Mathematical Analysis of Information Systems through Technology

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Abstract

We live in a world submerged with more information than ever before. We express information or data mathematically and it is growing faster than ever. If the data is imperfect, out of context or otherwise contaminated, it can lead to decisions that could undermine the competitiveness of an enterprise or damage the personal lives of individuals. Therefore, knowledge representation plays an important role in dealing with many aspects of problem solving. In order to obtain complete information systems, we use a mathematical tool, namely, Rough Set Theory (RST), which was introduced by Pawlak in 1982 as a way to deal with data analysis based on approximation methods in information systems. RST is a novel approach to cope with imperfect data analysis as well. In essence, the theory extends the classical crisp set to rough set by defining lower and upper approximations for any subset of a nonempty finite universe. The theory presupposes that with every object of the universe some information is associate with certain relationship. It has many applications in a number of different areas, such as engineering, environment, banking, medicine, bioinformatics, pattern recognition, data mining, machine learning and others. RST is intrinsically a study of equivalence relations on the universe. We intend to use some advanced computing technologies to implement the computations and find several properties of the characteristics of objects. We will show some advanced computing method that can solve our problems effectively. We also extend the results from one universe to two universes.

1 Introduction

The study or management of information systems is an important topic in today's Big Data era. Both complete information system and incomplete information system (IIS) are rich research subjects. An IIS is a system with partially known data or unknown data, which may be considered as missing data. In other words, an IIS is a system whose attribute values for objects may be unknown or missing. This kind of system can be regarded as a set-valued system. Some of its attribute values may be subsets of an attribute domain. Our objective is to find rules, relationships and classifications of such systems and to develop applications to Big Data analysis and data mining.

Information systems can be represented in various ways. One way to do this is to consider the representations in set theory. Two elements in a set can be related by certain rules so that the set is partitioned into different classes. An equivalence relation gives a partition and this can become the relational database. Binary relations are topological partitions which give sets some kind of structures. More precisely, we consider attribute systems in which each system can be interpreted by a set U and a subset R of the Cartesian product $U \times U$, where U is a non-empty set of all finite objects under consideration, and R is an equivalence relation on U derived from certain relationships among attributes. This approach is called the rough set approach. Rough Set theory (RST) originated from Pawlaks seminal work [6]. It has been conceived as a pathway to conceptualize, analyze and classify various types of data and has been developed as a tool to classify objects which are only roughly described.

The available information provides a partial discrimination among them, although they are considered as different objects. Distinct objects could happen to have the same or similar description, as far as a set of attributes is considered. The theory extends the classical crisp set to a rough (or approximate) set by defining lower and upper approximations for any subset of a non-empty universe. It is based on the concept that every object of the universe is associated with some information (data or knowledge). In other words, RST can be considered in the context of granular computing. It provides tools that make use of granules in the process of some given task. Granules are collections formed in the process of a semantically meaningful classifications of elements based on their indistinguishability, similarity, proximity or functionality. In RST, a granulation is obtained by identifying an indiscernibility or similarity relation over the universe of discourse. Therefore, each class of the relation is an elementary granule.

Objects characterized by the same information are considered indiscernible. Thus, an elementary set can be any set of all indiscernible entities, and it forms the basic granule of knowledge [3, 4, 5]. Information granulation is a collection of granules, with a granule being a clump of points which are drawn toward an object. Each object is associated with a family of clumps. An unstructured collection of clumps has some mathematical meaning in a crisp world having to do with the notion of neighborhood system. Basically, a neighborhood system assigns each object a family of subsets. Such subsets, called neighborhoods, represent the notion of 'near'. Therefore, one can define open sets, closed sets, the interior and closure of any subsets as we do in the content of topology or set theory. If all clumps are non-empty, then the neighborhood system is a covering. If there is at most one clump per object, then the neighborhood system is defined by a binary relation, and is called a binary neighborhood system. If we assume the binary relation is an equivalence relation, the neighborhood system is a rough set system. If we assume a neighborhood system satisfies certain axioms, then the neighborhood system defines a topological space; such neighborhood system is called a topological neighborhood system.

Research efforts to advance the classical rough set model have been made by generalizing the Pawlaks approximation space, particularly by exploring the use of a more general binary relation [2]. Such a generalization is usually called an approximation space. We were able to provide a clear and more general framework of lower and upper covering approximations [11]. We not only provide a duality property, but also obtain optimal lower and upper approximations of a covering of an approximation space. Therefore, we basically provide a clear understanding of a given universe. In addition, we developed several properties of the so called total pure reflexive binary neighborhood systems. We generalize our methodology to obtain variable precision of generalized rough sets. More precisely, reductions of redundant attributes in imprecise information systems, decision-making, data analysis, knowledge presentation, image processing, pattern recognition, data preprocessing, modeling complex system and many other applications can use our framework in the generalized rough set theory. The idea is to introduce a set of data, then we use our covering approximation techniques to find accuracy, classification, and reduction attributes of a given information system to pursue our objectives for incomplete information systems.

This paper is organized as follows. We provide some preliminary backgrouns in Section 2, which includes eqivalence classes, lower and upper approximations, as well as the inclusion degree. In Section 3, knowledge representation is interpreted as decision tables. We also introuce the variable precision model and describe its properties. In Section 4, we present the characterization of decision classes, and a methodology to determine the discernibility threshhold for a given decision table. In Section 5, we use advanced technology to deal with large dataset. In Section 6, we intend to explore the results in previous sections to two universes. We explain RST based on two universal sets, and fuzzy binary relations, as well as variable precision generalized rough set model over two universes. We conclude with several remarks in Section 7.

2 Preliminary

One of the fundamental tasks in RST is to understand a neighborhood system which assigns each object a family of non-empty subsets. In topology, we use neighborhoods to define open sets and closed sets, while a rough set concept can be illustrated by means of topological operations, interior and closure, called approximations. As we described earlier, U is the universe and R is an (indiscernibility) equivalence relation on U. Considering a subset, X of U, a fundamental task in this theory is to characterize the set X relative to R. To do so, we need additional structures and concepts in RST. The indiscernibility relation R helps us describe our lack of knowledge about the universe U. We therefore study equivalence classes of the relation R, called granules. Each equivalence class represents an elementary portion of knowledge we are able to perceive due to R. However, we are not able to observe individual objects from U in general. In what follows, we introduce these structures which are also illustrated in Figure 1.

To approximate the set X, we consider all the subsets of X and subsets containing X. More precisely, the set of all objects which can be classified with certainty as members of X with respect to R is called the R-lower approximation of a set X. The set of all objects which can be classified as possible members of X with respect to R is called the R-upper approximation of a set X with respect to R. The set of boundary region can be classified as members of the R-upper

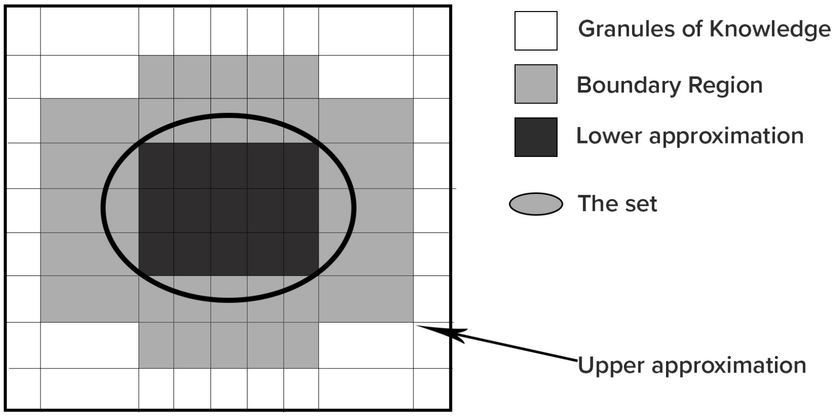

Figure 1: Illustration of set, region, lower and upper approximations

approximation of X but not members of the R-lower approximation of X. A set X is called rough with respect to R if and only if the boundary region of X is nonempty. Furthermore, we consider two different subsets and how they are related in a universe as follows.

Let U be a finite and nonempty set, known as the universe of discourse. We use the symbol " $\subseteq$ " (" $\subset$ ") to denote set inclusion (strict set inclusion, respectively). The cardinality of a set $S \subseteq U$, denoted $|S|$, is the number of elements in S. The power set of U is the collection of all the subsets of U, namely, $2^U = \{S \mid S \subseteq U\}$.

The inclusion degree of a nonempty set $X \subseteq U$ with a set $Y \subseteq U$ is defined as

$$I(X, Y) = \frac{|X \cap Y|}{|X|}. \tag{1}$$

The proportin of X in U is defined as $Pr : 2^U \longrightarrow [0, 1]$:

$$Pr(X) = \frac{|X|}{|U|}, \quad \forall \ X \subseteq U. \tag{2}$$

Consider the complete lattice $[0, 1]$, i.e., the unit interval. Given any set $S \subseteq [0, 1]$, we will write $\sup_{[0,1]} S$ or $\inf_{[0,1]} S$ for the supremum of S in $[0, 1]$, and infimum of S in $[0, 1]$, respectively. If $\sup_{[0,1]} S \in S$, then we also denote it by $\max S$ and call it the maximum of S, and if $\inf_{[0,1]} S \in S$, then we also denote it by $\min S$ and call it the minimum of S. From the definitions of $\sup_{[0,1]}$ and $\inf_{[0,1]}$, we have

$$\sup_{[0,1]} \emptyset = 0 \quad \text{and} \quad \inf_{[0,1]} \emptyset = 1. \tag{3}$$

3 VP-models under decision tables

In what follows, we use four pieces of information to describe some events or phenomena by applying rough set theory. In other words, the knowledge representation in the rough set model is often structured in a decision table which is a 4-tuple $(U, Q = C \cup D, V, f)$, where U is a nonempty finite universe, C is a nonempty finite set of condition attributes, D is a nonempty finite set of decision attributes, $C \cap D = \emptyset$, $V = \bigcup_{q \in Q} V_q$ and V_q is a value domain of the attribute q, and

$$f : U \times Q \longrightarrow V$$

is an information function such that $f(x, q) \in V_q$ for every $x \in U$ and $q \in Q$.

Every nonempty subset P of condition attributes C, or decision attributes D, generates an equivalence relation on U, denoted by $\hat{P}$ and defined as follows [8].

$$\hat{P} = \{(x, y) \in U \times U \mid \forall\, q \in P, f(x, q) = f(y, q)\}. \tag{4}$$

Let $P^* = \{P_1, P_2, \cdots, P_{|P^*|}\}$ denote the partition on U induced by equivalence relation $\hat{P}$. Each member of D^* will be called a decision class. The decision table $(U, C \cup D, V, f)$ is consistent if $\hat{C} \subseteq \hat{D}$; otherwise, the decision table is inconsistent [7].

Considering the partition of U, for any $X \subseteq U$, we can define the P-lower approximation $\underline{P}(X)$ and P-upper approximation $\overline{P}(X)$ of X, in the classical rough set model as follows [1, 13]:

$$\underline{P}(X) = \cup\{P_i \in P^* \mid P_i \subseteq X\} = \cup\{P_i \in P^* \mid I(P_i, X) = 1\}, \tag{5}$$

$$\overline{P}(X) = \cup\{P_i \in P^* \mid P_i \cap X \neq \emptyset\} = \cup\{P_i \in P^* \mid I(P_i, X) > 0\}. \tag{6}$$

The variable precision model (VP-model) was first introduced by Zirako [14]. The tool of varaiable precision was later used to provide a comprehensive analysis of equivalence classes by using infimum and supremum of inclusion degrees of equivalence classes in a given set with smaller (or greater) values than 0.5 instead of minimum and maximum, respectively [13]. Following this idea, we let β be a parameter such that $0.5 < \beta \leq 1$. For $P_i \in P^*$ and $X \subseteq U$, we define

$$P_i \subseteq^\beta X \quad \text{if and only if} \quad I(P_i, X) \geq \beta, \tag{7}$$

$$P_i \cap^\beta X \neq \emptyset \quad \text{if and only if} \quad I(P_i, U - X) < \beta. \tag{8}$$

Then, we can define the P^β-lower approximation $\underline{P}^\beta(X)$ and P^β-upper approximation $\overline{P}^\beta(X)$ of X, in the VP-model under the threshold β, as follows [1]:

$$\underline{P}^\beta(X) = \cup\{P_i \in P^* \mid P_i \subseteq^\beta X\} = \cup\{P_i \in P^* \mid I(P_i, X) \geq \beta\}, \tag{9}$$

$$\overline{P}^\beta(X) = \cup\{P_i \in P^* \mid P_i \cap^\beta X \neq \emptyset\} = \cup\{P_i \in P^* \mid I(P_i, X) > 1 - \beta\}. \tag{10}$$

Evidently, we have

$$\underline{P}^\beta(\emptyset) = \overline{P}^\beta(\emptyset) = \emptyset \text{ and } \underline{P}^\beta(U) = \overline{P}^\beta(U) = U, \tag{11}$$

$$\underline{P}(X) = \underline{P}^1(X) \subseteq \underline{P}^\beta(X) \subseteq \overline{P}^\beta(X) \subseteq \overline{P}^1(X) = \overline{P}(X), \tag{12}$$

$$\overline{P}^\beta(X) = U - \underline{P}^\beta(U - X). \tag{13}$$

Basically, we characterize the set by a partition with respect to some parameter variables so that this will give rise to some precisions in obtaining desired approximations in a given set.

4 Characterization of decision classes

In this section, we provide a method to choose an optimal parameter threshold for identifying lower and upper approximations to be the same object. It helps to give a clear picture of the nature of the sets.

As an immediate consequence of (9) and (10), we have the following [13]:

Lemma 1 *Given a decision table* $(U, C \cup D, V, f)$ *and a parameter* $\beta \in (0.5, 1]$*, let* $D^* = \{D_1, D_2, \cdots, D_{|D^*|}\}$*. For every decision class* $D_j \in D^*$*, we have*

$$\underline{C}^{\beta}(D_j) \subseteq \underline{C}^{\beta'}(D_j) \subseteq \overline{C}^{\beta'}(D_j) \subseteq \overline{C}^{\beta}(D_j), \quad \forall \;\; \beta' \in (0.5, \beta]. \tag{14}$$

Ziarko [14] states that a decision class $D_j \in D^*$ is said to be β-discernable if

$$\underline{C}^{\beta}(D_j) = \overline{C}^{\beta}(D_j). \tag{15}$$

According to Ziarko [14], a decision class which is not discernable for every $\beta \in (0.5, 1]$ will be called absolutely indiscernible. A decision class $D_k \in D^*$ is absolutely indiscernible iff its absolute boundary

$$M(D_k) = \cup\{C_i \in C^* : I(C_i, D_k) = 0.5\} \neq \emptyset. \tag{16}$$

A decision class which is not absolutely indiscernible will be referred to as weakly discernable. More precisely, a decision class $D_j \in D^*$ is weakly discernable iff $\underline{C}^{\beta}(D_j) = \overline{C}^{\beta}(D_j)$ for some $\beta \in (0.5, 1]$. The greatest value of β which makes D_j discernable is referred to as discernibility threshold. We recall the following Lemma in determing threshold.

Lemma 2 *Given a decision table* $(U, C \cup D, V, f)$*, let* $C^* = \{C_1, C_2, \cdots, D_{|C^*|}\}$ *and* $D^* = \{D_1, D_2, \cdots, D_{|D^*|}\}$*. If a decision class* $D_j \in D^*$ *is weakly discernable and its discernibility threshold is equal to* ζ_j*. Then*

$$\zeta_j = \min\,\{\eta_j, \lambda_j\}, \text{where}$$
$$\eta_j = \inf\,\{I(C_i, D_j) \mid C_i \in C^* \;\; \& \;\; I(C_i, D_j) > 0.5\}, \tag{17}$$
$$\lambda_j = 1 - \sup\,\{I(C_i, D_j) \mid C_i \in C^* \;\; \& \;\; I(C_i, D_j) < 0.5\}. \tag{18}$$

5 Data process

Dealing with a large dataset, one may work out the computation with a parallel and distributed solution implemented on Apache Spark. To do this, we consider a given decision tale, $(U, C \cup D, V, f)$. Let $m = |C^*|, n = |D^*|$, and

$$\begin{aligned}
&C^* = \{C_1, C_2, \cdots, C_m\}, \quad D^* = \{D_1, D_2, \cdots, D_n\},\\
&M(D_j) = \cup\{C_i \in C^* : I(C_i, D_j) = 0.5\},\\
&H(D_j) = \cup\{C_i \in C^* : I(C_i, D_j) \geq 0.5\},\\
&\eta_j = \inf\,\{I(C_i, D_j) \mid C_i \in C^* \;\; \& \;\; I(C_i, D_j) > 0.5\},\\
&\lambda_j = 1 - \sup\,\{I(C_i, D_j) \mid C_i \in C^* \;\; \& \;\; I(C_i, D_j) < 0.5\},\\
&\zeta_j = \min\,\{\eta_j, \lambda_j\},\\
&\eta = \min\,\{\eta_1, \eta_2, \cdots, \eta_n\}, \quad \lambda = \min\,\{\lambda_1, \lambda_2, \cdots, \lambda_n\},\\
&\zeta = \min\,\{\eta, \lambda\} = \min\,\{\zeta_1, \zeta_2, \cdots, \zeta_n\}.
\end{aligned} \tag{19}$$

In practice, we provide the following algorithm to obtain the optimal threshold.
Algorithm
INPUT: pathClassvector, m, n
DECLAIR: min1, min2, max
classvector - loadFromFile(pathClassvector)
broadcast(classvector)
ReduceFromSlaves
    for i from 1 to m do
    min1 - I_i^1
    max - I_i^2
    end for
EndReduce
ReduceFromSlaves($\eta_1, \eta_2, \cdots, \eta_n, \lambda_1, \lambda_2, \cdots, \lambda_n$)
    for j from 1 to n do
    η_j
    λ_j
end for
EndReuce
save
    $\zeta = \min \{\eta, \lambda\}$
Terminate

6 Two universal sets

In this section, we will extend the results on one universal set to two universal sets. Let U and W be two nonempty (may be finite or infinite) universes, and let 2^U and 2^W denote the power sets of U and W, respectively. A (binary) relation R from U to W is a subset of the Cartesian product $U \times W$. It's common to use uRw to mean that the ordered pair $(u, w) \in R$. If $U = W$ then we simply say that the binary relation is over U.

If R is a binary relation from U to W, and $u \in U$, define

$$R(u) = \{ \ w \in W \mid (u, w) \in R\}. \tag{20}$$

The set is called the image of u under R.

6.1 Rough set theory based on two universal sets

Generalized definition of Pawlak's lower and upper approximations [6] has been considered by T.Y. Lin [3] for a relation R from U to W instead of equivalence relation as follows: For any $X \subseteq W$, the lower and upper approximations, $\underline{R}(X)$ and $\overline{R}(X)$, respectively, of X under R are defined as

$$\underline{R}(X) = \{u \in U \mid R(u) \subseteq X\}, \quad \overline{R}(X) = \{u \in U \mid R(u) \cap X \neq \emptyset\}. \tag{21}$$

Accordingly, we have

$$\underline{R}(\emptyset) = \{u \in U \mid R(u) = \emptyset\}, \quad \overline{R}(W) = U - \{u \in U \mid R(u) = \emptyset\}, \tag{22}$$

$$\underline{R}(X) = U - \overline{R}(W - X), \quad \forall \ \ X \subseteq W. \tag{23}$$

The above identity natually gives rise to the duality between the lower and upper approximations.

6.2 Fuzzy binary relations

A relation $R \subseteq U \times W$ can be identified with its characteristic function $\mu_R : U \times W \to \{0,1\}$ defined as:

$$\mu_R(u,w) = \begin{cases} 1, & \text{if } (u,w) \in R, \\ \\ 0, & \text{otherwise.} \end{cases} \tag{24}$$

This function can be generalized to allow ordered pairs to have degrees of membership.

A fuzzy binary relation $\tilde{R}$ from U to W is a fuzzy subset of $U \times W$ with the membership function $\mu_{\tilde{R}} : U \times W \to [0,1]$

Denote by $\mathcal{F}(U \times W)$ the collection of all fuzzy relations from U to W. For $\alpha \in (0,1]$, the *α-cut* of a fuzzy relation $\tilde{R} \in \mathcal{F}(U \times W)$, denoted as $\tilde{R}_\alpha$, is defined as

$$\tilde{R}_\alpha = \{(u,w) \in U \times W \mid \mu_{\tilde{R}}(u,w) \geq \alpha\}. \tag{25}$$

Note that the α-cut $\tilde{R}_\alpha$ is a binary relation from U to W.

Considering (20) and (25), we natually define α-cut image of u under $\tilde{R}$, $\tilde{R}_\alpha(u)$. This gives rise to a continuous version of the characterization of the relation instead of discrete version of the characterization.

We assume in the remainder of this paper that W is a finite universe. The *inclusion error* $e(Y,X)$ of a set $Y \subseteq W$ in another set $X \subseteq W$ is defined as

$$e(Y,X) = \begin{cases} 1 - \frac{|Y \cap X|}{|Y|}, & \text{if } |Y| > 0, \\ 0, & \text{if } |Y| = 0, \end{cases} \tag{26}$$

where $|\cdot|$ is the set cardinality. Let $\nu(Y,X) = 1 - e(Y,X)$. Then

$$\nu(Y,X) = \begin{cases} \frac{|Y \cap X|}{|Y|} = I(Y,X), & \text{if } |Y| > 0, \\ 1, & \text{if } |Y| = 0, \end{cases} \tag{27}$$

is the standard rough inclusion function on the set W [9].

6.3 Variable precision generalized rough set model over two universes

Assume that W is a nonempty finite universe, and that $\beta \in [0, 0.5)$.

Considering a relation R from U to W, motivated by [10, 12], we define its induced β-lower approximation operator $\underline{R}^\beta : 2^W \longrightarrow 2^U$ by

$$\begin{aligned} &\underline{R}^\beta(X) = \{u \in U \mid e(R(u), X) \leq \beta\} \\ = \ &\{u \in U \mid R(u) = \emptyset\} \cup \{u \in U \mid R(u) \neq \emptyset, \ 1 - \frac{|R(u) \cap X|}{|R(u)|} \leq \beta\} \\ = \ &\{u \in U \mid R(u) = \emptyset\} \cup \{u \in U \mid R(u) \neq \emptyset, \ \frac{|R(u) \cap X|}{|R(u)|} \geq \beta\} \end{aligned} \tag{28}$$

and its induced β-upper approximation operator $\overline{R}^{\beta}: 2^W \longrightarrow 2^U$ by

$$\begin{aligned}
\overline{R}^{\beta}(X) &= U - \underline{R}^{\beta}(W - X) = \{u \in U \mid e(R(u), W - X) > \beta\} \\
&= \{u \in U \mid R(u) \neq \emptyset,\ 1 - \frac{|R(u) \cap (W - X)|}{|R(u)|} > \beta\} \\
&= \{u \in U \mid R(u) \neq \emptyset,\ \frac{|R(u) \cap X|}{|R(u)|} > \beta\} \\
&= \{u \in U \mid R(u) \neq \emptyset,\ e(R(u), X) < 1 - \beta\}. \qquad (29)
\end{aligned}$$

We shall refer to rough set theory with such approximations as the variable precision generalized rough set model (VPGRS-model) under R with threshold β.

It follows immediately from (28) and (29) that the operators $\underline{R}^{\beta}$ and $\overline{R}^{\beta}$ are order-preserving, that is,

$$X \subseteq Y \subseteq W \quad \Longrightarrow \quad \underline{R}^{\beta}(X) \subseteq \underline{R}^{\beta}(Y) \text{ and } \overline{R}^{\beta}(X) \subseteq \overline{R}^{\beta}(Y). \qquad (30)$$

According to (27), we have

$$e(R(u), \emptyset) = \begin{cases} 1, & \text{if } R(u) \neq \emptyset, \\ 0, & \text{if } R(u) = \emptyset. \end{cases} \qquad (31)$$

This, together with (28) and (29), gives

$$\underline{R}^{\beta}(\emptyset) = \{u \in U \mid R(u) = \emptyset\}, \quad \overline{R}^{\beta}(\emptyset) = \emptyset. \qquad (32)$$

Then, by duality, we have

$$\underline{R}^{\beta}(W) = U, \quad \overline{R}^{\beta}(W) = U - \{u \in U \mid R(u) = \emptyset\}. \qquad (33)$$

Let $\beta = 0$ then it follows, from (21), (27) and (28), that

$$\begin{aligned}
&\underline{R}^{0}(X) = \{u \in U \mid e(R(u), X) \leq 0\} \\
&= \{u \in U \mid R(u) = \emptyset\} \cup \{u \in U \mid R(u) \neq \emptyset,\ 1 - \frac{|R(u) \cap X|}{|R(u)|} \leq 0\} \\
&= \{u \in U \mid R(u) = \emptyset\} \cup \{x \in U \mid R(u) \neq \emptyset,\ R(u) \subseteq X\} \\
&= \{x \in U \mid R(x) \subseteq X\} = \underline{R}(X).
\end{aligned}$$

Then, by (23), we have $\overline{R}^{0}(X) = U - \underline{R}^{0}(W - X) = U - \underline{R}(W - X) = \overline{R}(X)$. This gives:

Let $X \subseteq W$, and let $R \subseteq U \times W$. Then

1. $\underline{R}^{0}(X) = \underline{R}(X)$, where $\underline{R}(X) = \{u \in U \mid R(u) \subseteq X\}$.

2. $\overline{R}^{0}(X) = \overline{R}(X)$, where $\overline{R}(X) = \{u \in U \mid R(u) \cap X \neq \emptyset\}$.

Let W be a nonempty finite universe, $\tilde{R}$ be a fuzzy relation from U to W. For any $X \subseteq W$, $\alpha \in (0, 1]$, if $\beta = 0$ then

$$\underline{\tilde{R}_{\alpha}}^{0}(X) = \underline{\tilde{R}_{\alpha}}(X), \qquad (34)$$

where $\underline{\tilde{R}_{\alpha}}^{\beta}(X) = \{u \in U \mid e(\tilde{R}_{\alpha}(u), X) \leq \beta\}$.

6.4 Decision tables of two universes

From the above results, it is natural to extend our discussions to decision tables of two different universes. We consider the following settings: We consider a decision table which is a 5-tuple $(U_1, U_2, Q = C \cup D, V, f)$, where U_1, U_2 are nonempty finite universes, C is a nonempty finite set of condition attributes, D is a nonempty finite set of decision attributes, $C \cap D = \emptyset$, $V = \bigcup_{q \in Q} V_q$ and V_q is a value domain of the attribute q, and

$$f : U \times Q \longrightarrow V$$

is an information function such that $f(x, q) \in V_q$ for every $x \in U = U_1 \times U_2$ and $q \in Q$. It becomes a multi-dimensional task to extend the results from previous sections to this setting for two universes.

7 Conclusions

We use decision tables to represent the knowledge of information systems. In this paper, we have developed an algorithm to deal with a large data set in handling the determination of the variable precision model threshold, so that we can have a classification and characterization of the information system. We then present decision tables over two different universes which will be helpful to analyze more complex cases. We plan to further extend the results of a sigle universe to two different universes. We will also use our methods to couple data and knowledge in data mining and database in various contexts.

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