What's in a name? Using a scientific calculator for mathematical exploration in schools

Barry Kissane
b.kissane@murdoch.edu.au
School of Education, Murdoch University, WA
AUSTRALIA

Abstract: *This paper identifies a problem that calculators are often interpreted as devices whose sole purpose is to undertake numerical calculations, with the result that their educational significance in secondary schools is not understood well, in contrast to other forms of ICT, for which the software capabilities are recognised as the key features. It is suggested that the potential for educational use of calculators in many Asian contexts is undermined by this limited understanding of their capabilities. An important use of calculators in secondary schools beyond mere calculation involves mathematical exploration, which is described in the paper. Several examples of ways in which features of scientific calculators might be productively used for mathematical exploration are outlined, to indicate the range of contexts of relevance. Ways in which such features might be used in schools are described.*

1. Introduction

The Asian Technology Conference on Mathematics (ATCM) has focused on the place of technology in Asia since its inception in 1995. Some of that focus has been directed at the significance of technology of many kinds for mathematics education, including in particular the setting of secondary schools. In this paper, we explore and expand upon the significance of the scientific calculator for that setting.

As the title suggests, the paper is motivated in part by the ways in which the technology is described. There are various technologies given attention in this conference, under the generic heading of 'technology'. A common term, for example, is 'computer', while another is 'calculator' and a more generic description is 'Information and Communications Technology' (ICT), which generally refers to the addition of telecommunications capabilities to those of other kinds, most clearly the Internet. In general use, both within ATCM and in the wider Asian community, as well as beyond, these terms seem to be handled differently.

It is very rare at ATCM for computers to actually be described as computers, or for those using them to be described as engaging in 'computing'. Instead, attention is focused on what computers can do (or what mathematics people can do with them) and in particular there is a focus on the use of computer software of various kinds. A computer is recognized as a box of electronic things, the significance of which rests on the affordances provided by the software it uses, sometimes described in terms of 'technological tools'. Similarly, a website is not described as a (virtual) location of some sort, but rather is described in terms of what can be accomplished by those who access it, often in this context described in terms of 'digital tools' or 'digital objects'.

There are many recent examples of this, readily found by a perusal of the ATCM archives online. For example, [1] describes the use of dynamic geometry (DGS) software as well as computer algebra systems (CAS) in a *TI-NSpire* environment to explore various aspects of geometry. The focus is on the kinds of affordances provided by the technology and how these can be productively and creatively used by researchers, students or teachers. The paper illustrates how software of these kinds can be productively used, rather than on what computers can do. Similarly, [2] demonstrates a variety of ways in which a spreadsheet might be used to study and teach a range of mathematical topics; a search of the paper shows that the word 'computer' is not used once, although readers will realise that the software used (Microsoft *Excel*) is most frequently run on a computer. In the same sort of way, [3] highlights the benefits to students of using sophisticated CAS software (*Maple*); again the focus is on the software and its use, not on the device using it. In the domain of statistics, [4] describes and illustrates the benefits of the use of the computer software *R* for students and teachers in introductory statistics courses at the college level. Other examples are easily found, of course, by perusing the online archive of ATCM Proceedings at [5].

In contrast, a calculator is best regarded as a small computer, which also has similar functionalities (depending on the model) to various popular computer software packages, although generally on a smaller and more limited scale. Calculator software for student use has been developed for spreadsheets, geometry, CAS, graphing and statistical analysis, among other mathematical capabilities. The substantial limitations of calculators are best understood as related to the development of technology expressly designed for senior secondary school students, rather than for professional mathematicians and statisticians or even undergraduate students. Scientific calculators – as distinct from graphics calculators – are even more limited with small screens that reduce the capacity to represent visual objects well (such as graphs, statistical displays or geometric drawings), but nonetheless modern versions include significant software capabilities, as described in [6]. Despite these attributes, calculators continue to be regarded as merely devices for doing 'calculations' and getting answers – generally numerical answers in the case of scientific calculators.

This persistent use of the name 'calculator' is much too limiting, and leads to a widespread misunderstanding of the nature and significance of modern calculators. A calculator is not merely for 'calculation' any more than a computer is merely for 'computation'. As for other devices, calculators derive their educational significance from the software and related affordances that they make available to users. In this paper, we will first elaborate on the consequences of this problem, and then explore some examples of more appropriate educational use of scientific calculators.

2. Why does the name matter?

Calculation is often important in school mathematics, although it is too frequently mistaken for school mathematics, especially by people with a limited understanding of the nature of mathematics. Having a calculator available is convenient for some purposes (for efficient calculation) and even essential for others. Thus, when they have access to calculators, students can engage in real-world activity that involves measuring attributes, without being restricted to measurements that are conveniently integral or rounded, as they frequently are in textbooks. Similarly, as well as making personal measurements, students can access information published elsewhere to engage in mathematical modelling without limiting numbers to be convenient for calculation. In addition, students can undertake some numerical statistical analysis on real data, rather than imaginary data. So the argument of this paper is not that calculation is unimportant, but rather that it is overstated in importance when calculators are regarded as solely beneficial for that purpose.

In part to place calculation in context, [7] proposed a model for the educational use of calculators, identifying four kinds of activity: representation, computation, exploration and affirmation. While many productive uses of calculators in school involve several of these components simultaneously, those that involve *only* calculation are usually of very limited importance. Many activities include calculation in some way – in precisely the same way that computer software such as that for word processing, or dynamic geometry involve computation in some way, although the computation is not the key feature for learning, and indeed is often hidden from the view of the user.

When there is widespread misunderstanding of the potential benefits for calculators, they are less likely to be seen as an important means for students to gain affordable access to helpful technology, but rather misunderstood instead as merely devices for numerical calculation, and even be regarded as inimical to supporting student learning. [8] Indeed, they might not even be recognised at all as a form of ICT, even in curricula that claim to be 'encouraging' the use of ICT in mathematics. This might account in part for the widespread reluctance of school mathematics curricula in Asia to embrace and support the use of calculators as a routine tool for everyday student use, including for routine use in assessments, including high-stakes assessments, such as examinations external to the school. Most Western countries (including Australia, the USA and most of Europe), in contrast, have routinely encouraged the use of calculators in schools and on external examinations for many years, as have other countries (such as Singapore). It is argued in [9] that the scientific calculator remains the most appropriate device for widespread use in mathematics education, especially when national resources are limited or unevenly spread – as they frequently are in the southeast Asian region; regarding it as merely a device to assist with numerical computation is likely to inhibit its wider adoption.

Research studies such as [8] have provided clear and consistent conclusions that calculators can contribute positively to school mathematics education and are not harmful to student learning, despite common prejudices to the contrary. Indeed, a comprehensive research summary in the USA almost a decade ago summarised the situation as follows:

> Few areas in mathematics education technology have had such focused attention with such consistent results, yet the issue of whether the use of calculators is a positive addition to the mathematics classroom is still questioned in many areas of the mathematics community, as evidenced by continually repeated studies of the same topic. As a result, we concluded that future practitioner questions about calculator use for mathematics teaching and learning should advance from questions of whether or not they are effective to questions of what effective practices with calculators entail. ([10], p.2)

While it is unrealistic to change the everyday name of devices now in widespread use, this paper now turns attention to describing better ways in which they might be used in schools, to address the problem, consistent with the above suggestion, and in order to help to broaden the conception of the role of calculators. We focus on mathematical exploration in particular, as this appears to be the most promising aspect for productive educational use of scientific calculators.

3. Mathematical exploration

Like other kinds of exploration, a key feature of mathematical exploration is that it occurs mostly under the direction of the explorers themselves. When students engage in mathematical exploration, they will often be addressing open questions, rather than closed questions with one specific (and probably numerical) 'answer'. Explorations might be started by external observations or prompts, but often proceed with students asking their own questions and seeking their own answers. The process of exploring is often similar to scientific work, involving experimenting and predicting. Thus it differs from some routine mathematical work in classrooms that involves undertaking practice with using a specific method to address a given problem in a predictable way; while routine work and practice of such a kind is helpful to develop mathematical expertise, it is rarely helpful to develop insight into mathematical ideas and relationships between them. Mathematical exploration is often a productive form of learning when ideas are new to students, and not yet well-formed.

Some mathematical exploration might be beneficial to understand more deeply a concept (such as that of a function, or of convergence or a logarithm), while other explorations might be directed at students finding for themselves consistent patterns among mathematical objects and even discovering for themselves important and useful generalisations and properties (such as the laws of indices or the significance of the slope of a linear function). For many years educators have understood the benefits of learning by discovery; indeed, the celebrated mathematician George Pólya observed in this regard that there can be no mystery for students in a result that they have found for themselves.

When considered in comparison to other forms of exploration (such as geographical and scientific), it is not surprising that mathematical exploration is often especially beneficial when students are not working alone, but are part of a small group. The opportunity to describe observations with others, describe conclusions with others, share ideas, listen to the views of others and challenge the thinking of others is helpful for learning, so that many experienced teachers prefer mathematical exploration to take place with at least one partner, rather than with students working alone. This is a different experience for students, who are frequently expected to undertake mathematical work by themselves, especially school work that is concerned with the development of mathematical skills and significant organised practice of learned material.

Mathematical exploration often results in conclusions that need to be formalised and then understood differently; in that sense, this represent a means to learning, rather than the ends of learning. However, experience of mathematics in school that provides little opportunity for mathematical exploration will be much less interesting and fulfilling for students. In the next section, we look briefly at selected examples of the kinds of explorations that are made possible through access to modern scientific calculators.

4. Outlines of some mathematical explorations

In this section, brief glimpses of some kinds of mathematical exploration are offered and a context for their use described. These might be used by teachers with their students in any of several ways, briefly described in the next section of this paper. The explorations described here mostly use a recent scientific calculator, the *CASIO fx-991 EX* (generally referred to simply as *ClassWiz*). The examples have been chosen across a range of increasingly sophisticated secondary school mathematical contexts to indicate that the potential for mathematical exploration is widespread across the school curriculum.

4.1 Fractions, decimals and percentages

These fundamental number representations are commonly encountered in all school curricula late in the primary school years and early into the secondary school years. So it is not surprising that modern calculators routinely provide good representations of them, and hence afford students opportunities to understand relationships among them. Figure 1 shows a key property of *ClassWiz*, that numbers entered (on the first line of a screen) are routinely represented in conventional mathematical form on the bottom of the screen.

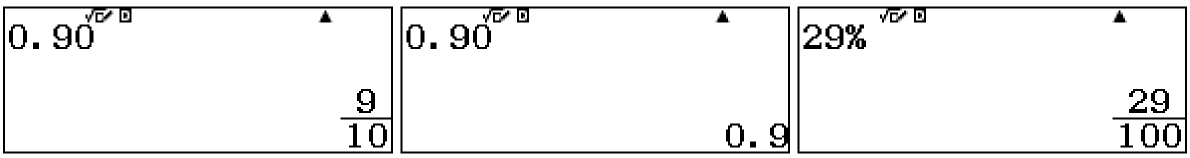

Figure 1: Exploring relationship between fractions, decimals and percentages

The first screen in Figure 1 shows that a decimal is routinely represented as a fraction in lowest terms, while the second screen shows that the user can see the result as a decimal if they prefer, using a dedicated calculator key allowing representations of fractions or decimals to be chosen by the user. In this case, the screen also shows that the trailing zero does not alter the number, likely to be a surprise to young students reading 0.90 as “zero point ninety”, a common error. Exploring this rich environment will help students to explore properties of decimals to understand for example that 0.900 = 0.90 = 0.9, which at first seems an unreasonable result to young students. The third screen might help students to see that a percentage is a number, in the same way that fractions and decimals are numbers, and that it is not necessary to restrict thinking to percentages of some quantity. This is a fertile environment for young students to explore the meanings of these various number representations, easily exploited in a classroom.

An environment of this kind offers opportunities not previously available to learners. For example, Figure 2 shows three *ClassWiz* screens that students exploring different number representations might encounter, to see that the same number can be represented as a fraction in several (in fact, ultimately, an infinite number of) ways.

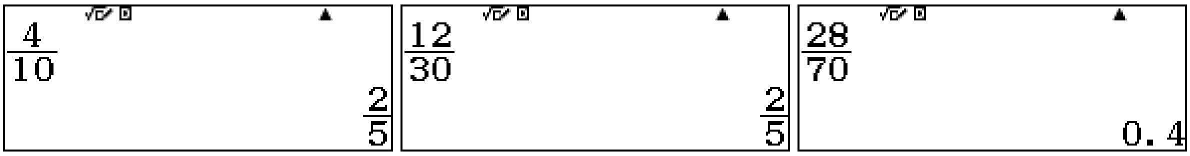

Figure 2: Exploring equivalent fractions

Students can see for themselves that there are many ways of representing the fraction 2/5, and that (as the final screen in Figure 2 shows, after the conversion key has been used), they are all representations of the same decimal number 0.4. While this is of course already familiar to any teacher, it is a revelation to many young children, even many of those who have been instructed how to reduce fractions to lowest terms, or even required to do so routinely. What the calculator provides – but the techniques of cancelling do not provide – is a sense of what the processes mean and a path to understanding them.

Not all calculators are the same, of course, and some offer different features, also ripe for mathematical exploration. To illustrate this, Figure 3 shows some screen shots from a special Indonesian model, the *CASIO fx-991ID Plus*.

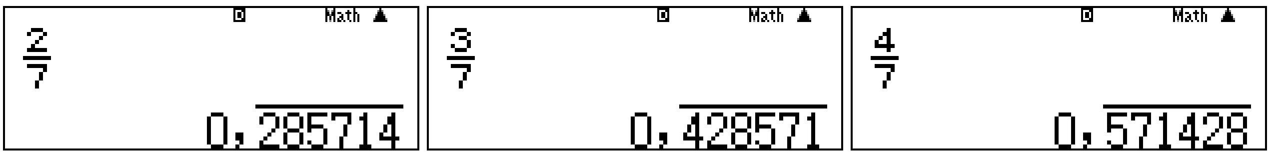

Figure 3: Exploring recurring decimals on the *CASIO fx-991ID Plus* calculator

Number capabilities of these various kinds have been included in scientific calculators for their educational benefits, not for their computational benefits, and it seems to be a misunderstanding of the devices to regard them as simply being used in order to complete calculations. They offer an opportunity for students to explore for themselves and together with others, the many relationships among the representations and to understand them well.

4.2 Logarithms

The idea of a logarithm can be addressed through helping students to see that positive numbers can be represented as powers of other positive numbers, especially powers that are not whole numbers. If students make a table of powers of 2, they can explore a question such as which power of 2 gives a result of 7:

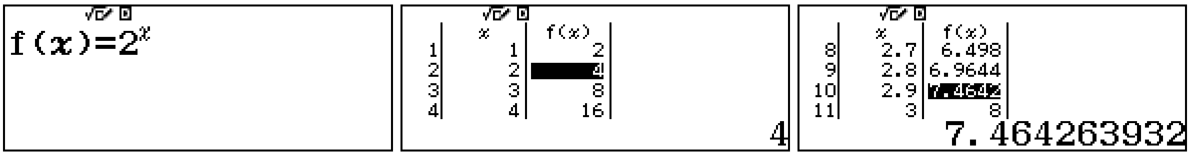

Figure 4: Exploring powers of 2

The middle screen of Figure 4 suggests that the power must be between 2 and 3, while the third screen suggests the necessary power is between 2.8 and 2.9. Successive systematic exploration allows students to get closer still to a value close to 7, building a good conceptual background for the concept of the logarithm of 7 to the base 2 and a suitable definition. Students can experience for themselves the fact that powers need not be restricted to integers.

Once the concept of a logarithm to a base has been defined, students can see on their calculator what this means, using the inbuilt logarithm functions. As Figure 5 indicates, the logarithms and powers are faithfully represented on a modern calculator like the *ClassWiz* and students can use the environment to generate other relationships by themselves that strengthen their understandings. Unlike work prior to calculators, students are not restricted to base 10 or *e* for logarithms, but can use any positive base.

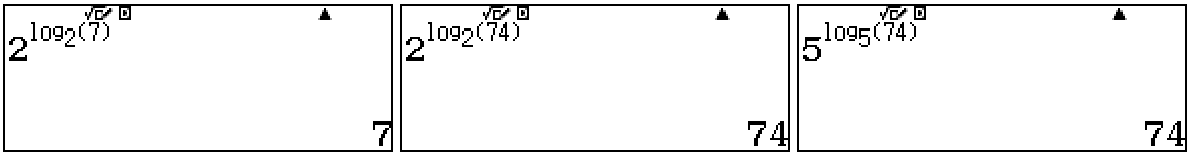

Figure 5: Exploring relationships between logarithms, bases and powers

Calculators offers other ways for students to explore and understand various relationships involved with logarithms. The first two screens in Figure 6 illustrate direct relationships, while the third screen extends this idea to roots. Of course, students can be directed to generate and study further examples like these for themselves, in order to strengthen their understandings.

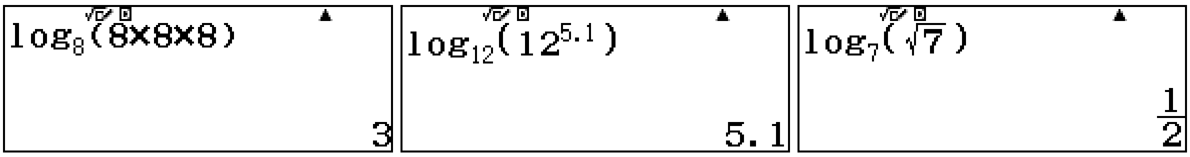

Figure 6: Exploring logarithms of products, powers and roots

4.3 Sequences and series

Many school curricula introduce students to sequences and series, emphasising the patterns involved. The most common sequences involved in early studies are arithmetic and geometric sequences, each of which are readily represented on calculators like *ClassWiz* either recursively or explicitly. For example, Figure 7 shows the first three screens for generating terms of the arithmetic sequence 5, 8, 11, 14, … After the second screen, a single key press generates each successive term, which in this case produces an arithmetic sequence.

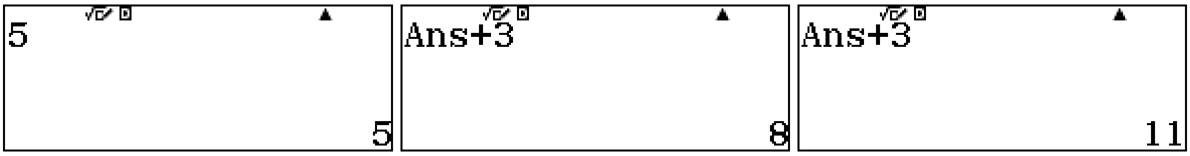

Figure 7: Generating terms of an arithmetic sequence recursively

Students can use these features to readily generate terms of a sequence that can be recursively defined, to discover its properties. This is a different from the kind of experience usually available in school, where sequences are typically described via algebraic expressions, rather than being generated. The *ClassWiz* allows students to generate terms of a sequence explicitly, too, via a suitable function and allows terms to be represented in a table, as shown in the example in Figure 8.

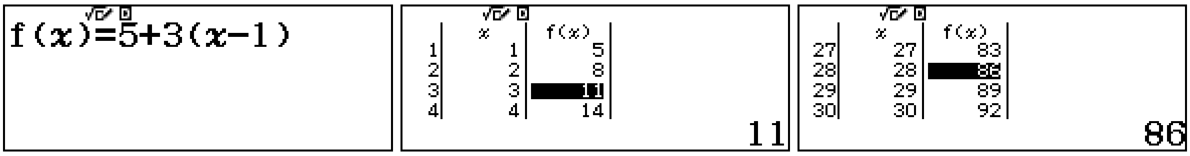

Figure 8: Generating terms of an arithmetic sequence explicitly

While the common difference can be seen from the recursive representation of the sequence, the explicit representation shows it more clearly. Importantly, students can see for themselves that the same sequence can be understood in two different ways – recursively and explicitly – while some ways are more convenient than others. In this case, the third screen of Figure 8 illustrates that the tabular representation makes it easier to see that the 28th term of the sequence is 86 than would be the case if a recursive approach had been used.

ClassWiz also offers students ways of exploring series, which are essentially the successive partial sums of a sequence. Thus, for example, the geometric series 1 + ½ + ¼ + … can be evaluated directly with a keyboard command, as shown in Figure 9, so that students can explore what happens as more terms are included, which provides strong experience in this case of the concept of convergence of a series.

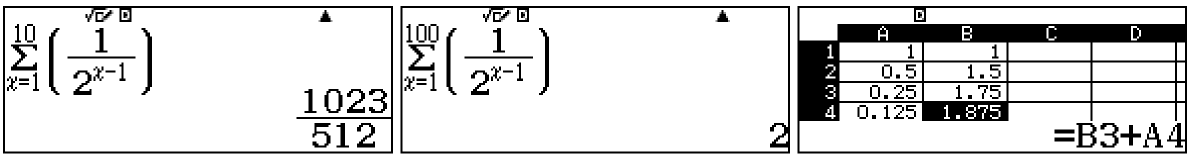

Figure 9: Exploring a geometric series

By choosing to evaluate different numbers of terms of the series, students can explore its behaviour, allowing them to see a process of convergence; within the limits of the *ClassWiz* decimal representations, the second screen in Figure 9 shows that the series seems to be converging to 2 (in this case, after 100 terms). Of course, exploratory work of this kind is likely to be followed by more formal study, and ultimately proof of results; but such work can be built on the solid foundation of practical experience. The final screen in Figure 9 shows that both the sequence and its associated series can be explored on *ClassWiz* through the use of a spreadsheet, providing a more comprehensive treatment for student explorers.

4.4 Limits and convergence

The idea of a limit is fundamental to much of the early study of calculus, even if it is not handled formally at school. A calculator offers powerful opportunities for students to explore what it means to get 'closer and closer' to a value, without actually reaching it. As suggested in Figure 10, for example, students can adjust table boundaries repeatedly to see that the values of the function get closer and closer to 1, as the variable gets closer to 0 (where the expression is undefined).

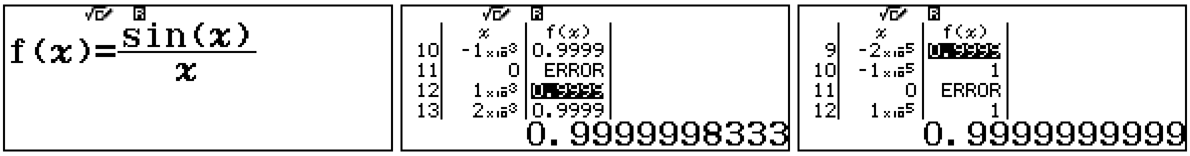

Figure 10: Exploring an important limit

As the *ClassWiz* is a finite device, it cannot deal fully with an idea that is essentially infinite in nature, but it can nonetheless offer students sound experiences to develop insights. As the final screen in Figure 10 indicates, the calculator will in this case sometimes give a (technically) incorrect result of 1 – because it is rounding a number to give a decimal approximation to many decimal places. Rather than this being problematic, however, such things offer worthwhile material for students to discuss amongst themselves or in the wider class.

Other kinds of limits can be explored in comparable ways. Figure 11 shows an example of finding the limit of an infinite series for the exponential function, *e*. As more terms of the series are included, the series can be seen to converge (in this case, very quickly) to the value, again with a numerical approximation to *e*, rather than the exact value.

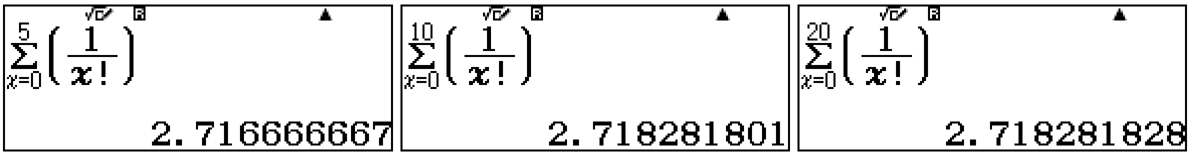

Figure 11: Exploring the limit of an infinite converging series

The *ClassWiz* environment makes it easy for students to edit the parameters of the sum to see the effects of taking more and more terms, so that attention is readily focused on the output.

4.5 Derivative function

Modern calculators like *ClassWiz* include some numerical calculus features, which can be exploited by teachers and students to explore basic concepts. For example, a key concept of differential calculus is that of the gradient of a curve at a point, and – subsequently – the concept of a derivative function. Possible explorations are suggested by Figure 12. The first screen shows that the calculator has a 'gradient at a point' function, while the second and third screens show how this might be used in a table of values.

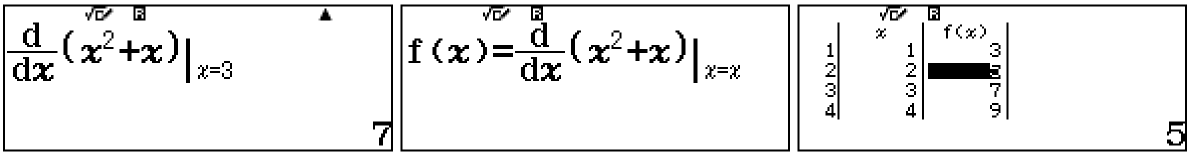

Figure 12: Exploring derivatives at a point to generate a derivative function

While *ClassWiz* does not include a CAS, students should be able to see that there is a clear relationship suggested between the value of the variable (x) and the gradient of the function $f(x)$ at various points. Students who are studying the calculus should have little difficulty recognising that there appears to be a functional relationship and, indeed, that it seems to be a linear relationship, suggesting in this case that the derivative function is $f'(x) = 2x + 1$. Further explorations can explore different values for the independent variable – to confirm that the relationship seems to hold more generally – or can investigate different functions.

5. Contexts for exploration

The selection of examples in this paper is intended to illustrate that the idea of mathematical exploration on scientific calculators can extend across the range of the secondary school years, and can be an important part of the learning experience for students in many components of the mathematics curriculum, rather than being restricted to use after the necessary mathematics learning has taken place. The examples were not written in a form that would necessarily be used directly in a classroom, but rather to suggest to the reader what kinds of student activity might be involved. There are many possibilities, and teachers will adopt those that best suit their preferred teaching styles, the age and sophistication of the students, and the mathematics involved. Some examples, successfully used in Indonesia, are shown in [11].

Students can encounter tasks in various ways. They might be written down as tasks (on a worksheet, or on a screen) or they might be used by teachers with verbal instructions, possibly including the use of a calculator emulator for demonstration or discussion purposes. Some explorations might be undertaken by the whole class together, while others might best be addressed in pairs or in small groups, to facilitate discussion among students. Some tasks might be handled as homework, which has the advantages of unlimited time and is likely to be possible when students have their own personal calculator (which is more likely than that students will have other forms of ICT). Exploration tasks will usually require some calculator expertise by students to get started, generally with only a limited portion of the calculator; this can be provided in the task itself or via a teacher demonstration when necessary. Some examples of tasks are available online in [12] and there are many suggestions included in the Activities sections in materials like [13].

6. Conclusions

The principal argument of this paper is that modern scientific calculators derive their educational importance in secondary schools from the ways in which they are used by students, rather than on the process of numerical calculation, which might be suggested by the word 'calculator'. While many calculator uses intrinsically involve calculations of some kind – in exactly the same way that many computer applications involve computation – it is argued that the main potential for calculators derives from using for mathematical purposes the software built into the devices, and not only their ability to provide answers to closed numerical questions. Rather than a device that only becomes educationally useful after mathematics has been learned, the scientific calculator has considerable potential in the learning process itself, through direct student exploration.

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