Designing Performance Tasks in Mathematics Using Technological Tools

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Abstract: In the Philippines, the performance task is one of the major summative assessments in the K to 12 curriculum. This paper discusses how performance tasks may utilize mathematical apps within the context of blended learning. Guidelines on designing performance tasks, as well as the GRASPS framework, are discussed. Performance tasks that cut across various grade levels and strands of mathematics are presented. These involve divisibility (Numbers), integer and polynomial operations (Algebra), triangle centers (Geometry), and statistics (Statistics). The performance tasks described in this paper can provide an initial idea for the design of other summative assessments and contribute to the literature on the use of technological tools in assessment and evaluation.

1. Introduction

The Covid19-pandemic has driven a shift of modalities in teaching and learning in schools worldwide. In the Philippines, the Department of Education has called for a blended learning modality for schools in lieu of face-to-face instruction in 2020. These modalities include printed modules,

radio, television, and the Internet. New challenges emerged from the sudden shift in educational practice, and some of these challenges relate to assessment. Traditional paper-and pencil assessment methods are no longer completely applicable. Schools are expected to adopt "assessment practices that can meaningfully support student development and respond to different contexts at this time" [1].

There is already a wealth of strategies for using technology in formative assessments even prior to the pandemic [2-7]. However, new schemes for summative assessments remain to be a key challenge for teachers, especially if these are to be done online [8]. This paper reports on the creation and design of performance tasks utilizing innovative technological tools such as mathematical applications as a potential strategy for summative assessment. Technology can provide new and more dynamic ways to face some mathematical tasks, with respect to paper and pencil methods, such as modeling real-life phenomena, making conjectures and proving properties based on experimentation, predicting, pattern-seeking [7, 9]. As Spector et al. [6, p. 60] said, "In addition to providing the means to support personalized learning and a smart education system responsive to the needs of learners and teachers and their learning environments, new technologies can support key 21st century skills ... – notably critical thinking and problem solving."

In the Philippines, one mandated summative assessment is in the form of a performance task, which comprises 40% of students' final grade [10]. This paper reports on the design and construction of sample performance tasks on numbers, algebra, geometry, and statistics using mathematical apps that a mathematics teacher can utilize at appropriate junctures in the curriculum. The use of mathematical apps in the design of each task is deliberate and aims to aid the students to carry out calculations that will be used in the analysis and solutions of the problems indicated in the tasks.

2. Performance Tasks

Philippine policy guidelines [10] define performance tasks as tasks that allow students to demonstrate their knowledge in various ways. These are typically done over a period of time and the purpose is to provide students the opportunity to integrate their understanding of the main topics covered during the academic quarter (about ten weeks). There is a wide range of possible formats, such as skills demonstration, group work, multimedia presentations, research or investigative projects, or written output.

The assessment guidelines were refined in response to the pandemic [1]. Assessment exemplars and rubrics were provided, together with suggestions for students with low, medium, or high access to technology. For mathematics, suggested performance tasks include the following: constructing graphs from conducted surveys, multimedia presentations, outdoor math, probability experiments, problem posing, reasoning and proof, using manipulatives to show math concepts and solve problems using measurement tools and devices. Performance tasks may also integrate multiple competencies not only in mathematics but also in the other subject areas.

Wiggins and McTighe's GRASPS model [11] is a recommended guide in carrying out performance tasks. This model is applicable when the performance task involves a real-life situation. GRASPS stands for: Goal (a task must state the problem or challenge to be resolved), Role (a task must explain who students are in the scenario and what they are being asked to do), Audience (a task must specify who the students are solving the problem for, who they need to convince of the validity and success of their solution for the problem), Situation (a task must provide the context of the situation and any additional factors that could impede the resolution of the problem), Product, Performance, and Purpose (a task must explain the product or performance that needs to be created and its larger purpose, and Standards and Criteria for Success (a task must dictate the standards that must be met and how the work will be judged by the assumed audience).

3. App-based Performance Tasks

The performance tasks in this section utilize apps that are part of Mathplus resources (mathplusresources.wordpress.com), which are products of an ongoing government supported project "Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment" by the Department of Science and Technology- Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD). All apps can be accessed through the website https://mathplusresources.wordpress.com/.

3.1 Grades 1 to 6

A divisibility rule is a shorthand way of discovering whether a given number is divisible by a fixed divisor without performing the division, usually, just by examining its digits. Knowledge of divisibility and factoring rules can facilitate computations needed for more advanced topics such as fraction operations. This topic is listed in the most essential learning competencies [12] for students in Grades 4 and 5. Students are expected to solve both routine and non-routine problems using rules on factors, multiples, and divisibility of numbers.

Integrating technologies into learners' activities has the potential to keep them more engaged in studying divisibility rules and factoring. Hence, the assessment activities and performance tasks on divisibility and factoring enumerated in this section begin with the use of technology.

One app designed to facilitate learning number sense is the *Divisibility Game*. Students may choose which divisibility rules they want to practice on and then either play to gain mastery, increase speed in answering questions, or simply engage in drill work. In the Divisibility Game, students can choose the divisor, for example 3, 6, 9 and the number of items (Figure 3.1 (a)). Then students select numbers that are divisible by the divisor.



Figure 3.1 Screenshots of the Divisibility Game

For example, in Figure 3.1 (b), among the numbers, which are divisible by 3? Students may also choose to play with 12 four-digit items such as in Figure 3.1 (c) or with 16 four-digit items such as in Figure 3.1 (d).

Another app is the *Factor Game*. Through the game, students demonstrate understanding of prime and composite numbers. The game addresses the need for the students, from various grade levels, to identify factors and multiples of a given number. Students select a number and all its factors. In the easiest level (Figure 3.2 (a)), numbers 1 to 30 are placed on a 5×6 grid arranged in order while in the 1-60 level (Figure 3.2 (b)), randomly selected numbers from 1 to 60, not in order, are placed on the 6×7 grid. Students may also select a time limit. The time limit can push students to strategize in selecting the numbers to get a higher score within the available time.





Figure 3.2 Screenshots of the Factors Game

In preparation for the performance task, a series of informal activities and formative assessments may be performed. Students may initially explore each game option in the app in a form of free play. At this point, the idea is to engage the students' interest and motivate them to reach as many levels as possible. Teachers may monitor their students by asking them to send screenshots or a written record of their progress. The students' outputs may enable teachers to see some learning patterns and provide timely feedback and interventions when necessary.

Students can also perform more structured activities. As a first activity, students can perform a "Shading Activity" using the divisibility rule. Given a grid of numbers, students are asked to shade all numbers divisible by 3 as in Figure 3.3 (a). In the end, the shaded numbers will form a familiar letter of the alphabet, object or an animal, as in Figure 3.3 (b).

25	11	56	77	37	19	49	25	11	56	77	37	19	49
71	91	42	18	36	95	83	71	91	42	18	36	95	83
38	85	24	73	75	32	28	38	85	24	73	75	32	28
28	59	57	27	48	13	40	28	59	57	27	48	13	40
16	10	84	29	63	80	34	16	10	84	29	63	80	34
22	76	87	20	30	82	65	22	76	87	20	30	82	65
31	61	92	55	25	26	52	31	61	92	55	25	26	52
			(a)							(b)			

Figure 3.3 Shading activity

A sample performance task related to this, is one where the teacher asks the students to design their own puzzles. A student creates a puzzle by filling-up the grid (Figure 3.4 (a)) with two-digit numbers so that the numbers in the shaded squares are divisible by a number of their choice and the numbers in all other squares are not.



Figure 3.4 (a) A sample grid (b). Fill in the blanks activity

A variation to this task requires students to create a puzzle by designing their own image, for example, an arrow, animal, or any object so that when students shade the correct squares with numbers divisible by their chosen number, the students will end up with the image. Furthermore, students may also use grids of different sizes, with a mixture of two-digit and three-digit numbers. In addition, in this task, teachers may require students to come up with three or more grids where the images would follow a story. At the end of the task, students write their reflection. Sample guide questions are i) How did you choose the numbers to be placed on the grid? ii) Any difficulties encountered? Explain these challenges. iii) What did you learn from this task?

Another activity is the "Fill in the Blanks Activity" where students fill in the blanks with numbers that will satisfy the given conditions on divisibility. For example, students fill in the blanks so that the 4-digit number is divisible by 9 as in Figure 3.4 (b). At the end of the activity, teachers may ask if there are other solutions and ask students to explain. A sample performance task related to this is requiring students to answer more difficult non-routine problems such as those appearing in national

competitions. Sample problems are: i) If the nine-digit number A1234567B is divisible by 45, determine all possible values of A and B (2019 Metrobank-MTAP-DepEd Math Challenge - Grade 6 National Finals), ii) How many ways are there to arrange four 3's and two 5's into a six-digit number divisible by 11? (19th Philippine Mathematical Olympiad - National Oral Stage). At the end of the task, students explain the steps they followed to solve the problem. Although this performance task does not involve a real-life situation that is central to the GRASPS model, the task is aligned with other alternatives indicated by the official guidelines [10] because it provides students an opportunity to integrate their knowledge of divisibility in an environment of inquiry and problem solving.

3.2 Algebra

Addition and subtraction of integers and polynomial expressions are fundamental algebraic skills that students need to master. Different models have been utilized to represent the concepts underlying integer operations [13, 14]. However, there seems to be no app that covers polynomial expressions. *AlgeOps* is an interactive visual mathematical app [15] which makes use of pictorial and symbolic representations of integers and polynomials (see Figure 3.5 for screenshots from the app). The app incorporates both the neutralization and number line models [16] which offer a more holistic representation of integers and eventually, strengthens instruction on integer and polynomial operations.



Figure 3.5 Screenshots of the different modes of an updated version of AlgeOps

To facilitate the operations, students use the different buttons found on the left and right panels of the screen to construct the expressions. For example, Figure 3.6 (a) illustrates the addition problem of (-2) + 5. It is represented by two red balloons and five green balloons. These visuals are displayed when the (-) left button is clicked twice and the (+) right button is clicked five times. Moreover, the

three green balloons representing the correct answer (Figure 3.6 (b)) remain after the animation of the neutralization method.



Figure 3.6 A representation of the addition problem: (-2) + 5 = 3

Performance tasks are usually open-ended and often yield more than one correct answer. This characteristic is manifested in the following task:

Students are to construct different situations where the sum of the balloons is the same as the given diagram. The diagram shows two green balloons plus two green balloons (or 2x + 2x) and three green apples plus two red apples (or 3x - 2y). The diagram (Figure 3.7) can be unique to each student since it is generated by the device used by the student.



Figure 3.7 Addition of polynomials

Indeed, the task involves providing different values of a, b, c, and d that satisfy the equation ax + by + cx + dy = 4x + y. As such there will be numerous responses that students can provide satisfying the equation. Additional constraints on the values of a, b, c, and d can be added to make it more challenging. For example, the values must range from -20 to 20 and the values need not be integers. In consideration of guidelines for assessment, this performance task can involve competencies in another subject area. For example, the student's output requiring creative storylines and visual representations of the polynomials can be used competencies in writing and the arts.

The use of *AlgeOps* in designing the performance task provides a digital medium that replaces the usual pen and paper format of assessment. The generation of unique exercises to students illustrates

its efficiency and the animation of the neutralization method provides a dynamic representation of the addition and subtraction of integers and polynomials.

3.3 Geometry

One of the advantages afforded by technology in performance tasks in geometry is the dynamic geometry environment [9]. As will be discussed here, a digital geometry environment in assessment can develop mathematical and technological competencies and connect mathematical ideas to everyday experiences and to other disciplines [17]. It can also promote mathematical and technological competencies, such as developing and applying new mathematical knowledge through problem solving; developing mathematical reasoning, developing visualization skills to assist in processing information, making connections and solving problems, and generalizing and exploring properties of objects [7]. In addition, the feedback provided by a dynamic geometry environment supports interaction during assessment [7, 18].

One example of a performance task for Geometry in Grade 7 employing the GRASPS model is a real-life application in which students gain experience and discover the significance of triangle centers such as the centroid, incenter, circumcenter and orthocenter. This can be facilitated with a *Geogebra* applet "Bisectors of Triangles" that allows the student to explore the various centers (Figure 3.8).



Figure 3.8 Screenshots from the Geogebra applet: Bisectors of Triangles

The GOAL of the performance task is for the students (their ROLE as a research team) to present and discuss a proposal to the Chief Executive Officer (AUDIENCE) of a local amusement park, *Enchanted Kingdom*, regarding the best possible location of a snack stand in the park so that it will be strategic to the three most popular rides in the park (SITUATION). The final output (PRODUCT) is a multimedia presentation (using a presentation software, video demo, poster) that showcases i) the location of the snack stand such that it is central to the three main popular rides in the park and how this was determined, ii) maps and pictures to support the recommendation, and iii) the advantages and disadvantages of the location, supported by sound geometric principles. The multimedia presentation will be one session (in an online environment, this will be one synchronous session) where the students will present the proposal to the teacher.

The performance task is divided into four parts.

- Students will work in groups. To get students to start thinking about this task, some questions
 posed could be: "Have you ever visited the amusement park?", "What was your experience in
 getting a snack in between rides?", "Where would be a location that would be "central" to most
 customers?". The location of the snack stall is critical in order to attract more potential
 customers. Using a Google Form, students then conduct a survey among classmates and friends
 of the top three favorite/popular rides in the park.
- 2. In order to be guided with regards to the strategic location of the stall in the park, explorations on the definition and properties of the various four (4) centers of a triangle will be carried out using the *Geogebra* app "Bisectors of Triangles" (see Figure 3.8) which will be given to the students. This is a very important part of the task where students explore the various centers by using the "drag feature" of the app. By being able to drag the vertices of a triangle, students develop visualization skills, which are important in processing information. For example, some key concepts and properties that can be explored for the incenter using the *Geogebra* app are: the incenter is the center of the triangle's inscribed circle; the incenter is always inside the triangle, the perpendicular segments from the center to sides of the triangle serve as radii of the inscribed circle. The incenter can be verified for various types of triangles.
- 3. For the discussion/analysis part, students will consider the four different types of circle centers when solving this task. They will also need to clearly establish the assumptions that are necessary in order to come to a solution, i.e., how did you define "best site of the stand," etc. In the proposal analysis, the student justifies the appropriate use of the triangle center depending on its application. (i) Is it important that the center be equidistant from the three vertices or sides of the triangle? ii) Should it be that the center, when connected by a line segment to each location (vertex) creates three portions that are equal in area? iii) Should the circle be

circumscribed about or inscribed in the circle? iv) Is this choice realistic in a given situation? Through the dynamic geometry software, the students can employ trial and error techniques, interpret the behavior of the triangle and its parts to make inferences about it, and test the validity of their conjectures. The students' conjectures can be communicated through email or Messenger app for quick feedback as they go through the task and seek confirmation of their work.

 Students will then find evidence of the distance between the food stand and the three rides using Google Earth (see Figure 3.9) or some other application.



Figure 3.9 Screenshots from the Geogebra applet: Bisectors of Triangles

For the STANDARDS part of the GRASPS framework, the following criteria may be used: i) data/assumptions used to arrive at the location of the stand (survey, geometric explorations using the app), ii) geometric principles used to arrive at location, iii) accuracy of the geometric solution, iv) supporting figures/screenshots from app/tables, and v) presentation (video, PowerPoint, poster) and delivery.

3.4 Statistics

The role of context in learning and teaching statistics has consistently been emphasized in the literature. For instance, according to Cobb and Moore [19, p. 801], "Statistics requires a different *kind* of thinking, because data are not just numbers, they are numbers with a context." Moreover, the Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) [20, p. 11] indicates that assessments "must require students to use statistical reasoning with context and variability at all stages of the statistical problem-solving process." In this section, in light of the role of context in Statistics Education, the design of performance tasks using data from the web-based application *Senso Eskwela Pilipinas* (SEP) is discussed. The SEP app provides teachers and students not only contextualized but also authentic and relatable data. The benefits afforded for the exploration and transformation of data with digital technology reveals fresh approaches to analyzing statistics [21].

SEP is "an online platform for gathering, storing, and accessing readily available and relatable data for learning and teaching statistics" [22, p. 174]. It is a web application patterned after *CensusAtSchool*, which was first established in the UK in 2000 with the "dual thrust to enliven data handling activities within the classroom while also educating children about the principles and processes involved in conducting a census" [23, p.1]. In SEP (Figure 3.10 (a)), students anonymously log in to the platform (freely available at mathplusresources.com) to answer a 31-item survey (Figure 3.10 (b)) involving questions on different topics such as food, hobbies, and interactive tasks. The students' responses become part of the SEP database, to which they have access. Students and teachers can retrieve the data of their own class/es or random samples from the entire database.



Figure 3.10 Screenshots from the SEP platform: (a) The landing page for student accounts, (b) A question from the SEP survey

It is anticipated that students' use of data from the SEP database in performance tasks can increase their engagement and interest as "the sense of belonging that participating pupils feel, purely because they know their own responses will become part of a database of responses from their peers, is fascinating and motivating for them" [24, p. 175]. Another clear advantage is the ready access that students have to the data from the SEP database. In situations in which the actual data collection process is difficult or is not integral to the objectives of the task, readily available data will prove to be convenient.

The performance task *From SEP to Infographic* discussed below recognizes the aforementioned role of context and adapts the GRASPS model. It addresses the following Grade 7 Statistics competencies in the Philippine K-12 curriculum [12, pp. 229-230]: "poses real-life problems that can be solved by Statistics, gathers statistical data, organizes data in a frequency distribution table, uses appropriate graphs to represent organized data: pie chart, bar graph, line graph, histogram, and ogive, draws conclusions from graphic and tabular data and measures of central tendency and variability, draws conclusions from graphic and tabular data (and measures of central tendency and variability)."

The task involves the creation of an *information graphics* or *infographics*, which has become more common in recent years. These infographics, which are visual representations of information, are mainly used to present complex data in a form that is more understandable and, at times, more impactful. This performance task allows learners to choose from the varied data available in the SEP database and use these data to create their own infographics. The GOAL is for the learners to retrieve data from the SEP database and organize, process, and analyze these data so that they can create an infographic on their chosen topic.

The students are given the following setting and instructions for the performance task: "In this task, you will play the ROLE of a researcher and writer for a social media page that mainly posts interesting information (e.g., their hobbies, activities, interests) about the Filipino youth. The administrator (or admin) of the social media page is asking the page's researchers and writers to submit new infographics (PRODUCT) that can be posted on the page. Naturally, the admin would like to receive submissions that have the potential for increased engagement from the page's followers (AUDIENCE). Moreover, the infographic should be based on real data, so the admin has advised the use of the SEP database.

As one of the researchers and writers, you are now faced with the following tasks (SITUATION):

- 1. Determine a suitable topic of the infographics based on the available data in the SEP database.
- 2. Process and organize the data into graphs or charts.
- 3. Analyze the data and the graphs/charts.
- 4. Design and create an infographic, with the graphs/charts integrated in a creative or meaningful way, that presents clear, concise, and interesting information on the chosen topic."

The students are also given the following criteria for grading (STANDARDS): i) topic of the infographic, ii) correctness of graphs or charts, iii) organization of the infographic, iv) design and creativity.

The performance task using the SEP platform offers a number of advantages. First, since the SEP database contains a large amount of readily available and varied data, students are given more space to think creatively in deciding the topic and direction of their performance task. This aspect is enhanced further by the fact that the students also answered the SEP survey themselves. Because the data is relatable to them, the students may easily develop a curiosity towards how their responses compare to others. Moreover, the Access Data feature (Figure 3.11) of the SEP platform offers the students flexibility and allows them to make their own decision/s in terms of i) using their own class' data or a random sample from the database, ii) which questions in the SEP survey they want to work with, iii) which grade level/s they should focus on, and iv) how small or large a sample size they

should use. These allow them to ask themselves questions like "Do we use data only for Grade 7, or Grades 7 to 10? Do we do a comparison between grade levels? Do we try to relate the data from two SEP questions?"

Mathplus Pilipin	Eskwela as	a 🧖	Hi, Student Logout						
	Download Your Class Data								
9. Which p	9. Which pet animals do you have at home? Write how many of each y X V Search								
	Downloa	ad a Random Samr	ble						
This allows	• This allows you to get a random sample from the SEP database								
School Year	2020 - 2021 +								
Enter sampl	e size 50	There are a total of 1341 respondents							
Select		~ 1	Search						
14. The fo these gan	bllowing are some Pinoy gam nes.	nes. Indicate how much you enjoy playing							

Figure 3.11 The Access Data feature of the SEP platform

Given sufficient time, students will also be able to create multiple infographics, from which they can choose their best work for submission. With the SEP database, this is possible for them without having to repeat the data collection process. Finally, accessing and processing data from the SEP database mirror steps in doing statistical analyses using data from actual databases. Students get raw data from the SEP database, then organize and process them using the skills that they have learned. This results in a more authentic context for the performance task.

Moreover, conforming to one of the thrusts of the Department of Education's Order No. 31 s.2020 [1], the performance task can integrate competencies in other subject areas. For example, the students may be asked to accompany their infographic with a feature article in English or in Filipino. The article may serve as an assessment for their written communication classes. The design aspect of the infographic may also be used as an assessment for their arts class. Lastly, their output may also be used as an assessment for their Qulture, Society and Politics class if the possible topics of the infographic are restricted appropriately.

The SEP database naturally leads to a number of performance tasks. The infographics task is suitable for Grade 7, but more tasks can be designed to assess more advanced statistical concepts. For example, in *Hypothesis Testing Using SEP Data*, which is designed for Grade 11 classes, students play the role of researchers for a non-profit organization that promotes the health and welfare of

Filipino youth. After performing data analysis and hypothesis tests, they are to prepare a written report for the officers of the organization to help in their planning of programs and campaigns. The complete details of this performance task are available in the SEP website (https://mathplusresources. wordpress.com/senso-eskwela-pilipinas-for-grades-1-11-statistics/).

Finally, the use of SEP in performance tasks offers a possible avenue for doing effective online assessment. In addition to the elements mentioned above, these performance tasks can be easily implemented in an online setting. Teachers can easily communicate the instructions to students and receive students' submissions online (e.g., via learning management systems, email, or official/school-authorized messaging platforms). In the same way, students can conveniently retrieve their data sets from the online SEP database. Moreover, depending on the involved topic/s, students may opt to use freely available online tools (e.g., a spreadsheet application) to organize and process their data.

4. Conclusion and Future Direction

Designing mathematics performance tasks for summative assessments must be adapted to the changes brought by the shift to a blended learning modality of education. The summative assessments in the form of performance tasks described in this article were designed to address the urgent educational difficulties driven by the Covid-19 pandemic and the need of the teachers to create forms of assessment while in a blended learning modality. The design of these tasks considered alignment with curriculum goals so that integrity of assessments is still met.

This study presented the design of mathematics performance tasks that creatively make use of mathematical apps to support student learning. The sample tasks address the need of making assessments in four key content areas: Numbers, Algebra, Geometry and Statistics. This set of tasks can provide teachers in basic elementary, junior and senior high school with creative ideas on designing performance tasks in the future. The designs of all performance tasks discussed in this paper considered the creation of an environment of inquiry and the integration with other subject areas whenever applicable. Further, the task designs for Geometry and Statistics were guided by the GRASPS framework by Wiggins and McTighe [11].

While these apps were designed to help students gain more insight and develop stronger mathematical skills, this paper described how the apps can also be integrated into performance tasks that teachers can assign their students. The performance tasks were presented in the webinar for Philippine teachers *Amplifying Development of Assessment and Performance Task Skills with Mathplus Resources*. Although the performance tasks and apps were designed for Filipino learners, these can assess learning outcomes that are present in many other national curricula as well. As a goal, the authors affirm the recommendations of [6] for an "open assessment repository (OAR) or an education observatory" that addresses the need for challenging but engaging technology-assisted performance tasks. The tasks designed by the authors can form part of this repository.

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