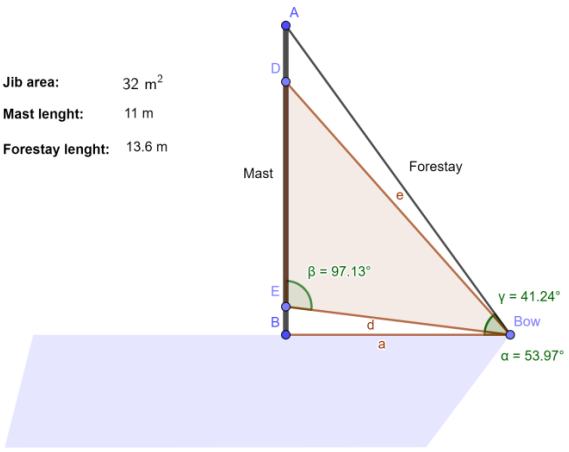

Figure 2: Proposed solution of the problem given by a group of students

3.3 Assessment

Every proposed problem is evaluated by the school formal teacher (70% of the task score), considering the point of view of its application in his classroom, in terms of complexity, quality of the problem as PBL, adequate level, duration, etc.; and by the teacher at University (30% of the task score), attending if the problem followed the given recommendations, analyzing the proposed solution, the inclusion of GeoGebra, and the statement of the problem and the given solution from the didactic point of view. Not every proposed problems were applied at school, formal teacher chose the ones who better fitted with the characteristics of his students (level of difficulty, extension, hours to be dedicated…).

4. Results

Once the set of all the videos was available, a qualitative analysis of them was carried out in a discussion group moderated by a professor from the Rey Juan Carlos University and integrated by 3 university professors from the area of Didactics of Mathematics, and 4 Grade 10 mathematics teachers. The discussion group held working meetings to analyze all the videos taking into account among other aspects if they conformed to the contents and learning standards for the stage, the curriculum approved in the legislation in force in Spain, and if they could be useful for the teachers of the subject [13].

Number of problems suitable to be used in the first stage (no revision needed)	1/9		
Number of problems with percentage of revision needed	20%-30% revision	30%-40% revision	More than 80% revision
	5	2	1
Number of problems suitable to be used in the second stage (after revision)	7/9		

Table 1: Number of problems suitable to be applied at high school in Grade 10

Most of the reviews suggested by the teacher of the high school, were related to the level of difficulty, knowledge necessary to solve the problem, necessary information, type of guide questions (writing, number), context of the problem.

To analyse the opinion of the students after the activity, and the view of the teacher about the experience, in-depth semi structured interviews were conducted with seven students from different groups, and with the teacher.

Some of the responses were:

Question: What advantages do you think using PBL has for teaching mathematics in secondary education?

"*I think it is a great way to bring children closer to what they will find both in their studies at University, in the event that they take them, and in their day to day, both professional and personal*"

"*When a problem is posed for us, we usually have the tools to find ways to solve it, but not directly how to solve it; with the PBL methodology, we work in the search for those ways to solve it, so I think it is a great enhancer of critical thinking and the autonomy of students*".

Question: What is your opinion of introducing this methodology as part of teacher training in STEM?

"*The work process to develop our PBL proposal has allowed me to better understand which teaching situations are appropriate for each age group.*

Question: Comment globally on your opinion about this activity (How do you value the possibility of carrying out with real students the proposed *Problems?* Has the feedback given by the teacher been important for you?).

"*In my case, after all my years as a student, I think it is very interesting to know these methodologies and ICT tools, not so widespread and that can contribute a lot to the student*".

"*It has changed my perception of teaching mathematics after working on this project*"

Teacher of the high school answered similar questions in another interview. Due the limitations of the article, we'll just transcribe the answer to questions 2 and 3:

Question: What is your opinion of introducing this methodology as part of teacher training in STEM?.
"*PBL learning cannot be conceived without the use of ICT tools. If only for the mere fact of having to search for information on the Internet (inherent in any PBL type problem), the use of this type of technology is essential.*

Beyond this, the environment that creates this method is ideal for the use of other ICT tools such as GeoGebra (especially in the areas of functions and geometry), Excel (in the management of tables, functions and statistics), Google Sketch-Up (for spatial geometry) or the usual Word and Power Point (especially useful the use and learning of the Equation Editor that incorporate both applications).

Question Comment globally on your opinion about this activity.

"*I have a positive opinion, no doubt. Historically, in all educational centers, the teachers of the science area have been the spearhead, both in the use of new technologies in the educational field and new teaching-learning methods. This is because of the very essence of STEM subjects, based on the scientific method, rigorous research, and the use of ingenuity to solve problems. What better for any subject in this field than to put students in simulated situations similar to those they will encounter in the future?*

I believe that every teacher in general, but in this area, must carry in "his curriculum" a training in this method of teaching and a willingness to put it into practice when exercising his profession.

5. Concussions and future work

The type of activities such as the one we present here, represent a significant improvement in the training of future teachers. After the experience, the students of the Master's degree learned about the PBL methodology, increasingly present in teaching STEM subjects, and received training in a fundamental tool for teaching mathematics, GeoGebra. The method of learning by doing was applied, proposing themselves a problem as a task with which to apply the methodology. We bring them the option to do it with real students, thus bringing the real practice in the classroom to the teachers in training. Finally, the participation of the high school teacher, who applied his professional criteria to correct the proposed problems and give us the feedback of the application in the classroom, was invaluable, being highly appreciated by the students.

The success of the experience is also measured in terms of motivation. The students of the Master valued very positively the experience, as reflected in the responses of the interviews carried out to a sample of them. The common opinion is that this activity had allowed them to know and apply new methodologies and technology, permitting them to act as practicing teachers. In addition, five of the students continued with the research in PBL methodology, dedicating their Master thesis to this topic.

An 89% of the problems proposed in this course 20-21 by the pre-service teachers, were considered suitable to be applied at Grade 10 mathematical classroom by the discussion group who analyzed the problems, as we presented in the results. The level reached of some proposed problems was so good, that the teacher included them in the library of PBL class, becoming part of the project of teaching mathematics of the high school.

As proposals for improvement for later courses, we would like to introduce elements of GeoGebra Classroom to manage the resolution of the proposed problems, once modified with the indications of the teacher of the center. In this way, the students of the center could work on the resolution remotely, tutorized by the corresponding group of pre-service teachers, who would be supervised both by the teacher of the center and that of the university. In this way, we could incorporate direct feedback to the activity, making it even more real.

The philosophy of sharing and collaborating in the educational community is present, so that another line of improvement in the future will be to publish the PBL made, together with the GeoGebra files and ICT tools used, so that they can be used and improved by other teachers, thus giving an unparalleled dimension to this type of experience.

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