A Tragedy at the National Gallery of Art: Problem-based learning approach to online teaching of mathematics.

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Abstract: *This paper examines an experience of mathematics preservice teachers of using problem-based learning (PBL) approach in an online course about teaching applied mathematics. The study results showed that PBL approach to teaching mathematics supports collaborative/cooperative work and increases PSTs' engagement in and out of class. The structure of the course that involved PBL experiences from students' perspective followed by analysis of PBL approach from teacher's perspective provided PSTs with sufficient background to develop their own PBL-based instructional units. The study results also showed that while PSTs' teams were successful in developing curriculum-based problematic situations with real-world connections, they were challenged to justify the use of technology in these experiences. PSTs also had difficulty with development of authentic assessments within PBL approach.*

1. Introduction

In March of 2020 the pandemic demanded that teachers design a 21st century learning solution that is scalable, adaptable, engaging, and meaningful. We were forced to untether our thinking and create a new world for learning. This transformation required educators at all levels to move quickly to create fundamentally different virtual environments that provided learning experiences that had never before been offered. Transforming the traditional in-person classroom to a virtual environment while maintaining student-centered teaching became one of the most difficult challenges for all educators during the COVID-19 pandemic. This paper examines experiences of mathematics preservice teachers' (PSTs) in an online problem-based learning (PBL) course about teaching and learning applied mathematics designed to address the challenges of the online environment, e.g. truly engaging students in learning mathematics in and out of class, providing space and time for collaborative/cooperative work, and rethinking formative assessment. The course was designed to meet the following goals:

i. Apply the philosophy, theory, and rationale behind PBL through the experience of a variety of mathematics modeling and applications PBL activities.
ii. Use a variety of resources, including the use of technology to facilitate and create PBL units that demonstrate mathematics modeling and applications.
iii. Use authentic assessments, techniques, and tools in planning and evaluating PBL lessons that reflect the needs of the students.
iv. Illustrate how to employ a PBL in mathematics courses through the creation of a collaborative and engaging environment.
v. Evaluate the effectiveness of a PBL lesson by using a set of criteria to determine if the lesson meets the educational needs of the students.

The purpose of this paper is to discuss my success with these goals. To do so, I address the following research question: Did PBL-based design of online course support PSTs' engagement in mathematics problem-solving, collaborative/cooperative work, and development of their own PBL-based instructional units for secondary mathematics classrooms?

2. Background

Studies show that PBL has been an effective approach to teaching and learning mathematics in the in-person settings. In [1] a meta-analysis of 217 studies showed that the implementation of PBL into secondary school mathematics courses significantly enhanced learners' higher-order thinking skills in mathematics. In higher education, the implementation of PBL in mathematics courses led to increased student achievement [2], interest, engagement, enthusiasm, and sense of ownership [3], and improved reasoning skills [4]. Research also shows that PBL is an effective approach to teacher education. Specifically, empirical evidence suggests PBL can deepen PSTs' mathematics content knowledge [5], and prepare PSTs for implementing PBL in their own classrooms [6]. While there is overwhelming evidence that PBL is an effective approach to teaching and learning mathematics and mathematics education courses in the in-person settings, very few studies examine the PBL approach in online mathematics or mathematics education courses. In [7] a study that compared academic achievement of undergraduate students in online and in-person mathematics PBL course, students in the online course performed significantly better than in the in-person course. Moreover, the online environment supported the use of social media for student discussion in a hybrid mathematics PBL course, which led to increased student interaction and development of the mathematics learning community [8]. However, studies are needed to examine how online PBL-based courses can support preparing mathematics PSTs to implement PBL in their future classrooms.

According to [9] students learn best when their teachers maintain a high-level of cognitive demand throughout the lessons, which can be achieved through PBL approach. Therefore, in order to provide an active learning environment with high cognitive demand tasks, I decided to use a problem-based learning (PBL) approach as suggested in [10]. In this approach each unit starts with a problematic situation that serves as the organizing center and context for learning. The problematic situation is usually ill-structured and messy, that students perceive as important and relevant. The PBL process steers students through the complex tasks of brainstorming ideas, identifying useful knowledge, asking appropriate questions, and crafting a strategy for finding answers [11]. However, PBL requires a high level of interaction and collaboration which is not naturally supported by the virtual environment. Thus, I needed to redesign a course about teaching applied mathematics for mathematics PSTs that would transform PBL approach used in the in-person settings to a virtual environment.

3. Course design and implementation

In order to create a highly active learning environment where PSTs could discuss and share ideas, and collaboratively solve problems, I analyzed high-impact influences on student learning [12]. John Hattie [12] examined and synthesized more than 1,600 meta-analyses comprising more than 95,000 studies about factors affecting student learning and ranked them according to their effect size (Cohen's d). Among teaching strategies with the largest effect sizes that could be enhanced by open source instructional technology and effectively used in an online PBL environment I selected Cognitive Task Analysis (CTA, $d = 1.29$), classroom discussion ($d = 0.82$), and providing feedback strategies ($d = 0.62$). I knew I could support these strategies in an online environment by using Nearpod (https://nearpod.com/), a multimedia student engagement platform for designing interactive lessons using various multimedia content, e.g. PhET simulations, interactive videos, VR field trips, Desmos graphing calculator, etc. Nearpod also includes a variety of activities that could be used for formative assessment, e.g. Collaborate board, Draw-it, different types of questions, and FlipGrid video. Live Nearpod presentations could be used in a synchronous mode of teaching enabling teachers to control student progression through the lesson. An asynchronous mode can provide self-paced Nearpod presentations so students can experience the same interactive features at their own pace. All my classes were conducted live, so it was very important for me to have an asynchronous option for students who might have difficulties accessing the Internet during classes. I also wanted to be able to integrate various resources in my lessons, and Nearpod enabled me to do that within one platform, so students would only need to join a single Nearpod lesson to have access to all resources that I had planned for them.

The goal of the redesigned online course was to explore and experience the ways in which applied mathematicians approach practical applications, from understanding the underlying problem, creating a model, analyzing the model using mathematical techniques and digital technology, and interpreting the findings in terms of the original problem. In order to provide PSTs with first-hand PBL experience, I started each unit of the course with a problematic situation on a topic aligned with 7 – 12th grades mathematics curriculum. The course outline is shown in Table 1.

Table 1. Outline for the course *Teaching applied mathematics*

Week	*Focus*	*Topics*
Week 1	Introduction	Introductory problem: A greenhouse extension
Week 2	PBL philosophy and theory	What is PBL? Why use PBL?
Week 3		Teacher role in PBL.
Week 4		Student role in PBL.
Week 5	Middle school	Problem 1. Crime scene investigation: Classroom experience from students' perspective.
Week 6	PBL model	Developing a PBL.
Week 7		Criteria for good applied mathematics PBL.
Week 8	Algebra, geometry	Problem 2. Moving a ladder: Classroom experience from students' perspective
Week 9		Evaluating PBL in the classroom.
Week 10	Precalculus, calculus	Problem 3. Designing a detector: Classroom experience from students' perspective.
Week 11		Brainstorming PBL scenarios for the final project.
Week 12	Assessment in PBL	Alternative assessment options.
Week 13	Gains and pains of PBL	Teachers, students, parents.
Week 14	Student presentations	Individual/group presentations of problems. Peer evaluation.
Week 15	Final project	Reflection on experience. Course evaluation.

3.1. PBL design

Designing a problem scenario that results in the targeted learning outcomes is critical. You want to make sure that the problem scenario encourages students to take charge of their learning, that it emphasizes critical and creative thinking, problem-solving, collaboration, and self-directed learning. The following criteria were used to evaluate problematic situations [:

- Does the problem approximate reality? Will your students perceive the problem scenario as realistic? Avoid creating a scenario that is unbelievable. Avoid designing a problem that is so global and generic that students will throw up their hands in frustration and dismay. If students think that the scenario could never happen in a million they won't be bought into solving it.

- Is the problem curriculum-based? You want to ensure that the problem addresses the target knowledge and skills. Avoid getting carried away writing a problem that you think is intriguing and thrilling, when in reality the problem doesn't match teaching objectives.
- Is the problem engaging? How well will the problem stimulate and sustain curiosity, or the need to explain or understand, or the desire for order, or the need to resolve the situation, or the need to improve a condition? The problem must sustain the motivation to continue interaction and learning from beginning to end.
- Does the problem allow inquiry? The problem presentation should further inquiry and analysis. Is it sufficiently open-ended, ill-structured, and messy?

The problem design is an iterative process, it takes time and energy to write a good problem. The *Tragedy* problem scenario went through multiple iterations before it took its final form shared in this paper as an example of a problem that could be used at the beginning of a middle school unit to teach proportionality and similarity in scale drawings. Here is the excerpt from the problem scenario [13]:

> *A Tragedy at the National Gallery of Art*
>
> *As a team of forensic specialists from Lloyds of London, you have been asked to investigate a theft of a priceless Picasso painting that took place at the National Gallery of Art in Washington, D.C. The painting, titled The Tragedy was housed in the East Building, Upper Level, Gallery 99. The theft of this painting marks the first successful removal of a painting from the National Gallery of Art. The National Gallery had The Tragedy insured by Lloyd's of London for $15,000,000, but of course, the painting is priceless.*
>
> *Neither the audible alarm nor the police alarm had been activated. The wall, which had just hours earlier housed several of Picasso's greatest works, was now missing The Tragedy, one of the most prized paintings of Picasso's "Blue Period". To the officers' surprise, there was no one in the gallery. Black paint was found on the floor beneath the area where the Picasso had hung, and spatters of black paint were found throughout the gallery. In addition, footprints from the paint were found leading from the missing painting to the gallery exit door and outside.*
>
> *Digital images were taken of the paint spatters and footprints. After examining digital images of the trace evidence left at the scene, your team should be prepared to present quantitative evidence that can lead to the warrant for the arrest of the suspected thief. Lloyd's of London and the National Gallery of Art are relying heavily on your team of forensic specialists to determine who stole the Picasso.*

The problem then provides a list of suspects identified by the police, digital images, background on the painting, and requirements for the dossier to be submitted by the forensic team (you can see the full text of the problem at http://bit.ly/3uWcCMg). Considering the fact that this problem was developed for an online environment, instead of a 'real' crime scene I used digital images 'taken at the crime scene' that were 'sent' to the forensics lab, thus creating an opportunity for students to conduct an investigation using online resources only. This problem becomes the context for student learning about proportionality and similarity in scale drawings. The whole unit for this topic could take up to ten days with the plan shown in Table 2.

Table 2. Teaching plan for the middle school unit on similarity in scale drawings.

Day 1	Introduction of the CSI scenario. Whole class development of learning needs using problem-solving grid.
Day 2	Review of ratios and proportions. Introduction of similar figures.
Day 3	Practice solving similarity problems, including problems with proportions in the human body.

Day 4	Learning to use GeoGebra software. Practice constructing similar figures and exploring their properties in GeoGebra.
Day 5	Practice solving similarity word problems dealing with indirect measurements. Group work with digital evidence – qualitative analysis.
Day 6	Introduction to scale drawings. Scale factor. Practice drawing objects to scale using ratios and proportions.
Day 7	Practice finding distances on the map or scale drawings given scale factor.
Day 8	Practice finding distances on the map or scale drawings given scale factor.
Day 9	Group presentations of evidence, results, and conclusions
Day 10	Peer review: Presentation of rebuttal analysis from the "defense attorneys".

3.2. PBL implementation

This problem places students in an active role of forensic specialists to investigate the theft of Picasso's masterpiece, *The Tragedy,* from the National Gallery of Art in Washington, DC. In order to provide authentic experience to PSTs, I modeled the first day lesson the way it would be taught to middle school students. The Nearpod slides for this lesson are shown on Figure 1. (You can access this lesson at join.nearpod.com with the code UZGN9 that is valid through September 4, 2022).

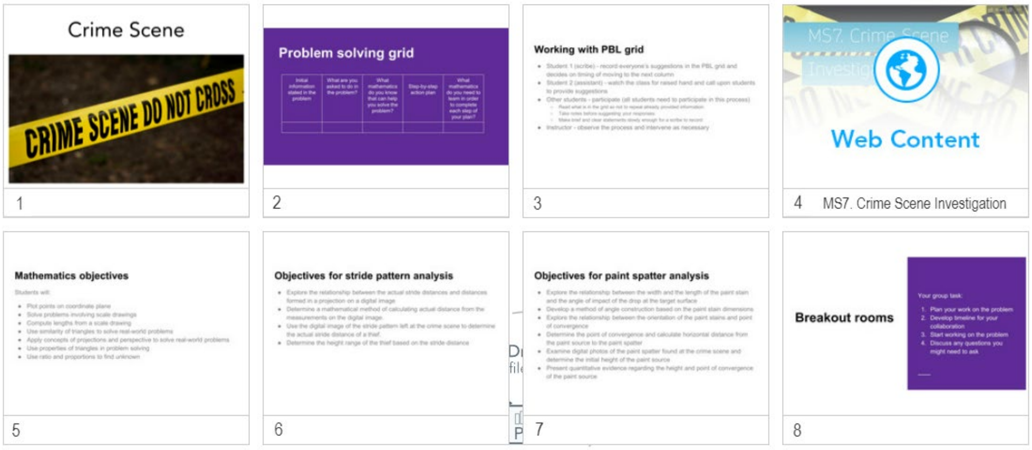

Figure 1. Nearpod slides for Day 1

In this lesson we first discussed the rules of working with the problem-solving grid, a graphic organizer for logical brainstorming (Figure 1, slides 2-3). This graphic organizer is used to help students unpack the problem, e.g. to identify the information given in the problem and translate it into mathematics language, then formulate problem questions in mathematics terms, determine what mathematics they know and develop a plan on how they intend to approach the problem. The final step leads to identifying learning needs for solving the problem that become learning objectives for the whole unit. The process of working with the problem-solving grid as a whole class involves assigning one of the students to be a scribe, a person who facilitates whole class discussion and records information provided by other students. The whole class starts working with the first column until the scribe decides to move to the next column. Students can still add information to the previous column, but they cannot move forward until the scribe decides to do so. One very important aspect of this whole class activity is recording all student ideas without judgment or evaluation.

After the rules were clarified, Nearpod took PSTs to a Microsoft Sway (Figure 1, slide 4) that included a full problem statement, digital images from the crime scene, link to editable Google Document with a blank problem-solving grid, as well as some additional information about the actual painting. At this point PSTs were given quiet time to read the problem and then each of them had an opportunity to use a 'raise hand' feature of Zoom in order to add information to the grid. Working with the problem-solving grid took about 30 minutes with adult learners, but I would have allocated one full class period for unpacking the problem if I were working with middle school students.

There were 18 PSTs in this course and every one of them contributed to the grid. They were truly interested in the plot and wanted to share their ideas about solving the problem. The Google Document that everyone could see replaced the white board we would have used in the physical classroom. At the same time, PSTs' ideas were saved in a document that was easily accessible to all of them. This is just one example of how Google Apps can be used as tools for collaborative problem solving, inquiry-based learning, and student discourse in online courses.

In a PBL classroom, a problematic situation is used as a context for learning the topic. Thus, it is critical that the learning needs that students identify through their work with the problem-solving grid match the unit teaching objectives. I consider it a successful problem design if students' learning needs and my teaching objectives match at 80% or more. In my previous experience teaching middle and high school students, it took about 1-2 months to get to this matching level. After the introduction of the problem on the first day of a unit, the problem becomes a student group project for the length of the unit. When a problem is engaging, students want to learn mathematics as they immediately see how they can use what they learn in class to resolve a given problematic situation, and I think that is the major advantage of the problem-based learning.

With PSTs we continued the lesson to discuss pedagogical aspects of the task, we compared their learning needs with my teaching objectives (Figure 1, slides 5-7), and discussed how the unit could continue in a middle school mathematics classroom. In the lessons that followed I continued using the Nearpod platform to embed inquiry activities, problem-solving, technology tasks to demonstrate how this platform could be used throughout the unit to maintain a high level of engagement and to provide students with collaborative activities and feedback. Nearpod has over 8,000 premade lessons available online, so teachers can find lessons that will meet their teaching needs for most of the topics and modify these lessons as needed. As an example, we reviewed a Nearpod lesson that I modified from the available premade lesson to demonstrate a possible way to introduce a Day 6 topic of scale drawings (selected slides are shown on Figure 2).

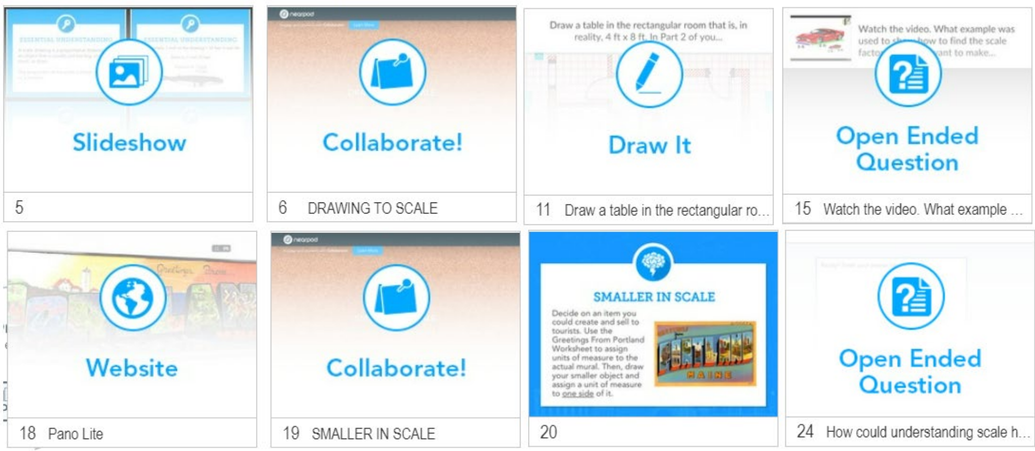

Figure 2. Selected slides from Day 6 Nearpod lesson

This lesson uses interactive features such as slideshow, video, and VR to present material to students in an engaging way. Nearpod's Collaborate board serves as a place for sharing ideas, and student drawings and answers to open-ended questions provide the teacher and the students with ongoing feedback. At the end of the lesson, students are asked to reflect on their learning and how what they learned could help them in their crime scene investigation. All student work is automatically saved in Nearpod reports, providing the teacher with another opportunity to review student learning and make necessary adjustments to the next lesson. (You can access this lesson at join.nearpod.com with the code ZEGCN that is be valid through September 4, 2022).

This lesson also helped me engage PSTs in a discussion of pedagogical aspects of teaching applied mathematics through a PBL approach. Throughout the course for each problematic situation that I introduced to PSTs, we continued discussions about PBL philosophy and theory, design of problems, models for PBL implementation, issues of assessment, as well as issues of using a PBL approach in teaching mathematics in an online environment.

3.3. Examples of PSTs' solutions of the *Tragedy* problem

In order to have a full PBL experience, PSTs working in groups of 3 were required to solve each unit problem as part of their out-of-class assignments. For the *Tragedy*, the final product had to include the dossier with quantitative evidence that could lead to the search warrant of the suspected thief (thieves). These were the requirements for the dossier content:

1. Summary of your findings based on your analysis of the digital evidence left at the crime scene that can be understood by the judge who is not a mathematics professor.
2. Appendix that includes:
    a. Step-by-step explanation of your analysis.
    b. GeoGebra files with completed constructions and calculations.
    c. Explanations of how you used GeoGebra with illustrations from the software.

When problems are ill-structured and messy, you can expect very different approaches and solutions from different students. Team A presented a flowchart showing their process of solving the problem (Figure 3).

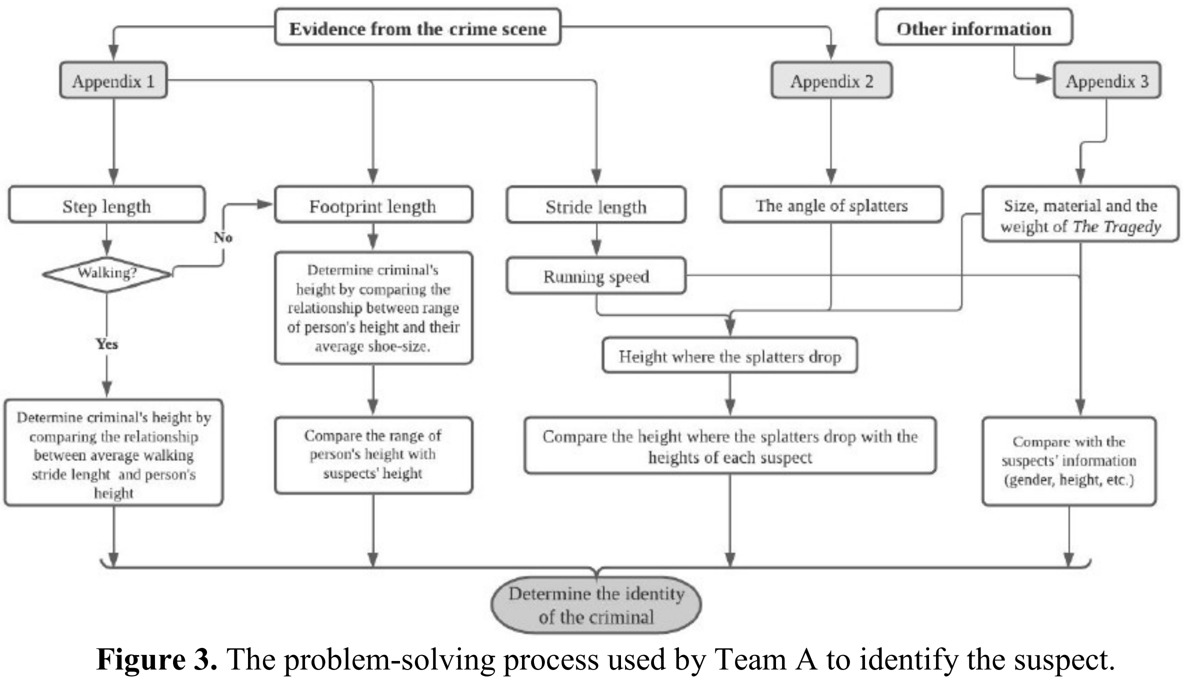

Figure 3. The problem-solving process used by Team A to identify the suspect.

Following this plan, team A produced mathematical analysis to estimate the size of the footprint, the stride and the speed of the criminal, the height where the paint spatter came from, and the weight of the framed painting. Using these data, they were able to identify a suspect most likely responsible for the theft.

Team B used GeoGebra to perform an analysis of the provided image of multiple footprints that were found on the floor of the gallery (Figure 4).

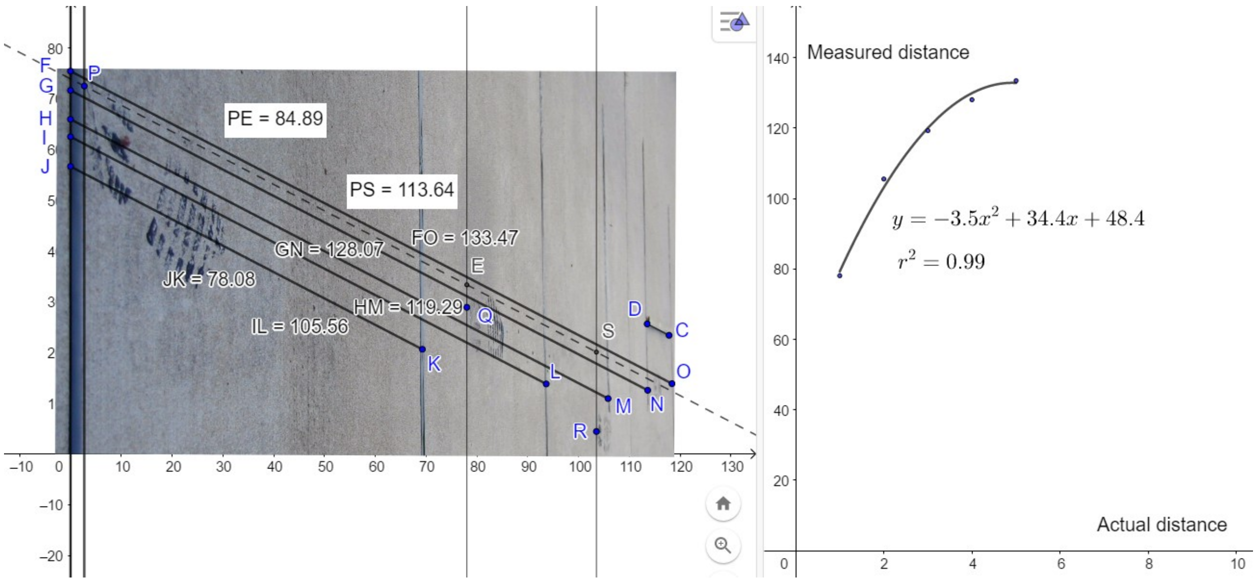

Figure 4. GeoGebra analysis of the footprints image by Team B. The black tape is placed at 1-meter intervals.

Here PSTs measured distances between the black tape on the image and then plotted the measured distance against the actual distance. They then used regression analysis and found a quadratic function to model this relationship. Using this function, students found the distance between the base of one footprint to the base of the next footprint, which led them to an estimation of the height of the suspect.

In order to understand the paint spatter, Team C conducted an experiment with spattering water onto a paper to find a spatter pattern similar to that in the digital evidence (Figure 5).

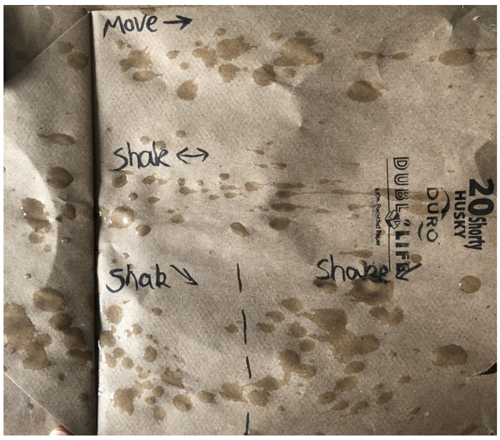

Figure 5. Water spatter experiment by Team C to understand paint spatter evidence

Team D used physics to model the shape of paint drops on the floor (Figure 6) and properties of right triangles to determine the height from which the paint was spattered (Figure 7) in order to estimate the height of the suspect.

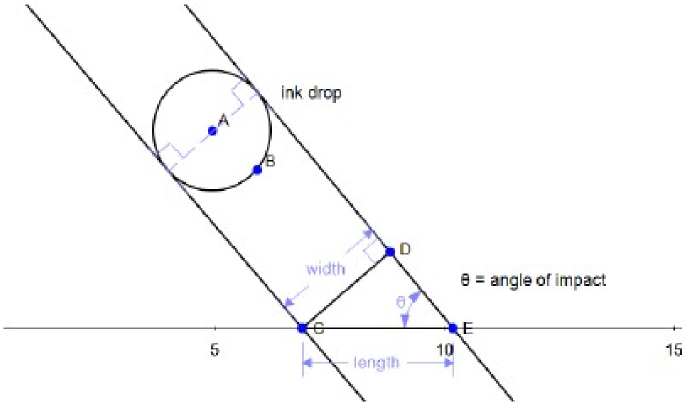

Figure 6. Physical model used by Team D

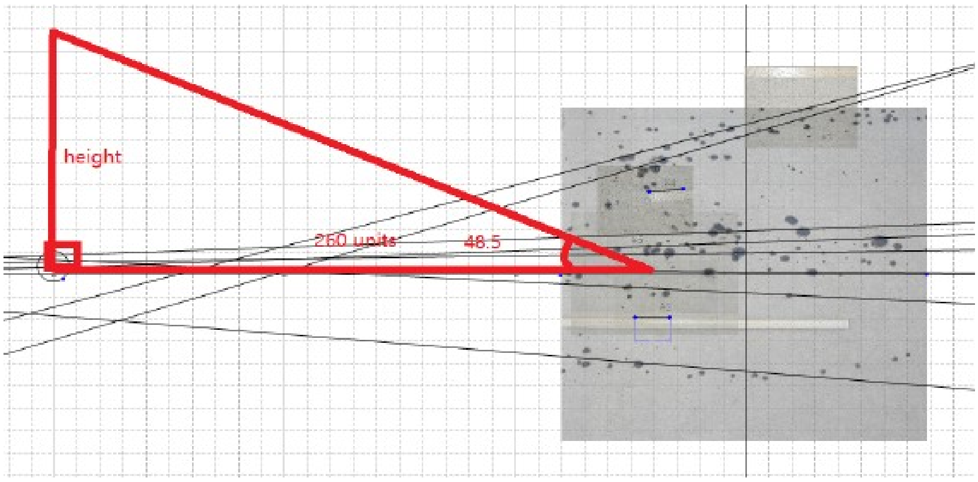

Figure 7. Paint spatter analysis by Team D

When time came to share their solutions, I wanted PSTs to engage in peer review in a meaningful way. Therefore, each of them was assigned to a suspect identified by a peer as a 'defense attorney'. The defense attorney had the following task:

> *Your job is to help your client, and therefore, you will be looking for flaws in the quantitative evidence provided by the forensics team. After a careful review of the documents provided to you by the forensic team, submit a summary of your comments and recommendations for the judge based on your analysis of these documents.*

This task created another problematic situation for PSTs that engaged them in much deeper analysis of the work developed by their peers. Here is an excerpt from one of the peer reviews:

> Your Honor,
> I respectfully ask that you disregard the forensic report's findings based on the following reasons:
>
> 1. First and foremost, there is a grave inconsistency in this report. The scientists affirm they have calculated a step length of 1.07 m. Later, they affirm the stride length of 2.86 m. This can mean one of two things. Either the suspect has a left step of 1.07 m

and a right step of 1.79 m, or the scientists miscalculated. If they made an error here, who is to say where else there are hidden mistakes? Note that Frothingan (2018) suggests the stride is usually 2 times the step length (meaning, the left and right steps are roughly the same size).

2. The report includes the calculations for only one footprint. In case an average was calculated (like in the case of the paint spatter), the scientists could have a stronger case.
3. This report simply dismisses the women suspects, affirming they are not strong enough to carry a heavy painting. This sort of stereotypical profiling is often deceptive. What if the women suspects practice CrossFit? Further, the report assumes that the footprints were left by the thief by chance, when everyone knows any woman could be smart enough to plant evidence to point to other suspects.
4. The calculations of the paint spatter assume that the suspect was running and dropped paint on the floor by accident. However, it could be that a suspect was actually dispersing paint deliberately with horizontal and vertical initial velocity.

Given the reasons above, please consider releasing my client from preventive custody. The prosecutor's case is very weak. (a member of Team B)

4. Analysis of PST-generated PBL units

As part of the course final project, PSTs engaged in designing their own problematic situations. The scenarios covered a wide range of topics, such as planning a school concert during pandemic (Team F) to address middle school mathematics standards, park design competition (Team C) and building accessible entryways to public schools (Team B) to address high school geometry standards, designing a soccer shooting machine (Team A) and planning an indoor intelligent traffic flow for social distancing (Team E) to address high school algebra standards, and Coca-Cola bottle design competition for Olympic games (Team D) to address calculus topics. Each team submitted written PBL package that included teacher's materials and student package. Teacher's materials had to include instructional objectives and research-based rationale for the PBL-based unit, emphasizing PBL design in relation to meeting instructional objectives, justification of the topic relevance and value for students, and explanation of connections of mathematics knowledge and skills with real-world problems or natural or social science applications. PSTs also had to justify essentiality and affordances of technology integrated into the PBL experience. The teacher's materials also included unit teaching plan and assessment strategies. Student package had to include PBL scenario with suggested reading and Internet resources, expectations for the final product, and detailed assessment instrument of the final product.

The team's PBL packages were assessed using 4-point rubric that measured quality of PBL package in five different domains: research-based rationale for PBL (RBR), analysis of real-world connections of mathematics knowledge in the PBL unit (RWC), technological essentiality and affordances (TEA), pedagogical approach to support and assess student PBL experience (PA), and quality of scenario/problematic situation according to PBL evaluation criteria described above in section 3.1 (QPS). Table 2 represents results of this analysis for the six team projects. These results indicate that the most difficult aspect of designing PBL experience for this group of PSTs was meaningful integration of technology and providing research-based rationale for the use of technology by students. For example, Team A included requirement for the students to use GeoGebra in order to find a mathematical model for the trajectory of a soccer ball, but they did not include any explanation of how students will be using the software and they did not justify why technology was essential for problem-solving. Team E just made a generic statement that students will use technology to "develop the plan of the situation, create a model, and perform calculations aligned with the social distancing guidelines". Team F included requirement for students to use GeoGebra "to sketch the school gymnasium, create a location of each audience seat, and calculate the distance between each seat" but did not explain how GeoGebra features could be used for this task.

Table 2. Descriptive statistics for PSTs' written PBL packages

	RBR	RWC	TEA	PA	QPS	Group Mean	SD
Team A	4.0	4.0	1.0	2.0	4.0	3.0	1.26
Team B	4.0	4.0	3.0	4.0	4.0	3.8	0.40
Team C	4.0	4.0	3.0	3.0	4.0	3.6	0.49
Team D	4.0	4.0	4.0	4.0	4.0	4.0	0.00
Team E	2.0	4.0	2.0	2.0	4.0	2.8	0.98
Team F	3.0	4.0	2.0	3.0	4.0	3.2	0.75
Class mean	**3.5**	**4.0**	**2.5**	**3.0**	**4.0**	**3.4**	**0.58**

The pedagogical aspects of integrating PBL into curriculum-based instructions were also challenging for majority of PSTs. Specifically, development of authentic assessment that was aligned with unit instructional objectives, and provided opportunities to assess individual students within groups was the most challenging task for the same teams. Team F failed to explain the expectations for the student final project and all three teams (A, E and F) developed rubrics that did not align with unit objectives and were mostly measuring completion and organization of the student project rather than their learning of mathematics.

However, the results also demonstrate that all teams of PSTs were able to develop an engaging problem-based scenario for the selected instructional unit, that represented good approximation of real-world problem, provided opportunities for student inquiry, and was sufficiently open-ended and ill-structured. All problematic situations were curriculum-based and the teacher's materials included thorough synthesis and analysis of the connections between mathematics knowledge and skills and real-world applications.

5. Conclusions

While the focus of the course was mostly on pedagogical aspects of teaching, my experience with PSTs demonstrated that they got deeply engaged in problem solving and actively participated in class discussions and activities, despite the physical separation of remote learning. The final projects developed by PSTs suggest that course design and structure supported their pedagogical content knowledge for using PBL approach in teaching mathematics online. These findings are also supported by PSTs' comments from the course evaluations:

- *Experiencing the PBL approach first hand and reading about the PBL approach in scholarly journals that specifically discuss the pedagogy behind the different decisions made in using a PBL approach helped to prepare for using PBL in the classroom.*
- *It was great to learn about problem-based learning. It was also interesting to work on PBL as students and also create our own PBL as teachers. It was motivating.*
- *PBL approach encourages students to go beyond the rote memorization aspect of learning. It motivates students to learn more. Students can answer the following question: why am I taking this course?*

- *PBL makes me understand that students could learn by themselves, and teachers could play more roles not only the "teacher".*

At the same time, the study identified challenges PSTs had in integrating PBL into teaching mathematics that related to use of instructional technology and assessment of individual students within collaborative settings of PBL approach. Future studies are needed to examine how PSTs could be supported in online pedagogy courses to develop skills necessary to effectively integrate technology and design assessment tools appropriate for PBL classrooms.

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