Designing Mobile Apps to Promote Numeracy and Statistical Reasoning

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Abstract: *Even with major advancements in the field of mathematics education, many students still do not attain the learning competencies prescribed by official curricula. This may be partially explained by students' immersion in classrooms characterized by superficial strategies or rote-learning methods. This paper reports on the design of mobile applications (apps) developed by the authors as part of an ongoing project funded by a national government agency and intended to promote structural thinking and statistical reasoning. It describes the general features of the apps, as well as the pedagogical principles upon which the apps' designs were anchored on. These principles are grounded on research and established practices on number sense and statistical learning. Collaborations with the Philippine Department of Education for widespread implementation and sustainability are also discussed.*

1. Introduction

Multinational benchmarks of mathematics learning indicate that many students still had not attained the learning competencies prescribed by official curricula [1]. Possibly this is because mathematics classrooms in several countries promote rules and procedures with little attention to underlying mathematical concepts [2, 3, 4]. Without effective intervention, children can remain weak in basic numeracy or number recognition [5]. For example, in the Philippines, elementary school children's place value knowledge is limited to knowledge of labels (i.e., hundreds, tens, ones) rather

than on quantity [6]. A similar problem occurs in the learning of statistics, where the focus is on computation rather than on statistical literacy or reasoning skills [7].

A potential solution is offered by mobile technology [8], which can provide a sustainable educational alternative for mathematical learning [9]. However, a large number of available apps are basically glamorized worksheets which are mostly of the "drill and practice" type [10]. These apps may just perpetuate the situation where rote learning is the norm. Thus, there is a need to design apps that are informed by theories emerging from the wealth of knowledge gained from research on numeracy and statistical learning. This paper reports on five purposely designed numeracy apps for Grades 1 to 6; and one web app, a statistical database for Grades 1 to 11, to develop these two broad areas of mathematics.

The apps are products of an ongoing supported project "Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment" by the Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD). The apps acquired additional relevance due to the urgent educational difficulties driven by the Covid-19 pandemic and the projected adaptations that teachers or learners may face as they transition into blended learning. Although the apps were designed for Filipino learners, they address learning outcomes that are present in many other national curricula as well.

This paper is divided into several parts. The next section describes some general considerations that guided the design of the apps. The third section presents the pedagogical basis for the numeracy apps and the web statistical learning app. The last section presents the collaboration with the Philippine Department of Education (DepEd).

2. Number Sense and Statistical Learning

This section discusses the rationale for designating number sense and statistical reasoning as target educational outcomes of the apps. These two broad areas of mathematics can be considered "big ideas" because they (1) consist of foundational concepts that are necessary for more advanced learning, (2) encompass a network of concepts and strategies, (3) provide an organizational structure for further learning, and (4) are apparent in learners' activity [11]. Thus, support for these "big ideas" or fundamental ideas is called for. Further, many of the existing mathematical apps overlook much of the current research about mathematical learning [10].

There are several countries where reports of poor number sense and statistical reasoning exist (see [12, 13, 14, 15, 16] and the references therein). However, since the apps were designed through a funded project by the Philippine government to develop innovative tools in supporting the implementation of the Department of Education's K to 12 curriculum, it was a priority to design the apps based on published research on the mathematical achievement of Filipino learners. Nevertheless, these apps would also be of interest to mathematics educators and researchers from many other countries with similar educational contexts.

Number sense is often neglected in Philippine classrooms. For example, conceptual place value considers two distinct ideas: *quantity value* (knowing that 45 is 40 and 5) and *column value* (knowing that the 4 in 45 represents 4 tens) [17, 18]. However, Filipino students do not seem to fully acquire these two fundamental concepts because their knowledge is often limited to labels (e.g., in a 3-digit number, the middle digit is called the "tens place") [6]. This outcome suggests that the treatment of place value in Philippine classrooms is on memorizing the labels "hundreds, tens, and ones" rather than on connecting these labels to number quantity (quantity value) or the quantity represented by each digit (column value). Possibly, this poor understanding of place value is the reason why Filipino students cannot apply mental strategies to solve tasks such as 208−10, where regrouping is involved

[6]. In view of this situation, the apps designed in this project focus on conceptual place value. These apps are different from previously designed apps for Filipino students, where traditional place value instruction and the learning of standard algorithms are emphasized.

Statistical learning is another major concern in the Philippines. The Philippine K to 12 program specifies a very rigorous statistics curriculum. The K to 12 program was implemented in 2012, with the spiral approach as one of its new elements. Whereas previously, statistics was taught in only a few grade levels, in the new program, all teachers from Grades 1 to 11 are expected to teach statistics. The problem is, statistics remains one of the most difficult topics to teach, and even local tertiary level textbooks are replete with statistical misconceptions [19]. The database for statistics learning seeks to address this problem by providing teaching guides that can support teachers who themselves have limited background in statistics. Further, in the spirit of Census at School programs in other countries, the database aims to collect authentic data so that Filipino students would find additional relevance in what they are learning and a better grasp of understanding statistical concepts. An additional obstacle is that in the Philippines, statistics is the last topic in the curriculum of each grade level from Grades 1-11. Any delay in curriculum implementation, by calamities such as floods or earthquakes, will mean that statistics is cut short or not taught at all. With the database for statistical learning, students and teachers may continue the learning of statistics through synchronous or asynchronous activities through the worksheets and teaching guides provided within the database.

3. General Design Considerations

The numeracy apps were designed following these guiding principles. The first principle was inclusivity. The apps were intended to be available to as many learners and teachers as possible. Thus, the project team decided that the apps should remain free, requiring only one-time access to the Internet (during download). Even when there is no Internet access, the app must be transferable wirelessly (Bluetooth) or through an external device (such as USB or memory card).

Second, the apps were designed to be ready-to-use or ready-to-play. Students should be able to figure out how to use the apps even with minimal instruction and limited supervision. Further, students can play or use the apps at their own time. Thus, teachers may assign the apps as supplementary material for mathematical learning.

Third, the apps were designed to align with the DepEd curriculum, which encompasses the five strands of mathematical learning, namely, numbers and number sense, measurement, patterns and algebra, geometry, probability and statistics [20]. The project team consulted teachers, math heads, and principals to ensure that the apps are suitable for Filipino learners.

Fourth, the apps were designed to be interactive and dynamic. These were unlike static worksheets where students primarily perform drill and practice. The apps were designed in a game-like environment. The idea is that, by doing better in the game, learners develop a stronger understanding of mathematical concepts. In all grade levels, the apps facilitate exploration, visualization, and justification.

The design of the web statistical app was guided by similar principles. First, the web app was designed to be mobile-ready so that this may be conveniently accessed and used not just on computers but also on mobile devices such as tablets and smartphones. Second, the interface of the app is simple, easy-to-use, and appropriately designed for young children. Moreover, since the web app requires access to the Internet, measures were put in place to address possible interruptions due to intermittent or limited Internet connectivity. Finally, the student worksheets and teaching guides, aligned with the Statistics and Probability competencies for Grades 1-11 in the DepEd curriculum, were prepared to complement the use of the web application.

4. Pedagogical Design

This section discusses the pedagogical basis of the design of the apps. The five numeracy apps are downloadable from https://mathplusresources.wordpress.com/ and the web app is accessible from mathplusresources.com.

4.1 Quick Images

The *Quick Images* app presents some dot patterns and the user has to figure out how many dots are shown (Figure 4.1). It is meant to address the issue that some learners still rely on counting by ones even well into their upper elementary school years. These learners think of numbers primarily as a count sequence starting from one, rather than as a more sophisticated part-whole relationship [21]. For example, they think of "four" as the end of the count sequence 1, 2, 3, 4, without realizing that "four" is the same as three and one or two and two. In [22], a recommended intervention is the use of subitizing (or "instantly seeing how many"). It was noted that certain arrangements of dots can be quickly recognized as a certain number, such as recognizing that one four-dot and one three-dot dice pattern give the number seven. This recognition involves **conceptual subitizing**, or a knowledge of numbers as a composite of parts. Familiar spatial arrangements and five or ten frames were used as organizing features because these can facilitate arithmetic and place value knowledge [22, 23]. It was further recommended that the dot patterns be shown very quickly, so that students are prompted to draw on the part-whole composition of number rather than on the unitary count sequence. *Quick Images* allows the user to choose between three speed levels: slow (2.5 s), medium (1.5 s) and fast (0.5 s).

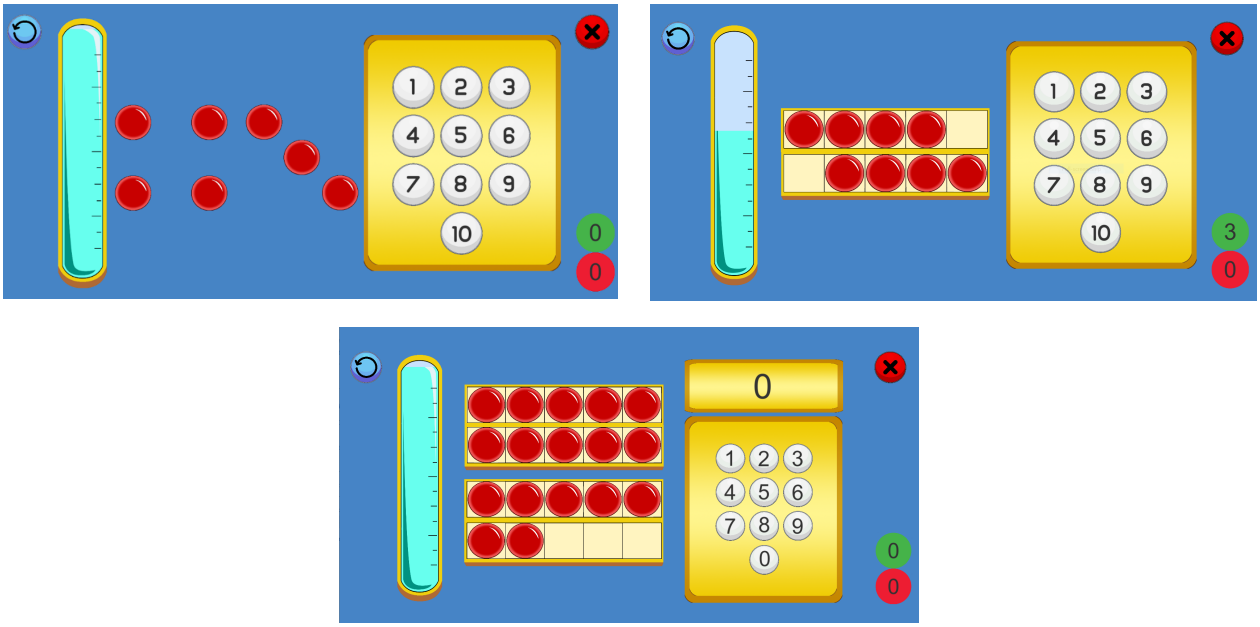


Figure 4.1 Screenshots from *Quick Images*

4.2 Frame Game

The *Frame Game* app (Figure 4.2) was designed to facilitate learners' skill in incrementing or decrementing by 10s, 100s, and so on, which is a foundational piece of knowledge for developing ten-structured thinking [21] and conceptual place value [18]. In *Frame Game*, a target number is given, and users need to press the buttons (±1, ±10) so that the target number is reached. Visual support is provided by a pictorial representation of the current number. As with *Quick Images*, the objects shown in *Frame Game* are arranged in tens to provide an organizational structure in the child's mind.

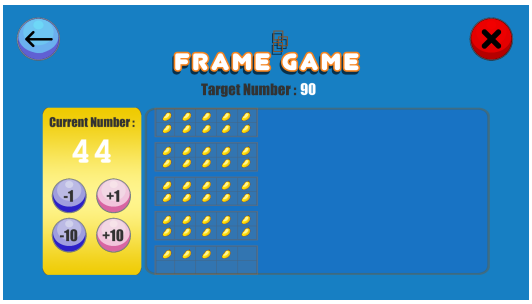
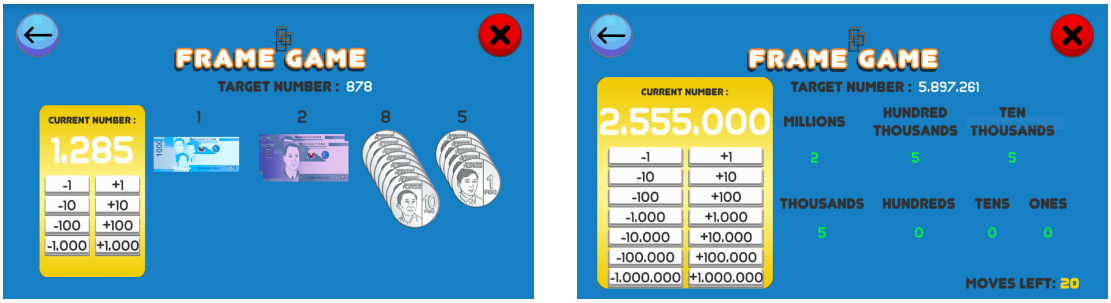


Figure 4.2 Screenshot from *Frame Game*

Frame Game also aligns with several grade levels in the official elementary school curriculum. In this app, the user has an option to choose the number range, from 1 to 100 for Grade 1 through 1 to 10,000,000 for Grade 5. It also addresses various levels of abstraction. Objects are initially shown using pictorial representations of each object (Figure 4.2), then using money (Figure 4.3a), and finally using purely symbolic or digit representations (Figure 4.3b).



(a) (b)

Figure 4.3 Screenshots of higher levels of abstraction from *Frame Game*

4.3 Grid Game

The *Grid Game* app is similar to *Frame Game* in that the user must reach a target number by pressing appropriate buttons. The main visual support is the hundred grid (Figure 4.4). There are also higher levels of abstraction in *Grid Game*. Initially, all numbers can be seen in the grid (Figure 4.4). At a higher level of abstraction, a blank grid is shown (Figure 4.5a). At the highest level of abstraction,

no grid is shown (Figure 4.5b). To address competencies in several grade levels, *Grid Game* offers the user an option to choose the number type: whole numbers (for Grades 1-2), decimals (for Grades 4-5), and integers (for Grades 6-7).

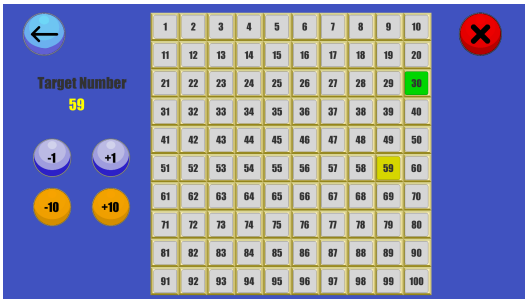
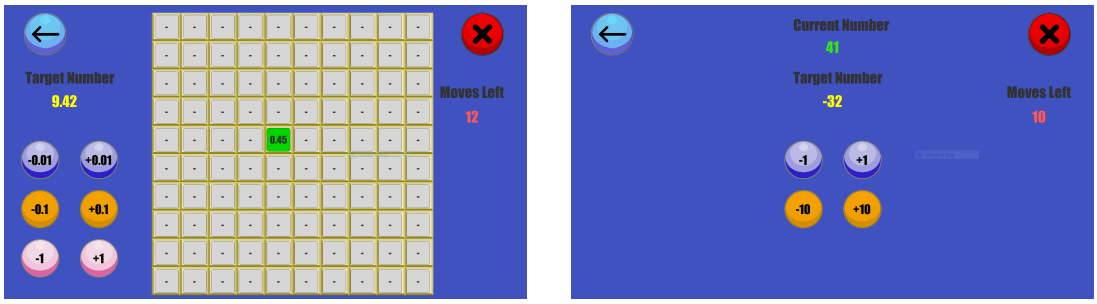


Figure 4.4 Screenshot from *Grid Game*

To ensure that users go beyond unitary counting (e.g., pressing +1 multiple times to go from 24 to 52), both *Frame Game* and *Grid Game* provide a higher level where users are required to apply the most efficient strategy to reach the target number. This is done by indicating the number of allowable moves ("Moves Left" in Figure 4.3b and Figure 4.5). When users run out of moves, they have the option to repeat the same problem and improve their solution.



(a) (b)

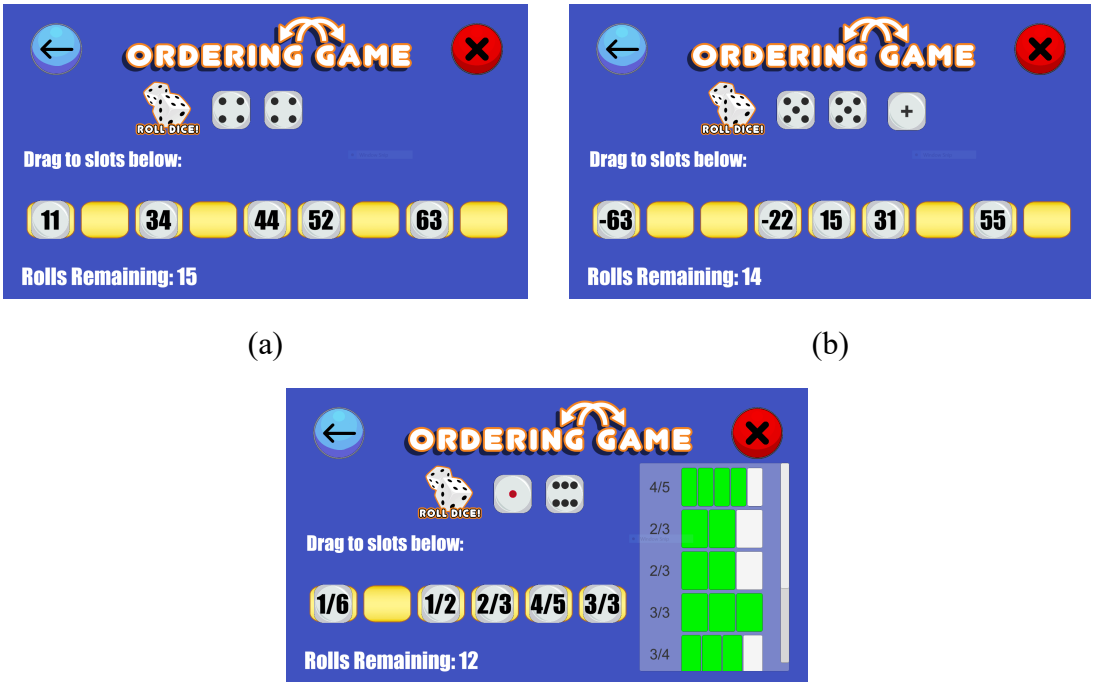
Figure 4.5 Screenshots of higher levels of abstraction from *Grid Game*

4.4 Ordering Game

The design of the *Ordering Game* app (Figure 4.6) was based on the premise that numerical development is hinged on an understanding of number magnitudes; and extending this understanding to increasingly large whole numbers and to a broader set of numbers including integers and rational numbers [24]. A simple proposed intervention is to incorporate activities that utilize the mental number line. It was found that learners from low-income families gained a more precise understanding of numerical magnitude after playing a numerical board game for just four 15-minute sessions [25]. Some resources [26] are already available online, so we developed another game, which was inspired by an activity by Marilyn Burns (cited in [27]).

In *Ordering Game*, users generate numbers by rolling the virtual dice. They place the generated numbers along a row, with the condition that the numbers are increasing from left to right (Figure

4.6). The goal is to fill up the entire row within the allowed number of rolls. The allowed number of rolls can be altered by choosing a level—a higher level allows fewer numbers of rolls before which the user must complete the task. The game becomes more difficult as it progresses because it becomes more likely that a generated number cannot be correctly placed in any of the remaining slots. As with the previous apps, *Ordering Game* also aligns with several grade levels because there are options for number range (e.g., from two- to five-digit whole numbers) and for number type (whole numbers, integers, fractions). In the case of fractions, a visual representation of each fraction is also shown (see Figure 4.6c). The goal is to strengthen the users' images of fraction magnitude. *Ordering Game* also gives feedback to indicate when a user places a number on an incorrect spot along the row.



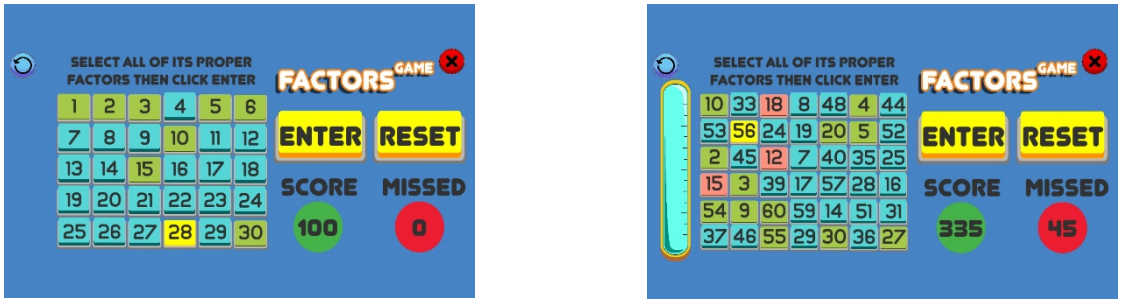
(a) (b)

(c)

Figure 4.6 Screenshots from the *Ordering Game* for (a) whole numbers, (b) integers, and (c) fractions

4.5 Factors Game

The *Factors Game* app was designed so that learners demonstrate understanding of prime and composite numbers in a game setting. There are four levels: 1-30, 1-40, 1-50 or 1-60 and three modes: unlimited time, two minutes or one minute. The player selects a number and then selects all its proper factors. The total score gained is the sum of the number chosen and the correct factors selected. The total missed score is the sum of missed factors and wrong factors selected. The game ends when all the numbers on the grid are selected or missed or the time is up. A summary appears after the game which gives the scored and missed points and correct, wrong or missed factors. The highest score attained by the player is also included in the summary. In the easiest level (Figure 4.7a), numbers 1 to 30 are placed on a 5 x 6 grid arranged in order while in the 1-60 level (Figure 4.7b), randomly selected numbers from 1 to 60, not in order, are placed on the 6 x 7 grid.



(a) (b)

Figure 4.7 Screenshots from the *Factors Game* (a) Level: 1-30 and (b) Level: 1-60

The *Factors Game* is aligned with the official elementary school curriculum. It addresses the need for the learners, from various grade levels, to identify factors and multiples of a given number. Moreover, it also trains the learners to differentiate prime from composite numbers. Without a time-limit, the goal is for the player to choose the factors of the number selected, correctly and avoid having missed points. The time limit will require players to strategize in selecting the numbers. Players will find out that they can score more points by selecting certain numbers.

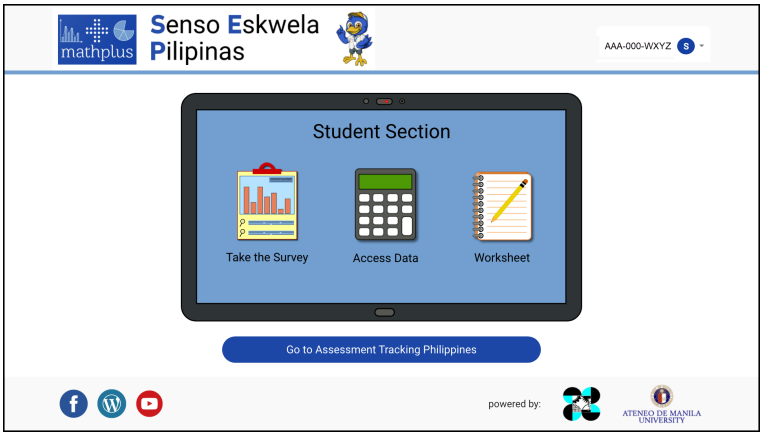
4.6 Senso Eskwela Pilipinas

A web app that has potential in improving statistics instruction in the early school years is the *Senso Eskwela Pilipinas* (*SEP*). The *SEP* is patterned after the international project called *CensusAtSchool*. First established in the UK in 2000, *CensusAtSchool* has the "dual thrust to enliven data handling activities within the classroom while also educating children about the principles and processes involved in conducting a census" [28, p.1]. Since then, *CensusAtSchool* has also been established in other countries like Australia, Canada, USA, New Zealand, and Japan. For instance, the Japanese *CensusAtSchool* project focused to address the observed needs of Japanese statistics teachers and students [29].

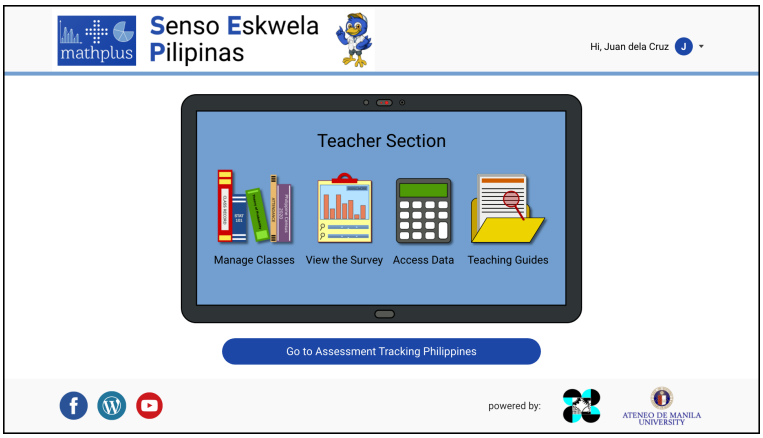
SEP serves as an online platform for gathering, storing, and accessing readily available and relatable data for learning and teaching statistics. In the login page, the user is asked if he or she is a student or a teacher. Grade 1-12 students, who are given access to *SEP* via a code given by their teacher, can answer a 28-question survey about themselves, such as their age, height and weight, favorite food and drinks, hobbies and interests, and others. Some questions involve puzzles and games that can captivate their interest. The students' responses are then stored in the *SEP* database and become part of the data that can be accessed by teachers and students nationwide. Although the same set of questions is used for all grade levels, a broad range of statistics topics can be studied depending on the kind of data the students will retrieve. Student worksheets are also provided to complement the teaching guides for teachers and also to allow students do activities independently (Figure 4.8a). At a basic level, the data can be used to teach students about creating pictographs and other charts or graphs. At a more advanced level, the same data can be used to teach the Central Limit Theorem or hypothesis testing. Moreover, since there are responses from various grade levels and locations, it is possible to do comparative analyses using different statistics topics.

Meanwhile, teachers get access to *SEP* by creating an account for free. The features designed for teachers allow them to manage classes, retrieve their students' survey results, access the data, and view teaching guides (Figure 4.8b). These teaching guides, prepared by the project team, are aligned

with the official DepEd curriculum. In general, teachers, students, and even unregistered guests may download a random sample from the database to be used for activities or tasks in their teaching or learning of statistics. Moreover, teachers and students can retrieve the data of their own class(es) (Figure 4.8c).



(a)



(b)

(c)

Figure 4.8 (a) Student page, (b) Teacher page, and (c) Students' Access Data page of *SEP*

Similar to the *CensusAtSchool* framework, *SEP* aims to enrich the experience of teachers and students in teaching and learning statistics. In particular, "[t]he sense of belonging that participating pupils feel, purely because they know their own responses will become part of a database of responses from their peers, is fascinating and motivating for them." [30, p. 175] Moreover, the questions in the *SEP* survey have been carefully drafted to appeal not only to the interests of grade school and high school students but also to emphasize experiences that are uniquely Filipino. For example, one question in the survey about games that the respondent likes to play is intentionally focused on Pinoy (Filipino) games (Figure 4.9). Through questions like this, the team aims to maximize the relatability of the *SEP* survey so that students can naturally develop a sense of curiosity about the responses of their classmates or other students elsewhere. In turn, this will hopefully help them formulate statistics questions more naturally and increase their engagement in answering these questions through their statistics lessons.

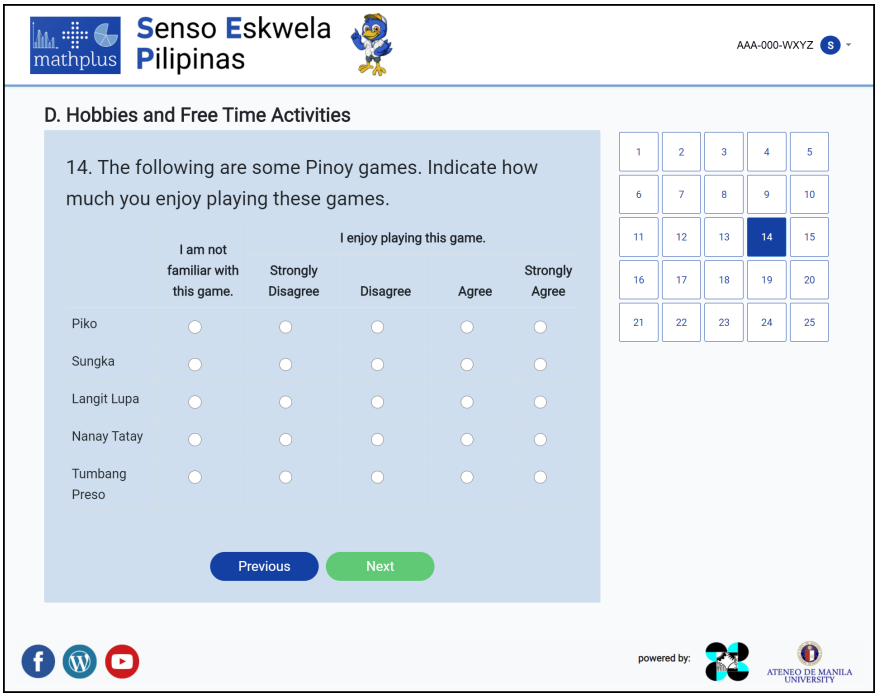


Figure 4.9 An example survey question of *SEP* on Pinoy (Filipino) games

5. Collaboration with the Department of Education

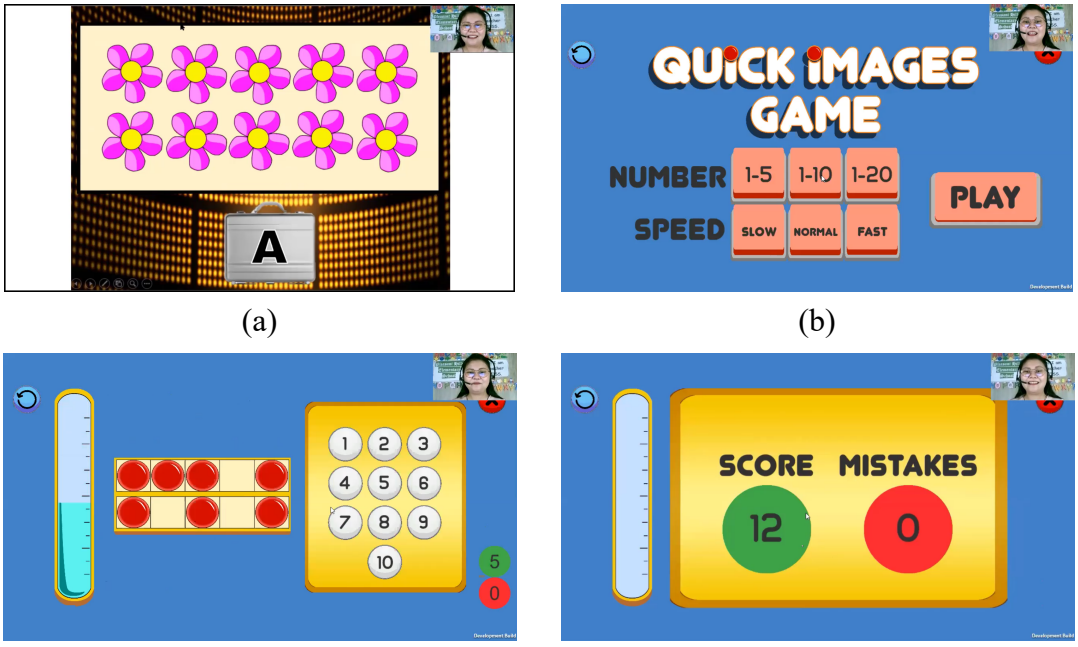
In 2020, the events arising from the Covid-19 pandemic propelled changes in modalities of teaching mathematics in schools in many countries around the world. In the Philippines the Department of Education (DepEd) has announced the shift to blended learning in schools from K to 12 in the school year 2020-2021. Radio, television, online and modular learning are being prepared and updated for use this school year. At the same time, teachers are being trained on utilizing newer platforms and innovative tools to prepare for this new mode of instruction. One of the methods of teaching that authors proposed to the Curriculum and Instruction Unit of the DepEd Central Office was the use of the mathematical apps as additional resources for teachers in blended learning to carry out instruction. One of the biggest opportunities for learning is that the apps can be integrated in the lesson wherein the students may explore and visualize concepts, use real data for exploration, practice drills and exercises remotely, even without much supervision and guidance of teachers or parents.

This approach will also provide students and teachers the opportunity to use their smartphone, tablet or laptop as a tool for mathematical learning. Since the numeracy apps are free and the web app requires no subscription; and the apps run on moderate system requirements, these make the resources very accessible to a wider range of teachers and students. The numeracy apps may be used offline; and will benefit students who do not have the resources and opportunities to engage in online learning due to Internet issues.

As the DepEd Central Office is working towards making the apps available in the DepEd learning resources and portal for the information and use of teachers nationwide, the authors already started partnerships with local DepEd divisions in Metro Manila so that teachers can start their training and exposure on the use of the apps. These partnerships produced an ongoing series of webinars to introduce the mathematical apps to teachers and familiarize them with how the apps are aligned with the most essential learning competencies prescribed by the curriculum. While the webinars were carried out in collaboration with local DepEd divisions, these were made available to teachers nationwide. The webinars were streamed live and were also made available shortly by the divisions in their social media channels so teachers could revisit the webinars at their own pace and time. Videos were also created to inform teachers about the apps to supplement the webinars. The apps were discussed in radio programs to reach the teachers in remote areas.

One collaborative effort worth mentioning was the project with Grades 1 to 6 teachers of one local DepEd division where the teachers created their own video lessons with the aid of the numeracy apps. In the video, teachers taught a lesson and introduced the game-based math apps as a means to strengthen the math concepts discussed. This endeavor was very fruitful as it provided other teachers the much-needed material to use for their lessons. Other teachers may use the videos as starting points to create their own lessons.

One of these videos is a 5-min video demonstration [31] of a Grade 1 teacher employing *Quick Images* as a complementary activity to a lesson on number identification. Screenshots of the lecture are shown in Figure 5.1. The video was posted in a city schools division portal and is available for use by teachers who may want to integrate the app in their own blended learning classroom.



(a) (b)

(c) (d)

Figure 5.1 Screenshots of a video lesson integrating *Quick Images*

In the first half of the video, the teacher reviewed the counting numbers from 1 to 10 with the aid of static images of various objects found in school. She guided the students in identifying the number of objects shown in each image (Figure 5.1a). The second half of the video shows the teacher using *Quick Images* to practice or enrich the students' quick number identification skills. She first introduced *Quick Images* and explained its interface and important features (Figure 5.1b) before playing the *Quick Images* app herself (Figure 5.1c). For students who may have missed identifying the number of objects before they disappeared, she occasionally pressed the "show again" button that allows the app to redisplay the objects. Towards the end of the video, the teacher encouraged the students to independently play with the app and send her a screenshot of the page which displays their score (Figure 5.1d).

Teachers who created the videos and prepared lessons using the apps answered feedback/ evaluation forms. They found the apps interactive, easy to use, and can cater to beginning, average and advanced students. Moreover, they believe students will be able to explore and practice mathematical skills through the applications. Some commented that the apps are helpful and adaptable into a blended learning setting because it offered a strategy to make mathematics interesting and accessible when the teacher is not always present as in a physical classroom setting. One teacher commented "The advantage is really to the young learners, as it is appropriate not only in learning math concepts but improves their mood/ attitude in learning."

The apps are currently undergoing field-testing; observations of the apps' contribution to conceptual development would be presented during the conference.

6. Conclusion and Future Direction

In this paper the design of five numeracy apps for various learning competencies in Grades 1 to 6, as well as the framework for a statistical web app for Grades 1 to 11 is presented which is part of a continuing project funded by a national government agency, to create a digital mathematics learning environment for implementation in schools. Studies on Grades 7 to 10 apps also created under the project have shown that the apps offer additional opportunities to learn mathematical content even with minimal supervision [32]. Example, a statistical analysis of the students' scores after using the proving app *Prove It* even for a short period of time showed evidence of a significant improvement in proof-writing [33].

As of this writing, more apps from Grades 1 to 10 intended for other strands in the mathematical curriculum, such as Geometry and Algebra are being created and field tested for refinements.

Initial feedback from teachers and students for the numeracy apps for Grades 1 to 6 indicates their huge potential for mathematical learning. Moreover, the apps have been showcased and discussed by the team in some webinars [34, 35, 36, 37, 38] for grade school and high school educators. The responses of the teachers in these webinars imply their appreciation towards the availability of the apps as well as their interest and intent to use these in their classes. One teacher echoed the value of technology in teaching amidst the pandemic: "With pandemic or not, technology is always there to help us deliver the knowledge [t]hat our learners deserve to receive." Another teacher commented "Instead of playing the usual games offered online, these mathematical apps can be a very good substitute beneficial to young children… with proper guidance and supported with other resource…" while another teacher affirmed that "gamification of the lessons is the new trend in education." In particular, within a week after the webinar *Senso Eskwela at iba pa* [38] where *SEP* was formally launched, more than 50 teachers registered a combined total of more than 350 classes for participation to *SEP*. Moreover, the participants were asked to evaluate the apps *SEP*, *Quick Images, Grid Game, Ordering Game,* and *Frame Game*. Majority of the teachers have indicated that

these apps are easy to use, adaptable to a blended classroom, and can be catered to beginning, average, and advanced students.

The next step will be to examine more closely the educational effectiveness of these apps in facilitating student performance. Studies will also be carried out on how *SEP* impacts statistical learning. Among the planned activities in collaboration with DepEd schools in the next months will be for these purposes.

There is also the assessment database that is currently under construction. The database will store assessment data gathered within (or outside) this project for evaluation and analysis. It is anticipated that the database can be useful to teachers and researchers who seek to improve the mathematical learning of students. Research-based assessments used to track the mathematical performance of thousands of pupils in Australia [39] will inform the implementation of the assessments in the Philippine setting. It is anticipated that the research-based assessments will provide a richer description of the strategies used by Filipino learners than can be gleaned from the more traditional multiple-choice assessments such as the Philippine National Achievement Test (NAT). In particular, these assessments can determine whether Filipino students are merely imitating procedures demonstrated by their teachers or using their critical thinking to solve mathematical tasks.

The authors invite the readers to visit the project website at https://mathplusresources.wordpress.com/ where future updates and refinements to the developed resources will be made available.

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