An Accurate Time Measurement of Short Interval Using an Electronic RC-Charging Circuit

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Abstract: *Some experimental data need to be carried out in a short span of time. It is not possible to measure the time manually using a normal stopwatch. Some can be done by using a highspeed photography. An alternative way of a measurement of short time accurately with the help of an electronic RC-charging circuit is designed and presented in this work. The time measurement starts by charging a capacitor C through a resistor R. The voltage across a capacitor, Vc is an exponential function of time t with the constant values of R, C and a constant voltage supply Vs of the circuit. The voltage across a capacitor C can be measured by using a datalogger with high sampling rate. The voltage charging across capacitor Vc is very fast and the charging time t can be calculated from the equation* $V_c = V_s(1 - e^{-\frac{t}{RC}})$. *This time measurement can be applied to a free-falling object experiment in order to calculate the acceleration due to gravity, g.*

1. Introduction

One of the four important fundamental quantities of scientific measurement is the time. In many science experiments we can normally use a stopwatch to measure the time. But in some experiment like measurement of a speed of a moving object, we cannot measure the time manually using a stopwatch. We have to find some other means to measure the time of short interval. A high-speed photography is one of the methods to measure the time from a photograph or a video clip. But there are some other ways to measure the time of a short interval.

Instead of measuring the time directly, we can easily measure another quantity which is a function of time. Then we can calculate back to the quantity of time using a mathematical relation of these two quantities.

In an RC charging circuit, when a voltage source is applied to an RC circuit, the capacitor, C charges up through the resistance, R.

The voltage Vc is a function of time, t. Vs is a constant voltage source. If the values of the resistor R and capacitor C are known, then the falling time, t, can be obtained. The circuit diagram for RC-charging is shown in the Figure 1. The time t can be obtained by calculating from following equation.

$$V_c = V_s\left(1 - e^{-\frac{t}{RC}}\right) \tag{1.1}$$

The electrical charge stored on the plates of the capacitor is given as: Q = CV. This charging (storage) and discharging (release) of a capacitors energy is never instant but takes a certain amount of time to occur with the time taken for the capacitor to charge or discharge to within a certain percentage of its maximum supply value being known as its time constant (τ).

If a resistor is connected in series with the capacitor forming an RC circuit, the capacitor will charge up gradually through the resistor until the voltage across it reaches that of the supply voltage. The time required for the capacitor to be 99% or almost fully charge is equivalent to 5 times of time constants or 5τ. Thus, the transient response or a series RC circuit is equivalent to 5 times of time constants (5τ).

This transient response time T, is measured in terms of τ = R x C, in seconds, where R is the value of the resistor in ohms and C is the value of the capacitor in Farads. This then forms the basis of an RC charging circuit were 5τ can also be thought of as "5 x RC".

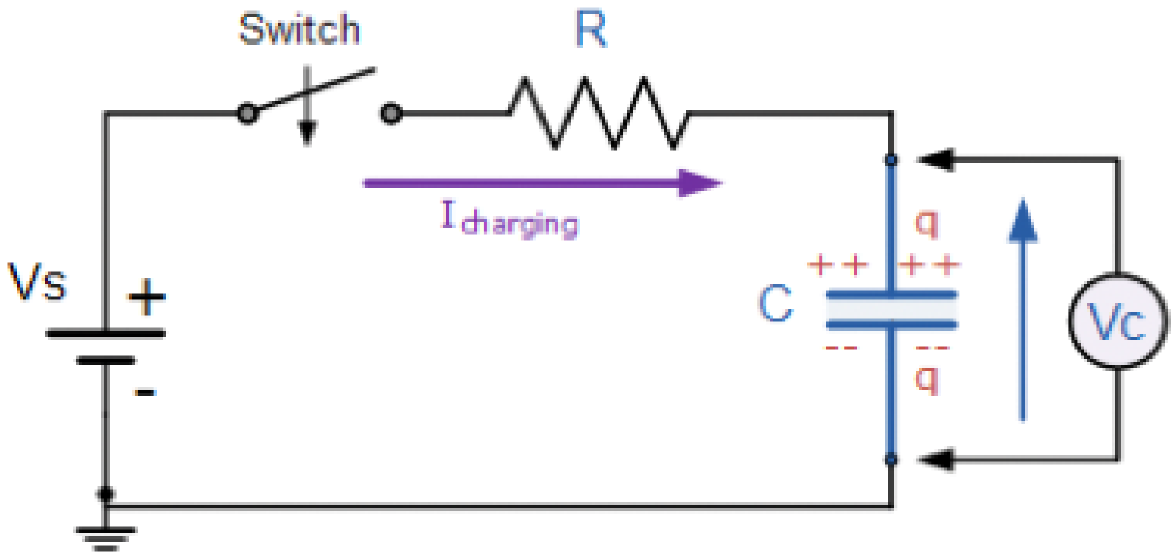

Figure 1.1 Showing A Circuit Diagram for RC-Charging

In the Figure 1.1, let us assume that the capacitor, C is fully "discharged" and the switch (S) is fully open. These are the initial conditions of the circuit, then t = 0, i = 0 and q = 0. When the switch is closed the time begins at t = 0 and current begins to flow into the capacitor via the resistor.

Since the initial voltage across the capacitor is zero, (Vc = 0) at t = 0 the capacitor appears to be a short circuit to the external circuit and the maximum current flows through the circuit restricted only by the resistor R. Then by using Kirchhoff's voltage law (KVL), the voltage drops around the circuit are given as:

$$V_s - R \times i(t) - V_c(t) = 0 \quad (1.2)$$

The current now flowing around the circuit is called the charging current and is found by using Ohms law as: i = Vs/R.

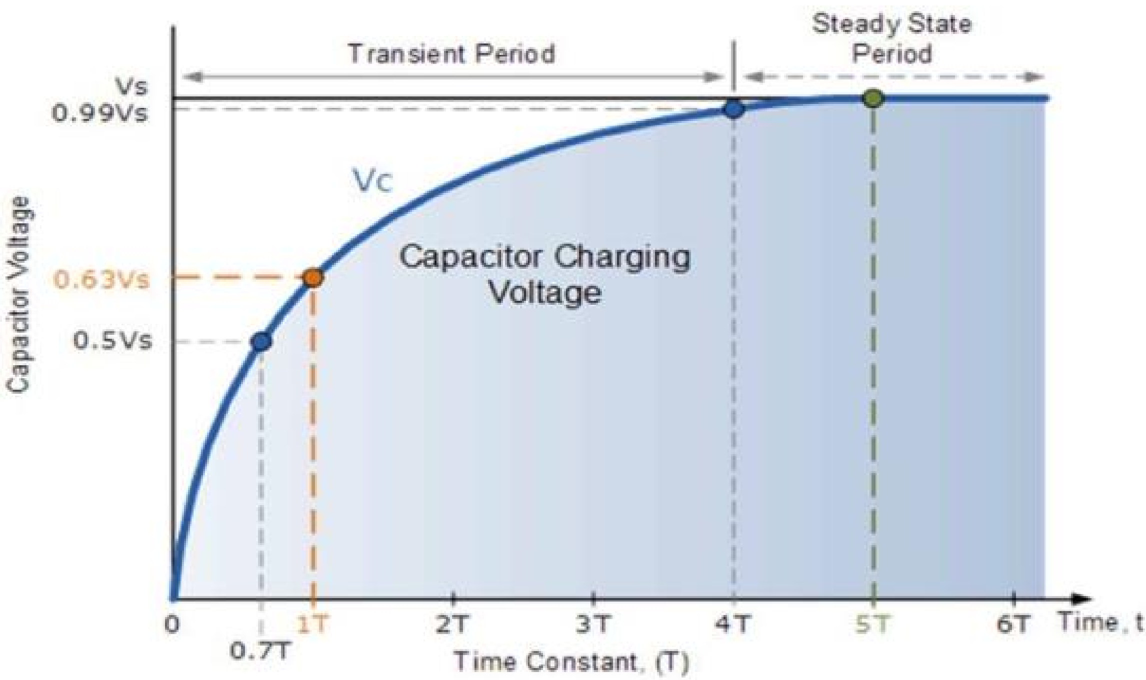

Figure 1.2 Showing a Curve of Charging Voltage as a Function of Charging Time of RC Circuit

The capacitor (C), charges up at a rate shown by the graph in Figure 2. The rise in the RC charging curve is much steeper at the beginning because the charging rate is fastest at the start of charge but soon tapers off exponentially as the capacitor takes on additional charge at a slower rate.

As the capacitor charges up, the potential difference across its plates begins to increase with the actual time taken for the charge on the capacitor to reach 63% of its maximum voltage possible fully charged voltage, in our curve 0.63Vs, being known as one full Time Constant, (τ).

This 0.63Vs voltage point is given the abbreviation of 1τ, (one time constant).

The capacitor continues charging up and the voltage difference between Vs and Vc reduces, so too does the circuit current, i. Then at its final condition greater than five times of time constants (5τ) when the capacitor is said to be fully charged, $t = \infty$, $i = 0$, $q = Q = CV$. At infinity the charging current finally diminishes to zero and the capacitor acts like an open circuit with the supply voltage value entirely across the capacitor as Vc = Vs.

So mathematically we can say that the time required for a capacitor to charge up to one time constant, (1 τ) is given as: RC Time Constant, Tau (τ)

$$\tau = RC \tag{1.3}$$

This RC time constant only specifies a rate of charge where, R is in ohm and C in Farads.

Since voltage V is related to charge on a capacitor given by the equation, Vc = Q/C, the voltage across the capacitor (Vc) at any instant in time during the charging period is given in equation 1.

2. Construction

The example of an accurate time measurement of short interval using an electronic RC-charging circuit is a determination of a gravitational acceleration "g" value by measuring time of a free-falling object. Direct measurement of time of a falling object manually using a normal stopwatch is not accurate. The accuracy of using a stopwatch depends on human reaction time when pressing the button. Simple human reaction times averaged 0.220 seconds and recognition reaction times averaged 0.384 seconds [8,2]. If we measure a time of a falling object in a short distance with a travelling time of about 0.3 seconds, it can give an error of about 100%. This is the reason why a measurement of time using an RC-charging circuit is introduced.

The experiment can be performed by dropping a object from rest at the top point of the tube. The experiment chamber is made from a transparent plastic tube. An iron nut is used as a falling object. This falling object is held on the top of the tube using a magnetic holding lever as shown in the Figure 2.1. When a lever is lifted, the RC-charging circuit starts to work.

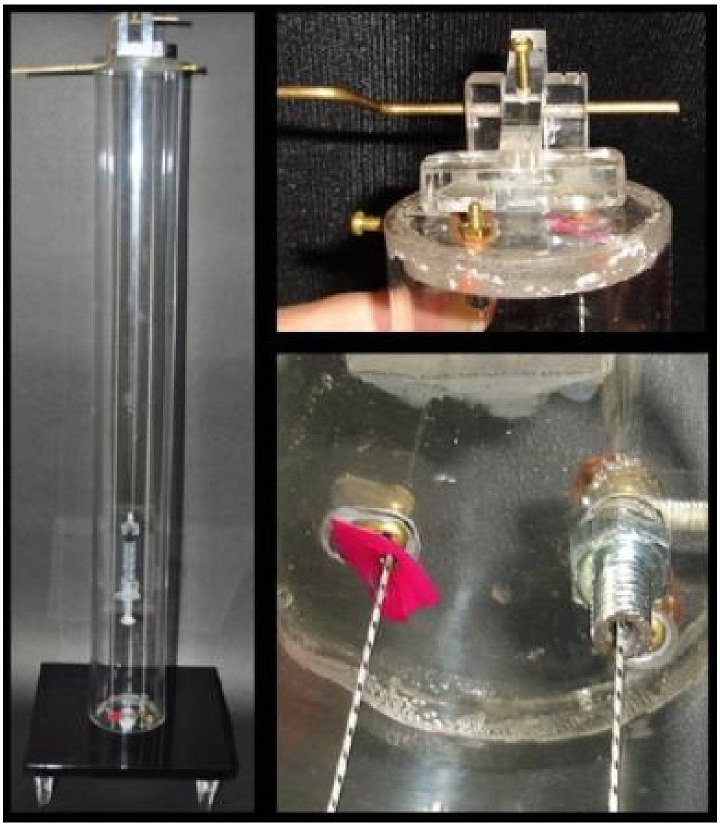

Figure 2.1 Showing the Experiment Chamber Made from a Transparent Plastic Tube with a Magnetic Holding Lever for Dropping an Iron Object

In order to measure the falling time of the objects for calculation of gravity g, a micro-key switch underneath is needed to trigger the electronic circuit of time measurement to stop charging the capacitor as shown in Figure 2.2.

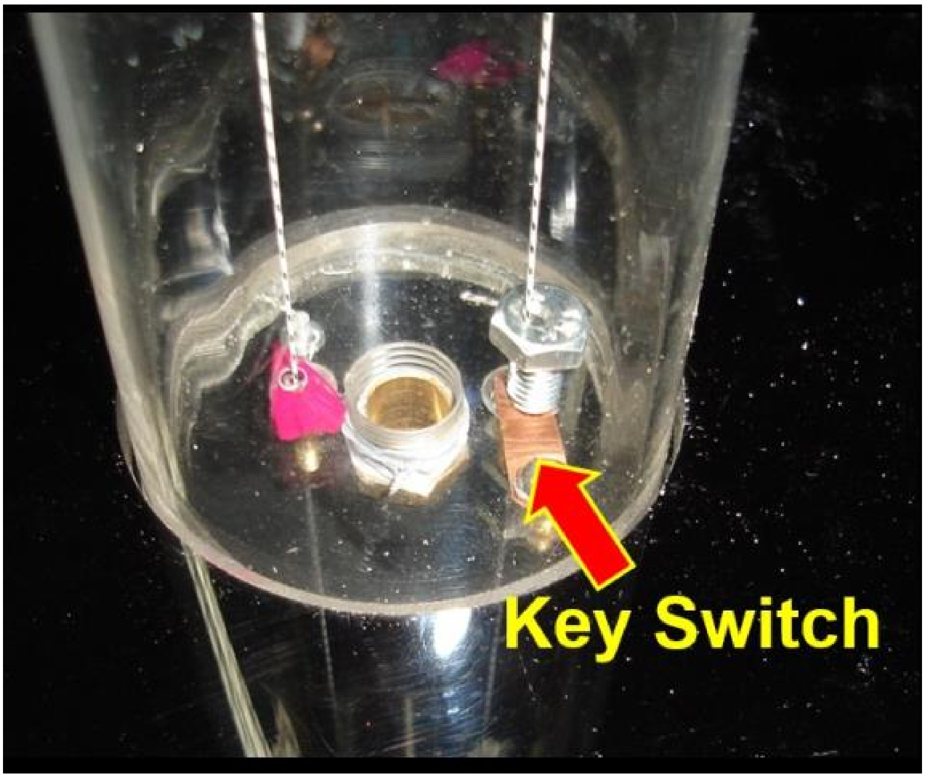

Figure 2.2 Showing a Micro-Key Switch in the Chamber for Triggering the Electronic Circuit of Time Measurement

The charging voltage Vc can be measured by a data logger CBL2 and displayed by a graphic calculator TI84 from Texas Instruments [4].

3. Experiments

An object falling from rest, the traveled distance S can be calculated from the following equation of motion:

$$s = \frac{1}{2}gt^2 \tag{3.1}$$

Where g is an acceleration due to gravity and t is a traveling time. Knowing the length of the chamber tube S and traveling time t, an acceleration due to gravity, g, can be calculated from equation (3.1).

The variation of Vc as a function of time t for different R and C in the circuit can be seen by simulation using Geometer's Sketchpad [1].

The charging voltage Vc can be measured by a data logger CBL2 and displayed by a graphic calculator TI84 from Texas Instruments [4]. There is a stop key switch at the bottom of the tube. When a falling iron nut touches the switch, the voltage will drop, and the time can be measured.

4. Results

Determination of a gravitational acceleration "g" value by using an RC charging circuit

The result of this experiment can be seen on the display screen of the data logger CBL2. We started the datalogger first then released the object to start charging right afterward. We can see that the RC charging circuit started at $t_1 = 0.21$ s and stopped at $t_2 = 0.51$ s. The starting and stopping times can be obtained from the screen of a CBL2. Time difference is 0.30 s as shown in the Figure 4.1. The travelling distance S in this experiment is 0.45 m. We can then obtain the value of g from the equation 3.1.

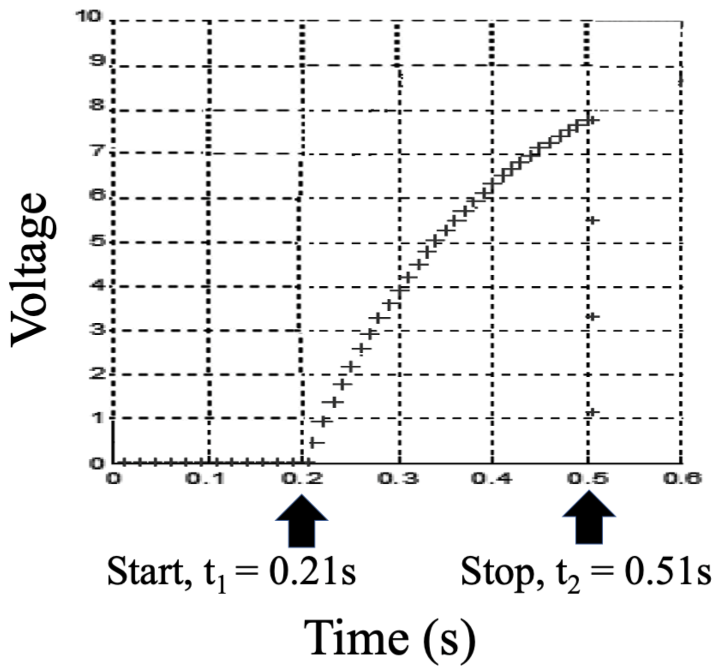

Figure 4.1 Showing a Charging Voltage as a Function of a Time for a Falling Object

It follows from (3.1) of

$$s = \frac{1}{2}gt^2,\ 0.45 = 1/2g\,(0.30)^2$$

that g = 10.0 m/s.

In addition, another interesting experiment using a measurement of time by this method is a measurement of Instantaneous velocity using photogate timers [6]. The experiment can be done by an experiment setup as shown in Figure 4.2.

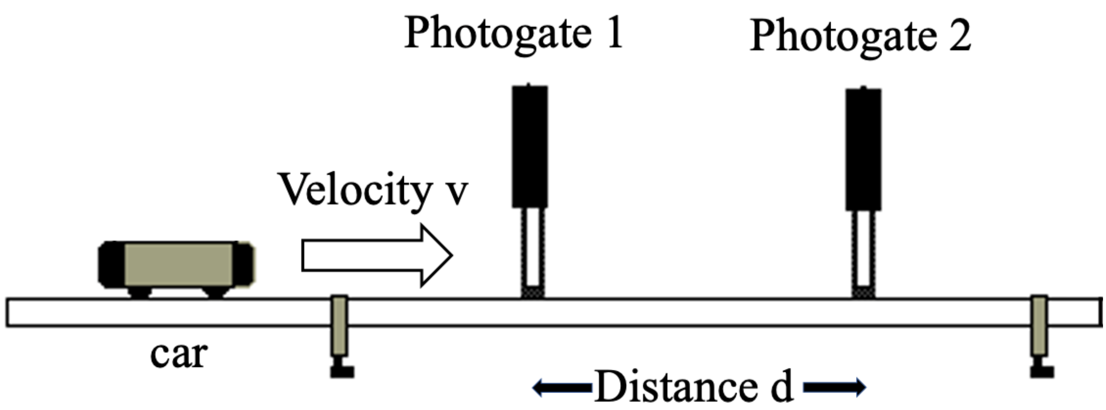

Figure 4.2 An Experiment Setup for a Measurement of An Instantaneous Velocity Using Photogate Timers

A moving object (an experiment car) moves with a velocity v through photogate 1 triggering a supplement electronic circuit to start an RC charging circuit. A can moves with a distance d reaching photogate 2 triggering an RC charging circuit to stop charging. With the help of a data logger CBL2, a travelling time t can be measured. An instantaneous velocity of a car can be calculated by $v = d/t$.

5. Conclusion

In order to study a free-falling object falling under the influence of gravity in a normal classroom is not that easy. The big chamber is expensive for longer distance to measure the falling time using a normal stopwatch. By using an RC-Charging circuit, the value of an acceleration due to gravity, **g**, can be easily obtained in a very short falling distance. We have tried this experiment several times with first year university physics students as well as high school students. The result showed that the proposed method is better than the traditional one using a stopwatch to measure the time in terms of a short range of time.

This method can also be used in several courses in Physics and Mathematics. It will become an interesting project for student using an RC charging circuit and a datalogger. The students can compare result from variation of V_c as a function of time t for different R and C in the circuit with simulation using Geometer's Sketchpad for better understanding [1]. This will make the classroom more interesting and meaningful.

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