Technology in Mathematics Education: A Stocktake & Crystal-Ball Gazing

Greg Oates
Greg.Oates@utas.edu.au
Faculty of Education
University of Tasmania, Australia

Abstract
This paper seeks to conduct a stocktake of the current use of technology in mathematics education, and engage in some crystal-ball gazing as to how it might be used in the future. First, it briefly discusses the history of the use of digital technologies in mathematics education, focusing primarily on the period of growth from the mid-1990's. It will consider some of the theoretical perspectives that have emerged over that period, and using the framework developed by the author [50; 53], will attempt to describe the 'current state of play' for the effective integration of technology into the teaching and learning of mathematics. Then using this position and framework as a starting point, the paper postulates what might be some significant challenges ahead for teachers and institutions in the continuing search for effective meaning-making in mathematics with technology.

1. Introduction: Setting up a Stocktake & Polishing the Crystal Ball

As described in many commentaries [e.g. 55; 71], the 1990's saw a rapid growth in the development and use of digital technologies in mathematics education, with greater access to more powerful desktop computers, more advanced graphics calculators and CAS-capable technologies. More studies also appeared researching the use of technology in mathematics education, especially those theorising how technologies may most effectively be used, and how they might fit with, or build on existing teaching and learning theories. By the mid-1990's the technology revolution in mathematics education had gained considerable momentum, as typified by the theme of the first Asian Technology Conference in Mathematics (ATCM) in 1995 entitled *Innovative Use of Technology for Teaching and Research in Mathematics.* Since 1997, the annual ATCM conferences have showcased a great number of studies describing novel and exciting ways in which technology has been used internationally to advance mathematical thinking, and in the classroom, to motivate and promote effective learning. International curriculum documents from around this period also reflected this trend, with explicit references to incorporating information technologies, although at this time there limited examples or guidance on how this might be best achieved [see e.g. 46; 48]. In 1996, Penglase and Arnold [55] published their meta-critique of recent research involving graphics calculators, which reviewed a large number of studies from the first decade of use (1985-1995). They posed two critical questions which they expected the studies might reasonably address:

1. In what ways can graphics calculators be used to maximise learning and achievement; and
2. What teaching practices and what types of learning environments best complement their use in order to bring about maximum benefits for students? [55, p. 59].

However, they were critical of many of these studies, contending that they did not address these questions because they failed to distinguish the role of the tool from the learning process:

> …many studies reviewed could be more appropriately classified as "program evaluations" rather than as research on the graphics calculator *per se*… Sadly, the answers offered by research to these questions at the end of this first decade remain elusive and conflicting [55, pp.58].

The book *Windows on mathematical meanings: Learning cultures and computers* [49], published the same year as the Penglase and Arnold review exemplifies the emergence of studies explicitly addressing the different roles of technology as a tool and an instructional aid. In this seminal book, Noss and Hoyles introduce the notion of the computer as a mediator in learning, based on socio-cultural perspectives of technology as a tool, which has underpinned much of the theoretical research in computers and technology to this day. Their central heuristic is "the idea of the computer as a window on knowledge, on the conceptions, beliefs and attitudes of learners, teachers and others involved in the meaning-making process" [49, p. 5]. In considering the potential for technology to reshape and remould mathematical activity, Noss and Hoyles take pains to point out the significance of keeping mathematics as our principal focus, with technology viewed as a background tool, when they emphasise that their "focus of interest is not on the computer, it is on what the computer makes possible for mathematical meaning-making [49, p.5]. This emphasis can be seen in many of the studies theorising the role of technology that followed, for example Kaput [31] who continues the *windows* metaphor to consider representational issues with technology, and the work by Guin and Trouche [19] and Artigue [3] which considers the complex process of converting tools into mathematical instruments and introduces the concept of *instrumentation* and *instrumental genesis* which in turn inform much of the current research. The early studies on instrumentation have since been used to develop a more extensive framework of instrumental orchestrations to describe and inform teachers' facilitation of student learning [13; 45; 70].

The distinction between the value of technology as a tool for doing mathematics, especially research mathematics, and its pedagogical value in helping students access and understand mathematics has continued to dominate studies in technology to this day. It is aptly demonstrated in the sub-division of papers in the first 1995 ATCM conference, categorised under the headings of *Pedagogy*, *Computer Algebra & Computation Mathematics* and has continued in an ever-growing number of studies and papers since the mid 1990's which have showcased innovative and interesting methods of using technology to solve specific mathematics problems or described examples of how to engage students in particular aspects of the curriculum. These have appeared in such forums as ATCM, the Electronic Journal of Mathematics & Technology (eJMT), the *International* Journal *of Education in* Mathematics, *Science and* Technology (iJMEST), the *International Journal for Technology in Mathematics Education* (IJTME) and the *International Journal of Computers for Mathematical Learning* (IJCML). Consider for example two presentations from the 1995 conference: a plenary lecture delivered by Conder [10], a research mathematician who considered the impact of semi-automated theorem proving on research in pure mathematics; and Abbot [1], who used the computer algebra system *Mathematica* to discuss teaching simulation, visualisation and modelling, or two recent articles from eJMT: McAndrew & Yang [44], who explored locus and optimisation problems in lower and higher dimensions; and Siew, Geofrey and Lee [60] who examined the effects of the effects of game-based learning using Dragonbox 12+ App on students' algebraic thinking and attitudes towards algebra.

There have been a number of reviews and studies considering the overall value and place of technology in mathematics education and research itself. Hoyles and Noss [25] ask what digital technologies might take from, and bring to research in mathematics education; Lagrange, Artigue, Laborde and Trouche [39] conduct a multidimensional study of the evolution of research and innovation in technology and mathematics education; the 17th ICMI-study edited by Hoyles and Lagrange titled *Rethinking the Terrain* [27] covers a wide-ranging number of practical and theoretical issues and Drijvers [14] asks the question "Why digital technology in mathematics education works (or doesn't)? At the secondary level, Li and Ma [42] conducted a meta-analysis of the effects of computer technology on school students' mathematics learning, while at the tertiary level, Thomas and Holton [67] consider the broader role of technology as a tool for teaching undergraduate mathematics; Oates [53] and Lavicza [40] examine the integration of technology into undergraduate mathematics teaching and Buteau, Marshall, Jarvis and Lavicza [8] describe several emerging themes from their literature review of 326 papers regarding the use of CAS in tertiary mathematics education. Buteau eta al. for example highlight differences in use across the educational levels, when they conclude that previous frameworks for examining technology at the school level [39] may need revising at the tertiary level

However, despite such impressive growth in the volume of research and studies showcasing the contemporary use of technology and the challenges it poses, several researchers [2; 27; 53] observe there is still a lot to do in respect of the questions Penglase and Arnold posed in 1996 [55].

> Even today, there remain widespread reservations regarding not only the best ways to incorporate such tools into teaching and learning, but fundamental questions regarding their appropriateness…The questions which practitioners have been asking (*What will be left to teach if students have access to tools which factorise, solve, and do calculus? What about their manipulative skills? What will we ask them to do in examinations?*)…were precisely the same questions asked twenty years ago regarding student access to traditional calculators [2, p.21].

This view is reinforced by Hoyles and Lagrange [27] some twenty years after the first1985 ICMI study on technology, when they note in their discussion document preceding the 17th ICMI study that while some aspects have moved quickly (e.g. development of technological tools and diversity of use), others have remained essentially static, including little evidence of any significant impact on the mathematics curriculum of secondary schools and universities. Lavicza [40] also notes that integration of technology has been much slower than expected, recognising "teachers' beliefs and conceptions about technology use in teaching (as) key factors for understanding the slowness of technology integration". In particular, greater research is needed at the tertiary level, especially the potential pedagogical benefits of the technologies research mathematicians commonly use in their own practice [40, 53]. Further, while we may have progressed in many ways from the arguments and issues dominating earlier years, for example arguments surrounding hand-held versus desk-top technologies are changing in the era of *BYOD* devices and the ubiquity of *apps* on smartphones and tablets [43; 54; 72], it nevertheless seems reasonable to assume that new advances in technology have either introduced new issues, or further complicated the existing ones we have yet to fully resolve [5; 6; 71].

Oates [50; 53] develops a framework of six categories (*Access; Assessment; Staff Factors; Student Factors & Organisational Factors*) within which we might consider the complexity of issues involved in the effective integration of technology into the teaching and learning of mathematics that have been outlined here. This framework will be next be used to examine the progress we have made in integrating technologies into our mathematics education programmes to date, and postulate what challenges lie ahead.

2. Brief History of Technology and Current Trends: A Stocktake

Any consideration of the history of technology must choose a suitable starting point for its discussions. Clearly technology has influenced the development and teaching of mathematics for many centuries and there is a long history particularly of hand-held technology tools in mathematics, for example the abacus dating as far back as the Babylonians' Salamis tablet (circa 300 BC), the more familiar modern Chinese (*Suan-pan*), Japanese (*Soroban*) and Russian abacus (*Schoty*) from 1200 AD to the present; or the lesser known manually-operated device Napier's Bones, used to calculate products and quotients of numbers, published by Napier in 1617. Exactly when the first modern electronic device was developed is open to interpretation (e.g. the 'first' mechanical computer constructed by Charles Babbage in 1882), but the Electronic Numerical Integrator and Computer (ENIAC) developed by Penn Engineering in 1946 is often regarded as the first Turing-complete general-purpose electronic computer. However, at 30 tons, and occupying a room 30 feet by 50 feet, portable it is not! Thus I have decided to use as a starting point here my own experiences with electronic technology as a high school student in the 1970's, through my secondary teaching years in the 1980's & 1990's to the present since 1993, teaching undergraduate mathematics and teacher education. Thus the use of such technologies as slide rules and log-tables will not be discussed here, as important as they were in my formative years.

I well remember having to "persuade" my father to get my first ever simple calculator (+, -, ×, ÷, √, %. M); he opposed it because they were costly and he believed it would ruin my ability to add and multiply. I also recollect the subsequent enjoyment of an early classroom activity making "words" on the screen from calculations provided by the teacher. Although at the time I did not recognise this, the way our teacher used such a simple device to reinforce conventions of rounding, and procedural proficiency with for example order of operations (BODMAS) was also very effective. I remember too the exciting and 'weird' buttons on my first scientific calculator in 1975 (what a relief from having to read log tables) and clearly recall the agonising wait for our punch-card programmes to be returned from the central card-reader in Auckland (3 days to a week) to see if our *Fortran* programmes in numerical analysis had worked (1978)! I have no memory of using technology at all in my undergraduate years, probably because calculators themselves were banned (1979-1982), and technology was not considered at all in my teacher training in 1983. One of the most stirring memories I have is the excitement I felt when I was first shown an early graphics calculator (an HP28C) at a professional development day in 1988. This gave me the first inkling of the potential of such devices to influence the way we teach, when we considered introducing quadratic functions using a graph, table and zooming functionality of the calculator to solve the root of $f(x) = 0$ for a practical everyday example. It was around then too that I first introduced to the power of programming to develop reasoning skills, although whoever imagined my early fumbling's with turtle-maths [22; 24] on the Commodore 64 (sold for $595 in 1982 when it came out, with an astonishing 64K of RAM!) would one day lead to such beautiful and accessible (to students) examples as the fractal quilt from the modern Logo-Math site (Figure 1)?

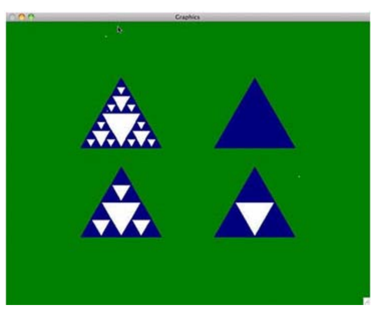

Figure 1 Fractal Quilt (Logo-Math, https://logo-math.wikispaces.com/Fractal_Quilt)

This was followed in 1991 by my first exposure to *Chaos Theory* and *Network Theory* at a workshop for gifted students hosted at Auckland University by Prof Ivan Reilly, an eminent research topologist with a dedication to student learning and quality education. In addition to the Königsberg Bridge problem which I use to this day as an introduction into intuitive and deductive reasoning, this workshop showed me how technology can make accessible beautiful and challenging new areas of mathematics, although it wasn't until 2013, at another seminar for teachers, that I fully realised the true value and accessibility of chaos equations for students. Consider for example the logistic difference equation $x_{n+1} = rx_n(1 - x_n)$, used by biologists to predict variability in animal populations. Few would expect from its apparent simplicity the fantastically and complex behaviour it exhibits. There are a range of wonderful activities on the internet for students at all levels (elementary to undergraduate) exploiting fractals and chaos theory, for example the *Fractal Challenge* (http://fractalfoundation.org/resources/). As well as their often beautiful images, fractal functions give great opportunities for students to visualise critical aspects of mathematics, for example zoom in on the "everywhere continuous, nowhere differentiable" Weierstrass Function, while technology allows us to spin 3-dimensional objects around to examine such features as level curves and surfaces for functions of two or more variables (see Figure 2).

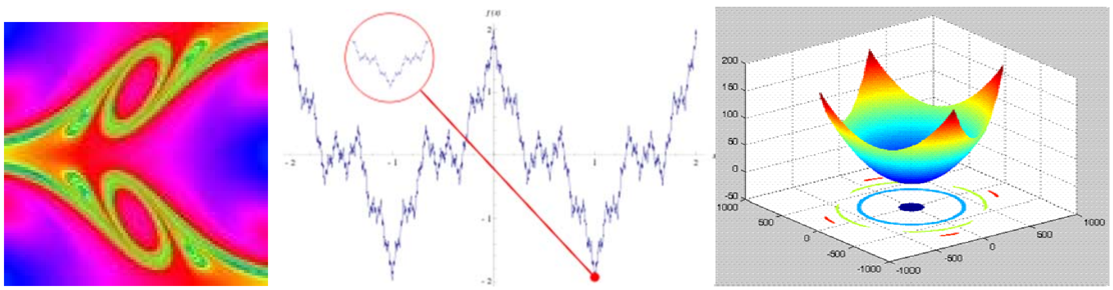

Figure 2: Chaotic Scattering (www.chaos.umd.edu/gallery.html); Weierstrass Function (https://commons.wikimedia.org/w/index.php?curid=5075959); & Random Rotated Griewangk Function (www.cil.pku.edu.cn/resources/benchmark_pso/)

Given the personal experiences stance adopted above and the many significant and acclaimed studies dedicated specifically to theoretical aspects of technology [e.g. 3; 7; 18; 19; 31; 39; 49], it seems neither appropriate nor feasible to present here an in-depth theoretical discussion of technology use in mathematics education. However, some theories that have impacted my own development since my return to university in 1993 are included in the subsequent discussions. My first revelation came with the arrival at Auckland University of Mike Thomas in 1993, when he introduced me to some work he had carried out, in part with David Tall and others, on representations and concept images [64; 65]. Using technology to *zoom-in* and observe local linearity was like a flash of inspiration [64], I finally realised just what the *Epsilon-Delta* proofs I had struggled to understand in my first undergraduate calculus course were actually getting at, and further, how such visualisations may help students understand introductory differentiation concepts (see Figure 2). This was followed in the mid-90's by obtaining our first truly CAS-capable calculator (TI-89), which initiated questioning of what changes we should make to how we teach and assess if students could use them. We struggled to get acceptance for their use in any of our courses, their use was and continues to be banned in nearly all tests and examinations, especially the powerful TI-92, whose QWERTY keyboard was deemed to contravene the university's calculator policy and was thus unable to be used even under the *Special Calculator's* category.

Exposure to more theoretical aspects and interesting and stimulating ideas for teaching on a variety of evolving technology platforms followed, especially through presentations at ATCM and Delta (biennial Southern Hemisphere conference on the teaching and learning of undergraduate mathematics & statistics which began in 1997) conferences. A few stand out in my memory from the many graphing, dynamic geometry and CAS-capable platforms (some of which have been discontinued) that I can remember over this period (e.g. *Autograph; Cabri; Derive; Desmos; GeoGebra; Geometer's Sketchpad; Grapher; Mathematica; Matlab & Maple; MuPAD; R* & *S* (stats); *Scientific Notebook*). One of these was a stimulating paper which in part described the benefits of the dynamic geometry package *Cabri* for developing students' reasoning skills [36] and a later plenary presentation at the 2003 ATCM conference which expanded on this [37]. This presentation resonated particularly with me as a means of addressing the perceived lack (in New Zealand at least) of students inability to understand and construct formal proofs. Another presentation at the 1999 Delta conference provided the example of using technology to first draw, and then zoom-in on a rational function $f(x) = \frac{x+2}{x^2+1}$, that crosses the *x*-asymptote, as a means to trigger cognitive conflict for developing understanding [e.g. see 24]. This and another example $f(x) = x^4 + 0.25x^2 - 18$, further demonstrated how the supposed constraints [7; 14] of scale in a calculator window may be used to stimulate learning when the graph is shown to conflict with how the students first perceived it. The power of technology to promote understanding through multiple representations also grew during this period [31; 49], for example with respect to the power of four (algebraic, numerical, graphical and verbal) as emphasised at the 1999 Delta conference by Hughes-Hallet [28]. Two visits to Auckland by David Tall to present at local conferences, and his continued research with Mike Thomas and others promoted further considerations with respect to the benefits of technology in visualisation and concept development, leading ultimately to the conceptualisation of representational versatility [65; 66].

Thinking about the need to re-examine curricular content, in particular the order in which we teach particular topics and ways in which we assess in an integrated technology environment which later formed a significant basis of my PhD study [51; 52; 53], began in earnest at a Delta 2001 conference (in Kruger National Park no less). Hillel [23] provided examples of how we might reconsider the teaching of inverse matrices and Gaussian elimination using CAS. At the same conference, David Smith from Duke University, a key advocates of technology during the 1990's math wars in the USA, described how he believed the traditional sequence of topics in teaching differential equations (the *raison d'être* of calculus) can be turned on its head using technology [61], an idea reinforced at the next Delta conference in 2003 when Harman [21] proposed teaching integration before differentiation using the historically more intuitive approach of accumulation. Unfortunately our subsequent attempts to trial this in a course at Auckland were somewhat unsuccessful, but these thoughts finally crystallised, again at the 2003 ATCM conference, when Stacey presented a range of examples to demonstrate how she perceives the value of some content had changed from a *pragmatic, epistemological* and *pedagogical (heuristic)* perspective [3; 62], as well as raising questions about assessment and teaching. She suggests for example that the *Product Rule* for differentiation may have lost its pragmatic value given we can now differentiate complex functions without it using CAS, however it retains its epistemic value because of connections to other concepts [62, p.7]. Building on this work, Oates [52; 53] gives examples of how technology changes the nature of questions we use in summative assessment, and in a later joint study [54], gives examples of how we might adapt existing questions to an active-technology model, or make use of collaborative online environments such as *Peerwise* [11] for formative assessment.

One significant theoretical perspective has influenced much of my research and teaching practice (including the values-framework used by Stacey [62]). It is the group of studies which conceptualise technology from the perspective of instrumentation, instrumental genesis and orchestrations [e.g. 3; 13; 19; 70]. A recent example comes from a study in which we adopted an intensive technology approach to teaching introductory undergraduate calculus at Auckland University. One particular finding was that students clearly favoured a graphing software platform *Desmos* largely because of its simplicity, and appeared to achieve reasonable instrumental genesis using it, even in the confines of a 12-week one semester course with limited teacher-privileging and no specific assistance in using the technology [32; 45; 54]. In another study, Stewart, Thomas & Hannah [63] examined student instrumentation using computer-based algebra systems (CAS) in a one-semester undergraduate linear algebra course. They conclude among other findings that student instrumentation raises important issues with respect to the time needed to learn the technology and interact effectively with the mathematics.

One final theoretical perspective included here is the conceptualisation of pedagogical technology knowledge (PTK, [68]), also characterised as technological pedagogical content knowledge (TPACK, [47]). PTK describes the ability of a teacher to realise the full potential of technology to effectively support learning. It requires the teacher for example to be sufficiently familiar with the technology to recognise its affordances and constraints [7], be aware of potential obstacles [14], and be able to switch seamlessly between the various representations [14; 31; 65; 66]. I have come to regard PTK as one of the most significant factors in teachers' ability to use technology effectively, and research mathematicians and policy makers to truly recognise the pedagogical value, as opposed to the value of technology as mostly a tool for doing mathematics. A model of how PTK is constructed is given in Figure 3.

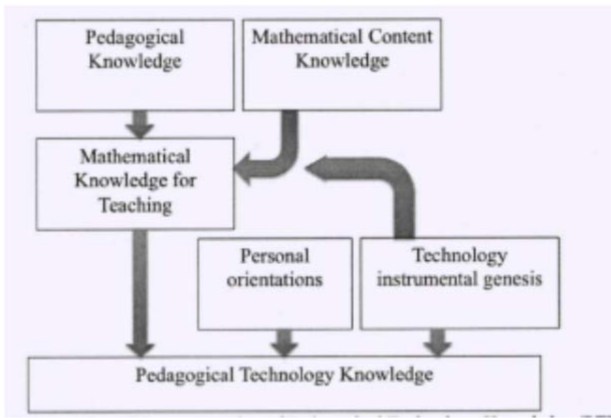

Figure 3: An outline of the construction of Pedagogical Technology Knowledge (PTK) [66, p. 70).

3. Issues and Challenges for the Future: Crystal-Ball Gazing

Despite the many benefits that have been shown in the literature for the use of technology in mathematics education, and the common acceptance and proliferation of its use that have been briefly described here, significant issues and challenges remain. Clearly there are implications across many of the elements of Oates [50; 53] framework: for example *BYOD* and an exploding range of potential learning environments [5; 6; 71] are revolutionising the *Access* component, while at the same time complicating *Staff* factors with respect to PTK and teacher privileging [32; 53] and computer-aided and online assessment models are potentially transforming the *Assessment* component [4; 59]. With respect to *Organisational* issues, unfortunately some of the emotive arguments associated with opposition to technology itself continue to affect adoption of technology, for example concerns about loss of skills, financial considerations and commercial interests. In the USA, the recent debate has exhibited emotive rhetoric similar to that from the 'maths wars' of the 1990's. Consider for example the arguments from Koblitz in 1996 [35, p. 10; p.14] that:

> …computers should not be a major component in math education reform…the intrinsic value of a pedagogical idea is not considered as important as its saleability…The technology-in-education movement has some of the characteristics of a religious evangelical campaign, fuelled by corporate and foundation money".

Compare the similarity of this statement with very recent dialogue some 20 years later when Kardaras [33], while not writing specifically about mathematics, observes in TIME magazine that

> We have accepted tech in the classroom as a necessary and beneficial evolution in education. This is a lie. Tech in the classroom … leads to worse educational outcomes for kids … Why have we allowed these "educational" Trojan horses to slip into our schools? Follow the money. Education technology is estimated to become a $60 billion industry by 2018…

Kardaras goes on to cite several examples of failed commercial technology initiatives, before as is common in many such commentaries, citing the counter-example of Finland as the world's leading educational system, which has according to him eschewed technology in favour of "(giving) students as many as four outdoor free-play breaks per day, regardless of the weather", while here (in the USA), children "sit in front of a glowing screen playing edu-games while over-scheduled and stressed by standardized testing seen as the Holy Grail" of educational practice [33]. While such debate may still be best viewed as "simplistic" and "unhelpful" as Koblitz's arguments were described back in 1996 by Dubinsky and Noss [16], it nevertheless highlights the need to continually research and evaluate the true value of, and most effective ways of using technology in mathematics education. Kardaras [33] for example cites the 2015 report by the Organization for Economic Co-operation and Development which concludes that heavy users of computers in the classroom "do a lot worse in most learning outcomes" and observes that "In the end, technology can amplify great teaching, but great technology cannot replace poor teaching." Mathematically, while playing devil's advocate to some extent, Devlin [12] questions whether technology developments may potentially cause the death of mathematics as we know it through the abandonment of pencil and paper techniques:

> There is no denying that advances in technology change the kind of mathematics that gets done. … the world is unlikely to ever again see the kinds of discoveries made in earlier eras by mathematical giants such as Fermat, Gauss, Riemann, or Ramanujan. The notebooks they left behind showed that they spent many, many hours carrying out long hand calculations…There are likely to be properties of numbers that will never be suspected (using) powerful computing technology. On the other hand, there is also a gain, since the computer has given rise to what is called Experimental Mathematics, where massive numerical simulations give rise to a different kind of conjecture—conjectures that likely would not have been discovered without computers.

So while technology can certainly change the direction of mathematical discovery, can it really cause its death? Devlin suggests it might when he states with respect to LaTeX that:

> We have, it seems, become so accustomed to working on a keyboard, and generating nicely laid out pages, we are rapidly losing, if indeed we have not already lost, the habit—and love—of scribbling with paper and pencil. Our presentation technologies encourage form over substance. But if (free-form) scribbling goes away, then I think mathematics goes with it. You simply cannot do original mathematics at a keyboard. The cognitive load is too great [12].

Issues of equity associated with access to technology also remain understandably prominent in contemporary discussions. In 1999, Zevenbergen [74] observed that while technology is sometimes seen as providing more equitable access to mathematics, for example by allowing students with weaker skills to attain higher levels of achievement, she believes that technology also introduces a "digital-divide" with respect to access that could see it replace mathematics itself as the historical gatekeeper for access to higher education and social success. Portable and easily accessible hand-held calculators with ever-increasing mathematical power have been seen to offer answers in this respect [34], but the costs of these have also increased with their increasing complexity. More recently, growing access to apps through phones and tablets and *Bring-Your Own Device (BYOD)* approaches in general are now seen as potential solutions to equity concerns with respect to access [43; 72]. However, even here the cost is still clearly an issue for many developing countries, and even in supposedly developed countries, where cost may not be the major issue, many believe there has been over-reliance on technology to ‘level-the playing-field’. Toyama, an associate professor at the University of Michigan's School of Information, once believed that technology in the classroom could solve the problems of modern urban education. However, he now refers to technology's ‘Law of Amplification’: “technology could help education where it's already doing well, but it does little for mediocre educational systems. Worse, in dysfunctional schools, it can cause outright harm" adding "Unfortunately, there is no technological fix; more technology only magnifies socioeconomic disparities, and the only way to avoid that is non-technological" [in 33].

Certainly such debates emphasise that we must not view technology as a cure-all panacea, but continue to seek effective means of integration, and this remains the major challenge confronting educators today. The proliferation of types of technology and the platforms available to staff and students presents complex issues and challenges, for example with respect to connectivity and the merging together of hand-held technologies (HHT), computers and the internet [5; 6; 26; 71]. Trouche and Drijvers [71] give the example of how using a google search can transform something as simple as multiplication, reducing it to an answer that “lies somewhere (just as it does if we ask ‘‘what is the main town in Transylvania?’’); we have to ‘search for the result and not imagine a constructive way to build it by our own means” [71, p. 678]. Students’ ability to download or access apps and added functionality for a multiplicity of devices also amplifies issues associated with instrumentation for students [71], and the associated PTK for teachers [53; 68]. This puts increasing responsibility on teachers to accurately evaluate and integrate such tools into their practice, and with the ever-expanding range of devices and online resources freely accessible to students, many previously unknown to teachers, this presents a major challenge. There is an urgent need for criteria to assess the mathematical and educational quality of available technologies and resources, which should be easily used by teachers, and preferably even available to students [53; 71]. In their summary, Borba et al. [6] highlight five important issues requiring further investigation:

1. Student access to mobile technologies creates a student-mathematics relationship that disrupts the traditional flow of mathematics knowledge from teacher to student. How can we offer guidance and influence the quality of these interactions?
2. The potential of MOOCs, varying in quality, to disrupt the institutional and hierarchical nature of traditional education, offering students opportunities to access courses without prerequisites, without fees (unless they require a record of course completion);
3. The availability of online mathematics learning resources (as the digital libraries and learning objects) means that many students now turn to these resources before they consult a teacher or a textbook, and this raises questions about how the resources are organized to in order to facilitate access and how they are designed pedagogically to foster conceptual understanding;
4. The collaborative and social networking affordances of current technologies raise questions about the design and use of learning management systems as well as personal learning environments and networks;
5. Teacher use of blended learning to extend and supplement classroom learning with online exploration and discussion or to employ a flipped classroom model to make the classroom a place for extension and elaboration rather than direct instruction raises questions about the need to research the various models used. [p. 27]

There are significant attempts being undertaken to address these issues, for example Trouche and Drijvers [71] point to several current projects at the European level, and the study by Handal, El-Khoury, Campbell and Cavanagh [20], which categorises educational apps according to their specific role in the teaching and learning of mathematics, along with their media richness. Their framework, validated with examples from a K-12 context, proposes nine distinct categories of apps, grouped into three main clusters, namely, investigative, productivity and instructive tools. This framework provides a very useful means for teachers to evaluate the value and potential effectiveness of apps, at least those that they can control the use of. It is worth investigating whether some similar guidance for students accessing such apps independently might be possible.

Technology-active resources are also identified as an area requiring attention, for example the design of suitable tasks [6; 30; 38; 71]. There are actually a huge variety of interesting, stimulating technology-based activities specific to particular content and skills areas available to teachers and students. Consider for example the many conference papers presented over the years at ATCM conferences and studies published in eJMT and other journals [e.g. 23; 38; 56], as well as the wealth of tasks and activities designed by software developers, training and professional organisations associated with specific software platforms, for example *Autograph* [9], *Cabri* [36; 37; 38], *GeoGebra* [30], *Maple & Matlab* [4] and *Mathematica* [1]. Many such studies have an appropriately practical focus and are thus not examined closely from a theoretical perspective, so issues remain with how teachers and students might assess the mathematical and educational value of available tasks, or whether the teachers themselves possess sufficient levels of PTK to appropriately integrate the task into their teaching [14; 45; 68]. Several recent studies have examined theoretical perspectives of task design, for example Jaworski et al. [30] explored the use of inquiry-based tasks in engineering using GeoGebra, while Ruthven [58] considered the didactical tetrahedron as a heuristic for analysing digital technologies to support the use of investigative approaches in the classroom. Ratnayake, Oates and Thomas [57] explore ways we might enable teachers to design suitable tasks themselves, informed by theoretical perspectives of task design in general which are limited in this respect [38; 56; 71]. This is a complex issue, requiring high levels of PTK [53; 54; 68]. Trouche and Drijvers [671] ask what we might learn from history in this respect, concluding that:

> To be able to design such tasks themselves, teachers need to master both the functionalities of the artefact as well as the mathematical and didactical backgrounds of the mathematical topic to be taught… (We) must be less naïve about machines and mediation, primarily involving the learner who uses the tools and, at a second level, the teacher who learns to integrate HHT into her teaching. Machines are not neutral, but deeply influence activity, conceptualisation and, more generally, students' and teachers' development [pp.674; 679].

This leads us to ponder whether it might be feasible to expect teachers to keep up with such issues given the ever-expanding range of technologies and the changing learning environment [6; 71]. To some extent teachers are losing control over students' interactions in the modern technological environment of *BYOD* devices and widely available digital resources, dramatically changing the student-teacher dynamic as highlighted by Borba et al. earlier [6].

What do these discussions and trends suggest our classrooms of the future might look like and what are the implications of this? The windows metaphor first introduced by Noss and Hoyles [49] seems useful here, as we open an ever-increasing kaleidoscope of *multi-glazed windows* through which to view and engage in mathematical activity, for example the mobile technologies, Personal Learning Environments (PLN's), Massive Online Courses (MOOCs), digital libraries and Virtual Learning Environments (VLE's) described by Borba et al. [6]. The merging of mobile technologies, computers and the internet ensures that connectivity will be a significant issue between devices and users for both students and teachers, and the ubiquity of mobile technologies such as smartphones will clearly influence the landscape [5, 6; 26; 71]. These artefacts are "stretching" the classroom, transforming it to the extent that it can hardly be recognized as such. The lines between "inside" and "outside" the classroom, or "between study time and leisure time" are being blurred [6: 71]. In respect of smartphones and everyday platforms such as *Facebook*, Trouche and Drijvers [71] ask "What is a learning tool, and what is a learning activity?" As teachers, how might we ensure that new hand-held devices such as smartphones are used for effective learning, as opposed to the multitude of imperatives students use them for on a daily basis? They suggest that bringing the world of games [e.g. see 60] from outside into the classroom may be a good way of addressing this:

> …(Games) are quite usual in our research community. Usually, however, the teacher designs the game, its rules and its management. In the case of serious games, the game and its rules are designed outside of schools. In the early times of calculators, educators had to think about the integration of these HHT into class-rooms; perhaps, nowadays we should seriously consider the integration into school of serious games, which might be considered as ‘‘HHA’’ (handheld activities), due to their quite natural appropriation. A shift in focus from technologies towards activities requires a rethinking of forms of instrumentalisation and orchestration [71, p.679].

If teachers are no longer designing the rules of the game, and students lead the choices of game devices, what are the implications for the role of teacher-privileging of technology [32; 45], as evidenced for example with how *Desmos* developed as the technology platform of choice in two studies [44; 54]. Dynamic geometry might provide one way teachers can bring real-world problems into the classroom, providing a range of activities and simulations which dynamic geometry makes accessible to students [56]. Borba et al. [6] on the other hand emphasise the classroom itself has changed: online and blended learning approaches mean “couches, chairs, tables at students’ houses, cafes and Lan Houses are the ‘new classrooms’. Flipped classrooms change the notion of what is in and outside of the classroom and with it the roles of students and teachers” [6, p. 26].

There are many other implications of such developments. Engelbrecht and Harding [17] for example ask the possibly obvious but nevertheless critical question “*Are online learners learning or just online?* and they discuss how we might facilitate or orchestrate learning in online environments. Consider for example the world-wide trend in tertiary institutions to record live lectures. Inglis, Palipana, Trenholm and Ward [29] found that students who often accessed online lectures had lower attainment than those who often attended lectures or the learning support centre, while another study [73] found that students were largely making strategic decisions with respect to attending lectures and watching recorded lectures, in line with the changing dynamics described by Borba et al. [6]. Trenholm, Alcock and Robinson [69] conclude that the problems of lecturing in the digital age are clearly complex, with current findings suggesting “that student use of e-lectures in mathematics may encourage and enable a form of learning that is in conflict with the disciplinary nature of mathematics (i.e. encourage rote learning versus conceptual understanding) [p. 712]. This leads into a consideration of the *Mathematical Factors* of the framework [53], such as the continued relevance and value of content in an integrated-technology environment described earlier [3; 53; 62]. In 2009, Oates [51] concluded that an urgent re-examination of the curriculum is called for: “…the changing pragmatic and epistemic values [3, 62] of specific content, and the goals of mathematics education, within a rapidly evolving technological environment, remains a pressing challenge for mathematics educators”, and it seems this challenge remains equally valid today. Consistent with sociological perspectives of technology as a tool [3; 49; 70], Trenholm et al. [69] argue eloquently in this respect when they emphasise:

> While technology may drive, and be driven by, mathematics pedagogy to the benefit of student learning [53], we would argue that the nature of mathematical thinking needs to drive the adoption and use of technology, and not the reverse. In the realm of e-lecture practice, how this could be realized remains to be understood [p. 713].

Issues of *Assessment* have been discussed briefly earlier and Oates [52] identifies further implications, for example the issue of curriculum misalignment if students use technology in all aspects of their learning, only to have technology withdrawn from the summative assessment procedures that remain prevalent. A hugely unexplored area of research is thus how technology might be better used for formative assessment measures, especially given the increasing frequency of students accessing activities and resources outside the classroom. With their potential for direct feedback to students, Computer-Aided Assessment (CAA) systems [4; 54; 59], Computer Response Systems (CRS, [41]) and approaches encouraging collaboration between students as advocated by Borba et al. [6], for example Peerwise [11], may offer great potential in this respect. CRS and Peerwise have also been shown to have positive benefits for student engagement and promoting effective cognitive conflict [11; 41].

4. Summary: Prophecies from the Crystal-Ball

The thoughts presented here may thus lead us to conclude, that despite, or more probably as a direct result of the rapidly and ever-changing technological landscape, the critical challenge today and as we gaze into our crystal-ball for the future, remains as it was described in 1996 [55], 2004 [2], and again in 2010 [27]: How do we most effectively exploit the pedagogical, mathematical and motivational potential of the technologies available to our students and ourselves as teachers? While the power of the internet and the new technologies provide exciting new opportunities, and are themselves part of the solution, they also present new issues and complications [5; 6]. Thus, notwithstanding the recommendations and issues identified in the two recent reports cited here [6; 71], four challenges are highlighted from the preceding discussions with respect to:

1. *Pedagogical Technology Knowledge*: How can we develop suitable PTK for our teachers in an ever-burgeoning technology environment, so they can best realise the potential of the tools at their disposal, or the ones students' are using or bring to the classroom?
2. *Assessment:* How can we better integrate the use of technology into the still-predominant summative assessment systems, or more importantly develop better ways of using the power of technology in formative assessment techniques at students' disposal?
3. *Evaluation*: Teachers and students need easily used and accessible tools to assess the mathematical and educational value of the exponentially increasing selection of technological tools, apps, software, and digital resources available to them.

In answering these three questions, it is imperative we continue to explore and develop the theoretical lenses and frameworks considered here and elsewhere within which these questions may be examined, for example PTK [53; 68]; affordances and constraints [7], instrumentation and orchestrations [13; 45; 70]; zone theory and APOS theory [15; 18], and app evaluations [20]. Finally, we need to find better ways of making the results of these studies more accessible to teachers, curriculum developers, politicians and the general public alike, so collectively as a community, we may build a more integrated and connected mathematics learning environment which truly reflects technologies kaleidoscope of windows and their promise of improved meaning-making in the mathematics of the future.

References

[1] Abbot, P. (1995) In *Innovative Use of Technology for Teaching and Research in Mathematics, Proceedings of First Asian Technology Conference in Mathematics (ATCM)* (pp. 262-271). Singapore: The Association of Mathematics Educators.

[2] S. Arnold (2004). Handheld classroom technology, in I. Putt, R. Faragher & M. McLean, (Eds.), *Mathematics Education for the Third Millennium: Towards 2010, Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia,* Melbourne, pp. 16-28. MERGA.

[3] Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245–274.

[4] Blyth, B., & Labovic, A. (2009). Using Maple to implement eLearning integrated with computer aided assessment. *International Journal of Mathematical Education in Science and Technology*, *40*(7), 975-988.

[5] Borba, M. C., Clarkson, P., & Gadanidis, G. (2012). Learning with the use of the Internet. In *Third International Handbook of Mathematics Education* (pp. 691-720). Springer New York.

[6] Borba, M.C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S. & Aguilar, M. S. (2016). Blended learning, e-learning and mobile learning in mathematics education *ZDM Mathematics Education, 48*(5): pp 589-610. doi:10.1007/s11858-016-0798-4

[7] Brown, J., Stillman, G. & Herbert, S. (2004). Can the notion of affordances be of use in the design of a technology enriched mathematics curriculum? In I. Putt, R. Faragher, & M. McLean (Eds.), *Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia* (pp. 119-126). Melbourne: MERGA.

[8] Buteau, C., Marshall, N., Jarvis, D. H., & Lavicza, Z. (2010). Integrating computer algebra systems in post-secondary mathematics education: Preliminary results of a literature review. *International Journal for Technology in Mathematics Education*, *17*(2), 57-68.

[9] Butler, D., Jackiw, N., Laborde, J. M., Lagrange, J. B., & Yerushalmy, M. (2009). Design for transformative practices. In *Mathematics Education and Technology-Rethinking the Terrain* (pp. 425-437). Springer US.

[10] Conder, M. (1995). Semi-automated theorem proving-the impact of computers on research in pure mathematics. In *Innovative Use of Technology for Teaching and Research in Mathematics, Proceedings of First Asian Technology Conference in Mathematics (ATCM)* (pp 1-8). Singapore: The Association of Mathematics Educators.

[11] Denny, P., Luxton-Reilly, A., & Hamer, J. (2008). The PeerWise system of student contributed assessment questions. *Proceedings of the Tenth Conference on Australasian Computing Education* (ACE 2008), Vol. 78, pages 69-74, 2008 (Wollongong, NSW, Australia).

[12] Devlin, K. (2013). The death Of mathematics. Accessed 15 June 2015 from *Edge: https://edge.org/annual-question/what-should-we-be-worried-about*

[13] Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: Instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75, 213–234. DOI 10.1007/s10649-010-9254-5

[14] Drijvers, P. (2012). Digital technology in mathematics education: Why it works (or doesn't). In *Pre-Proceedings of the 12th International Congress on Mathematical Education*, Seoul, Korea, *Regular Lectures 2-8*, 1-17.

[15] Dubinsky, E., & McDonald, M. A. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. In *The teaching and learning of mathematics at university level* (pp. 275-282). Springer Netherlands.

[16] Dubinsky, E., & Noss, R. (1996). Some kinds of computers for some kinds of learning: a reply to Koblitz, *Mathematical Intelligencer, 18*(1), 17-20.

[17] Engelbrecht, J. & Harding, A. (2003). *Are online learners learning or just online? A status report on the teaching of undergraduate mathematics on the Web.* Plenary address

to the Remarkable Delta’03 Fourth Southern Hemisphere Symposium on Undergraduate Mathematics and Statistics Teaching and Learning, Queenstown, New Zealand.

[18] Galbraith, P. & Goos, M. (2003). From description to analysis in technology aided teaching and learning: A contribution from zone theory. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds.), *Proceedings of the 26th annual conference of the Mathematics Education Research Group of Australasia* (pp. 364-371). Geelong, Vic: MERGA.

[19] Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*, 195-227.

[20] Handal, B., El-Khoury, J., Campbell, C., & Cavanagh, M. (2013). *A framework for categorising mobile applications in mathematics education.* Accessed 30 August 2016 from http://researchonline.nd.edu.au/edu_conference/70/.

[21] Harman, C. (2003). Reform calculus – Yesterday, today, and tomorrow. In M. O. J. Thomas, & G. Oates (Eds.), *New Zealand Journal of Mathematics: Proceedings of Remarkable Delta’03 Fourth Southern Hemisphere Symposium on Undergraduate Mathematics and Statistics Teaching and Learning* (pp. 89-96). Auckland: New Zealand Mathematical Society.

[22] Harvey, B. (1997).*Computer Science Logo Style: Symbolic Computing* (Vol. 1). MIT press.

[23] Hillel, J. Computer algebra systems in the learning and teaching of linear algebra: Some examples. In D. Holton (Ed.), T*he Teaching and Learning of Mathematics at the University Level: An ICMI Study.* Dordrecht: Kluwer, 2001, pp. 371-380.

[24] Hoyles, C. (1985). What is the point of group discussion in mathematics?. *Educational Studies in Mathematics*, *16*(2), 205-214.

[25] Hoyles, C., & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.) *Second International Handbook of Mathematics Education Volume 1* (pp. 323-349). Dordrecht: Kluwer.

[26] Hoyles, C., Kalas, I., Trouche, L., Hivon, L., Noss, R., & Wilensky, U. (2010), Connectivity and Virtual Networks for Learning. In C. Hoyles, & J.-B. Lagrange (Eds.), *Mathematical Education and Digital Technologies: Rethinking the terrain* (pp. 439-462). New York: Springer.

[27] Hoyles, C. & Lagrange, L-B. (Eds.) (2010). *Mathematics education and technology: Rethinking the terrain. New York/Berlin: Springer.*

[28] Hughes-Hallett, D. (1991). Visualization and calculus reform. In W. Zimmermann, & S. Cunningham (Eds.), *Visualization in Teaching and Learning Mathematics*, *MAA Notes 19*, pp. 121-126. Washington, DC: The Mathematical Association of America.

[29] Inglis, M., Palipana, A., Trenholm, S., & Ward, J. (2011). Individual differences in students' use of optional learning resources. *Journal of Computer Assisted Learning*, *27*(6), 490-502.

[30] Jaworski, B., Robinson, C., Matthews, J., & Croft, A. C. (2012). Issues in teaching mathematics to engineering students to promote conceptual understanding: A study of the use of GeoGebra and inquiry-based tasks. *The International Journal for Technology in Mathematics Education, 19*(4), 147-152.

[31] Kaput, J. J. (1998). Representations, inscriptions, descriptions and learning: A kaleidoscope of windows. *Journal of Mathematical Behaviour*, *17*(2), 265-281.

[32] Kendal, M., & Stacey, K. The impact of teacher privileging on learning differentiation. *International Journal of Computers for Mathematical Learning*, ***6***(2), (2001), pp. 143-165.
[33] Kardaras, N. (2016). Screens in schools are a $60 billion hoax. *TIME Magazine*, 31 August 2016, http://time.com/4474496/screens-schools-hoax/?xid=fbshare.
[34] Kissane, B. (1995). The importance of being accessible: The graphics calculator in mathematics. In *Innovative Use of Technology for Teaching and Research in Mathematics, Proceedings of First Asian Technology Conference in Mathematics (ATCM)* (pp161-170). Singapore: The Association of Mathematics Educators.
[35] Koblitz, N. (1996). The case against computers in k-13 math education (kindergarten through calculus), *Mathematical Intelligencer*, *18*(1), 9-1.
[36] Laborde, C., & Laborde, J. M. (1995). The case of Cabri-géomètre: learning geometry in a computer based environment. In *Integrating information technology into education* (pp. 95-106). Springer US.
[37] Laborde, C. (2003). Technology used as a tool for mediating knowledge in the teaching of mathematics: the case of Cabri-geometry. In *Plenary speech delivered at the Asian Technology Conference in Mathematics*.
[38] Laborde, C. (2011). Designing Substantial Tasks to Utilize ICT in Mathematics Lesson. *Mathematics Education with Digital Technology*, 75-83.
[39] Lagrange, J. B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and mathematics education: A Multidimensional study of the evolution of research and innovation. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second International Handbook of Mathematics Education Volume 1* (pp. 347-390). Dordrecht: Kluwer.
[40] Lavicza, Z. (2010). Integrating technology into mathematics teaching at the university level. *Zdm*, *42*(1), 105-119.
[41] Lee, H., Feldman, A., & Beatty, I. D. (2012). Factors that affect science and mathematics teachers' initial implementation of technology-enhanced formative assessment using a classroom response system. *Journal of Science Education and Technology*, *21*(5), 523-539.
[42] Li, Q., & Ma, X. (2010). A meta-analysis of the effects of computer technology on school students' mathematics learning. *Educational Psychology Review*, *22*, 215-243.
[43] Mancilla, R. L. (2014). BYOD: Re-examining the issue of digital equity. *Teachers College Record, http://www.tcrecord.org ID Number: 17639, accessed: 8/15/2014.*
[44] McAndrew, A. & Yang, W-C. (2016). Locus and optimization problems in lower and higher dimensions. *The Electronic Journal of Mathematics & Technology*, *10*(2).
[45] McMullen, S., Oates, G. & Thomas, M.O.J. (2015). An integrated technology course at university: Orchestration and mediation. In K. Beswick, T. Muir, & J. Wells (Eds.), *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 249-257). Hobart, Australia: PME.
[46] Ministry of Education. (1992). *Mathematics in the New Zealand curriculum.* Wellington: Learning Media.
[47] Mishra, P., & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054.
[48] National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

[49] Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning cultures and computers* (Vol. 17). Springer Science & Business Media.

[50] Oates, G. (2004). Measuring the degree of technology use in tertiary mathematics courses. In W.C. Yang, S.C. Chu, T. de Alwis, & K.C. Ang (Eds.), *Proceedings of the 9th Asian Technology Conference in Mathematics (ATCM)* (pp. 282-291). Blacksburg, VA: Asian Technology Conference in Mathematics.

[51] Oates, G. (2009). Relative values of curriculum topics in undergraduate mathematics in an integrated technology environment. In R. Hunter, B. Bicknell & T. Burgess, (Eds.), *Proceedings of the 32nd Annual Conference of the Mathematics Education Group of Australasia, Vol 2*, Palmerston North, New Zealand: MERGA, pp. 419-427.

[52] Oates, G. (2010). Integrated Technology in Undergraduate Mathematics: Issues of Assessment. *Electronic Journal of Mathematics and Technology, 4*(2), pp. 162-174. Available at http://www.radford.edu/ejmt

[53] Oates, G. N. (2011). Sustaining integrated technology in undergraduate mathematics. *International Journal of Mathematical Education in Science and Technology*, *42*(6), 709-721.

[54] Oates, G. N., Sheryn, L., & Thomas, M. O. J. (2014). Technology-active student engagement in an undergraduate mathematics course. In P. Liljedahl,, C. Nicol, S. Oesterle & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 4, pp. 330-337). Vancouver, Canada: IGPME.

[55] Penglase, M., & Arnold, S. (1996). The graphics calculator in mathematics education: A critical review of recent research. *Mathematics education research journal*, *8*(1), 58-90.

[56] Pierce, R. & Stacey, K. (2011). Using dynamic geometry to bring the real world into the classroom. In L. Bu & R. Schoen (Eds.), *Model-Centered Learning Modeling and Simulations for Learning and Instruction* Volume 6, pp 41-55. Sense Publishers.

[57] Ratnayake, I., Oates, G., & Thomas, M. (2016). Supporting teachers developing mathematical tasks with digital technology. In B. White, M. Chinnappan & S. Trenholm (Eds.), *Opening up Mathematics Education Research: Proceedings of the 39th Annual Conference of the Mathematics Education Research Group of Australasia*, pp. 543-551. Adelaide: MERGA,

[58] Ruthven, K. (2012). The didactical tetrahedron as a heuristic for analysing the incorporation of digital technologies into classroom practice in support of investigative approaches to teaching mathematics. *ZDM*, *44*(5), 627-640.

[59] Sangwin, C. (2013). *Computer aided assessment of mathematics*. OUP Oxford.

[60] Siew, N. M., Geofrey, J., & Lee, B. N. (2016). Students' algebraic thinking and attitudes towards algebra: the effects of game-based learning using Dragonbox 12+ app. *The Electronic Journal of Mathematics & Technology, 10*(2).

[61] Smith, D. (1998). Renewal in collegiate mathematics education: Learning from research. *Documenta Mathematica,* Extra Volume ICM III, 778-786.

[62] Stacey, K. (2003). Using computer algebra systems in secondary school mathematics: Issues of curriculum, assessment and teaching. In S-C. Chu, W-C. Yang, T. de Alwis, & M-G. Lee (Eds.), *Technology Connecting Mathematics, Proceedings of the 8th Asian Technology Conference in Mathematics*, Taiwan R.O.C: ATCM.

[63] Stewart, S., Thomas, M. O. J., & Hannah, J. (2005). Towards student instrumentation of computer-based algebra systems in university courses. *International Journal of Mathematical Education in Science and Technology*, *36*(7), 741-750.
[64] Tall, D. O. (1985). Understanding the calculus. *Mathematics Teaching*, *110*, 49–53.
[65] Tall, D. O. & Thomas, M. O. J. (1991). Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics*, **22** 2, 125–147.
[66] Thomas, M. O. J. (2008). Developing versatility in mathematical thinking. *Mediterranean Journal for Research in Mathematics Education*, *7*(2), 67-87.
[67] Thomas, M. O. J., & Holton, D. (2003), Technology as a tool for teaching undergraduate mathematics, In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.) *Second International Handbook of Mathematics Education Volume 1* (pp. 347-390), Dordrecht: Kluwer.
[68] Thomas, M. O. J., & Hong, Y. Y. (2013). Teacher integration of technology into mathematics learning. *International Journal of Technology in Mathematics Education*, *20*(2), 69-84.
[69] Trenholm, S., Alcock, L., & Robinson, C. L. (2012). Mathematics lecturing in the digital age. *International Journal of Mathematical Education in Science and Technology*, *43*(6), 703-716.
[70] Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: guiding students' command process through instrumental orchestrations. *Educational Studies in Mathematics*, *9*, 281-307.
[71] Trouche, L., & Drijvers, P. (2010). Handheld technology for mathematics education: flashback into the future. ZDM, The International Journal on Mathematics Education, 42(7), 667–681. DOI 10.1007/s11858-010-0269-2
[72] Wong, G. K. (2014). Engaging students using their own mobile devices for learning mathematics in classroom discourse: a case study in Hong Kong. *International Journal of Mobile Learning and Organisation*, *8*(2), 143-165.
[73] Yoon, C., Oates, G., & Sneddon, J. (2013). Undergraduate mathematics students' reasons for attending live lectures when recordings are available. *International Journal of Mathematical Education in Science and Technology*, 1-14. doi:10.1080/0020739X.2013.822578
[74] Zevenbergen, R. (1999). Equity in Tertiary Mathematics: Imaging a future. In W. Spunde, P. Cretchley & R. Hubbard (Eds.), *Proceedings of the Delta'99 Symposium on Undergraduate Mathematics*, (pp. 18-26). Rockhampton: Central Queensland University.