Teaching and Learning of Geometry in Primary School Using GeoGebra

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Abstract

The purpose of this article is to discuss how GeoGebra can be used to teach the concept of angle in Geometry in elementary level. This result is obtained after 2 weeks of lesson exploration. Teachers used GeoGebra as a teaching tool to make the lesson more creative and innovative in order to show how geometrical shapes relate with different angles in different polygons. For students, they can use GeoGebra to construct, drag or apply the actual shape instead of drawing on a piece of paper. Besides that, all the works created by students can be saved as documents for future reference. At the end of two weeks' exploration, pupils were asked to answer survey question regarding their experience with using GeoGebra. Analysis of the survey showed that pupils are able to express their geometric imagination and understanding of mathematical concepts before and after the exploration. Hence, using GeoGebra can make the classroom lesson more enjoyable and interesting.

1. Introduction

Today, technology is becoming an important tool in everyday life. Many educational researchers have carried out studies on integrating technology into education in order to increase the quality of teaching and learning [1]. Digital technology use in the mathematics classroom deals with two main facets of mathematics education: teaching and learning [2]. Several studies have investigated how students use technology or how teachers integrate technology into their teaching strategies. According to Pannen [3], as an integrated component of teaching and learning, digital technology allows learning experiences to become innovative, accelerated, enriched, besides deepening skills acquisition. Technology use is motivating and engaging; hence it relates school experience to work practices and authentic contexts. Furthermore, digital technology use in education has the potential to positively enrich the teaching and learning environments [1]. All these potential uses of digital technology in education indicate the simple use of ICT as a tool in teaching and learning is no longer adequate and thus creative and innovative strategies are needed to help teachers and students embark on the student-centered learning instead of content-oriented or teacher-centered learning [2].

2. GeoGebra as Teaching and Learning Tools

According to the National Council of Teachers of Mathematics (NCTM) technology is one of the six principles for school mathematics, as it mentioned that technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning [4]. Furthermore, technology use in mathematics education not only helps students

construct their visual representation of mathematical ideas and concepts, summarize and analyze data, and interpret data, but also enables them to investigate every area of mathematics, such as geometry, algebra and statistics [5]. With the support of technology [6], schools can provide extensive opportunities for facilitating, supporting and enriching the learning environment and continuously enhance the quality of the teaching-learning process.

Numerous powerful technological tools (dynamic geometry software) are available for teaching mathematics such as Geometer's Sketchpad, Cabri Geometry, Cinderella, and GeoGebra. However, different dynamic geometry software support teaching and learning mathematics at a variety of levels such as elementary level, middle school level and high schools level. One of the most useful and versatile dynamic geometry software is GeoGebra which was selected for discussion in this article. GeoGebra was created by Markus Hohenwarter in 2001 and it is a free software which can be downloaded from www.geogebra.org [7]. This software had been translated into 36 languages as many users from all over the world can use it without any restriction. Hence, it appears to be a user-friendly software that can be operated intuitively and does not require advanced skills to apply [8].

GeoGebra is an interactive geometry software offering students and teachers ways to design teaching modules and enable mathematics learning in a meaningful way. GeoGebra is another innovative tool for integrating technology in teaching and learning mathematics [9]. A powerful teaching tool like GeoGebra supports constructions with points, vectors, segments, polygon, straight lines and all conic sections. Besides that, it also can motivate students to explore mathematics and offer opportunities for critical thinking, which is central to constructivism.

GeoGebra can be used in many ways in mathematics teaching and learning [9]. It offers the following advantages:

- Provides basic features of Computer Algebra System (CAS) to bridge gaps between geometry, algebra and calculus.
- For demonstration and visualization as it can provide different representations
- As a construction tool as it has the abilities for constructing shapes
- For investigating to discover mathematics and help to create a suitable atmosphere for learning mathematics
- For preparing the teaching materials as a cooperation, communication and representation tool.

Integrating GeoGebra in geometry teaching and learning requires special understanding in many aspects. In this article, we discuss how integrating GeoGebra can foster the understanding of angles in primary level geometry. We were interested in studying how using GeoGebra can contribute to the students' learning of geometry and understanding of the concept of angles.

3. Geometry with GeoGebra

Geometry is the study of shapes and spaces and at the same time it is defined as "it examines figures and their movements" in the elementary mathematics curriculum [10]. It is stressed in the curriculum that while the geometrical thinking is developing also knowledge acquired in geometry activities have to provide visual and analytical reasoning and inference with a hierarchical order within the required attention respectively. In Malaysia, geometry is taught in primary and secondary school [11]. Learning geometry is not easy and some research found that a large number of students fail to develop adequate understanding of geometry concepts, geometry

reasoning and geometry problem-solving skills [12, 13]. The lack of understanding of geometry concepts may often discourage students and thus it leads them to poor performance in learning geometry. As further explained by Idris [12], some factors that caused poor achievements among students in learning geometry are geometry language, poor visualization abilities and ineffective instruction.

In Mathematics teaching and learning, especially in geometry, it is important for students to be able to imagine, construct and understand construction of shapes in order to connect them with related facts [14]. While the study of geometry in some shape or form has existed for many millennia, it is within the past twenty years that a shift has occurred in how geometry may be learned through computer-based interactive geometric software. Software such as GeoGebra allows users to construct interactive representations of points, lines and circles. These geometric objects are interactive and may be resized and shifted around on screen through clicking and dragging actions. In this way, GeoGebra allows students to actively construct their own understanding of geometry.

Using GeoGebra, students can learn geometry more effectively such as [9]:

- Use the polygon and circle tools to draw shapes.
- Measure angles and distance
- Use GeoGebra sliders to adjust values of different problems.
- Insert images into the file to demonstrate mathematical problem solving.

4. Geometry in Malaysian Mathematics Curriculum of Primary Schools

A new curriculum for primary school in Malaysia is known as the Standard Curriculum for Primary Schools (KSSR). KSSR is a new system introduced by the Ministry of Education in 2011. The Malaysian primary Mathematics curriculum aims at providing pupils with a deep understanding of basic mathematical concepts so that they are able to relate, explain, and apply the mathematical knowledge to solve daily problems more innovatively. In recent years, changes in the mathematics curriculum can be seen clearly, particularly in geometry. In the current Malaysian curriculum, geometry was introduced starting from Year 1 to 2 in primary education where pupils are taught to learn about properties of two dimensional shapes and three dimensional shapes. Pupils of Year 3 and 4 learn to compare between two dimensional shapes and three dimensional shapes and sort them according to their geometric properties and describe the relationship between both dimensions. Furthermore, Year 5 Malaysian pupils learn to distinguish among polygons, regular polygon and other two-dimensional shapes; identify and classify the acute, right, obtuse, and the straight angles; measure and construct angles up to 90° with using a protractor. While pupils of primary Year 6 learn to measure and construct angles up to 180° using a protractor and classify them as acute, right, obtuse or straight angles. As the pupils are introduced to the concept of angles in primary level, it may enhance pupils' understanding of geometric properties and also enable them to construct the figures correctly and develop a deep understanding about the relationship of angles in different polygons.

Besides that, the elements of using information technology and communications (ICT) skills were also strongly emphasized in the current mathematics curriculum. Pupils are encouraged to use and handle mathematics tools (dynamic geometry software such as Geometer's Sketchpad or GeoGebra), develop an understanding of geometric concepts and explore mathematical ideas more deeply. When pupils are able to master the technology skills in mathematics education, they could apply the ICT skills and mathematical knowledge to solve more complex routine and non-routine daily problems more creatively.

5. Examples of Using GeoGebra: Teaching the Concept of Angles in Topic Geometry in Primary Schools

Example 1: Exploring Angles of Triangles

In this activity, pupils will explore the measure of angles in a triangle to prove that the sum of three angles in a triangle always add up to 180 degrees. Pupils can use the toolbar of GeoGebra to construct a visualization of triangle. (see Fig. 1)

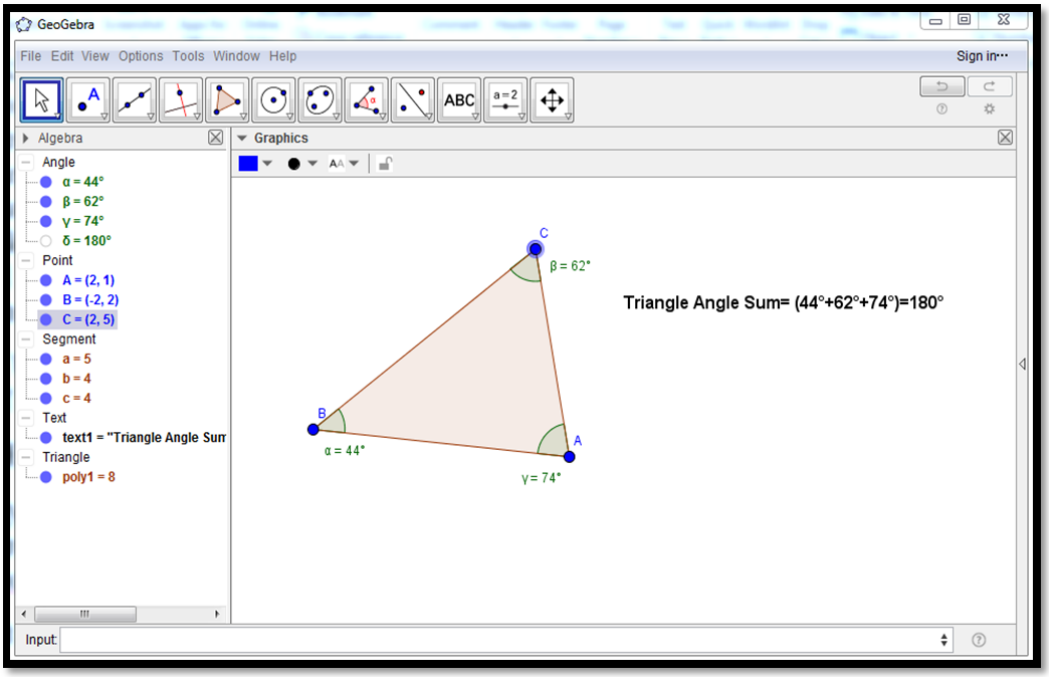

Figure 5.1 Triangle Sum Angle using GeoGebra.

Open a New GeoGebra File

Construction process:

Step 1: Select to hide the axes from graphics.

Step 2: Select Polygon tool and click any three points to construct a triangle.

Step 3: Hide the label of each segment with right click.

Step 4: Select the Angle tool and click on each angle to show its value, for example, ΔABC, ΔBCA, and ΔCAB.

Step 5: To round off the angle, select Options, choose Rounding and click on "0 Decimal Places". All the values show without any decimal places.

Step 6: To get the triangle sum angle, type the formula into Input (placed at the bottom of GeoGebra window). For example, type (α+β+γ), then enter, it comes out as a new symbol, δ to indicating that the triangle sum angle is equal to 180°.

Step 7: Select the Text tool, type the formula (same as above) into the Edit column. Then, the whole text will completely show on the screen.

At the end of this activity, pupils can drag and move one of the points of the triangle; the value of each angle will be changed when the move is made. Pupils can observe that when the point is being moved, the triangle angle sum would not be changed and remained as 180°.

Example 2: Exploring Acute, Obtuse, Right and Straight Angles

An angle is made up of two intersecting lines. The four types of angles such as acute, obtuse, right and straight angles are strongly emphasized in the primary mathematics curriculum. With the use of GeoGebra, angle can be constructed and moved with the slider. It helps pupils to see how an angle behaves and that each different angle has its own properties. An acute angle is an angle whose measure is less than 90°, an obtuse is an angle whose measure is greater than 90° but less than 180°. Furthermore, right angle is measured at exactly 90° and straight angle is measured exactly 180° and it is represented as a straight line. (see Fig. 2)

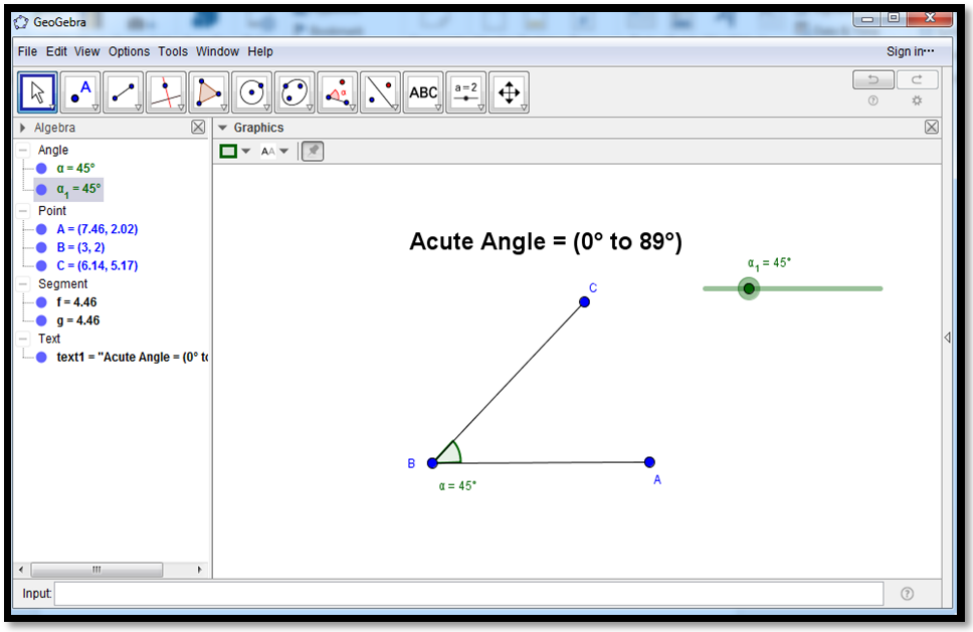

Figure 5.2 Acute Angle with Slider

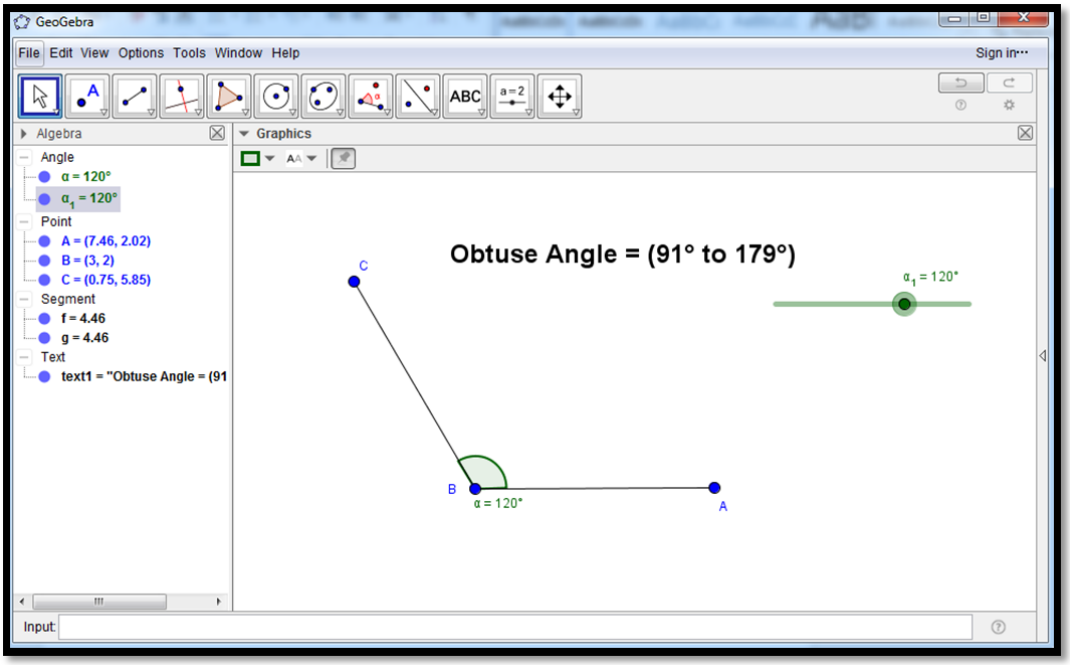

Figure 5.3 Obtuse Angle with Slider

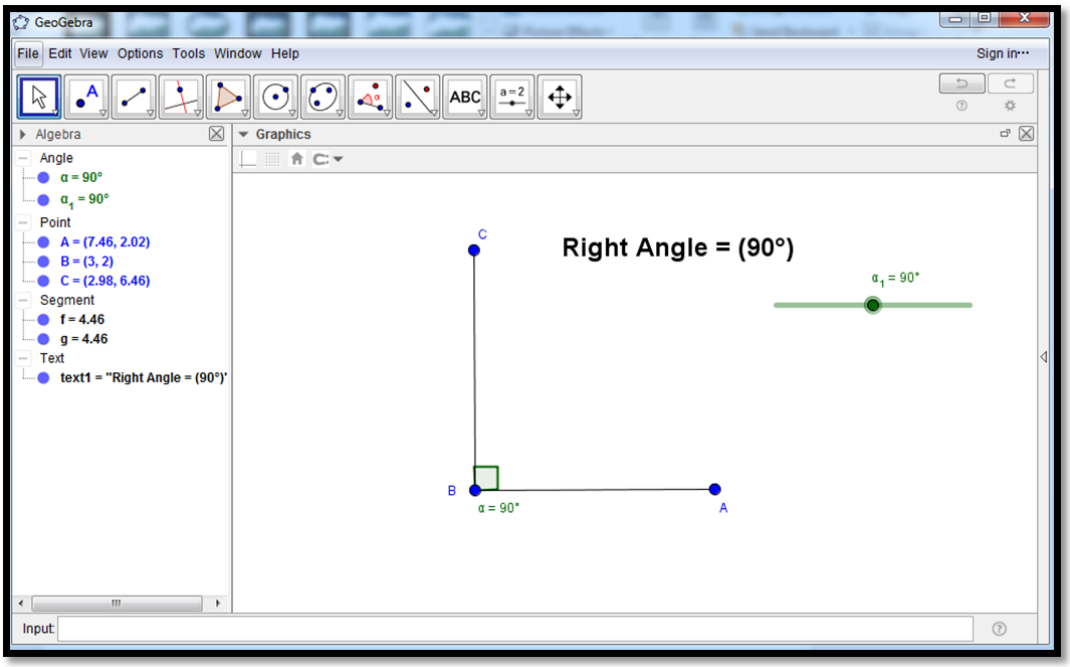

Figure 5.4 Right Angle with Slider

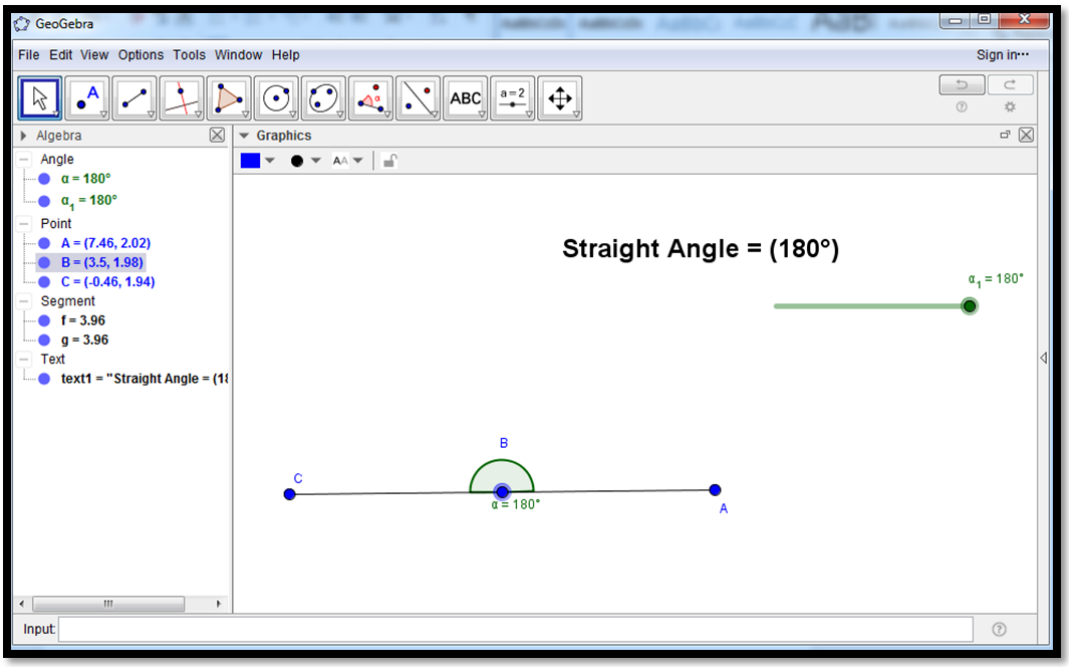

Figure 5.5 Straight Angle with Slider

Open a New GeoGebra File

Construction process:

Creating angles with GeoGebra involves the following steps:

Step 1: Select to hide the axes from graphics.

Step 2: Select the Slider tool from the toolbar.

Step 3: Name the slider as α, then tick the "angle", as the min and max we fixed it from 0° to 180°, with the increment of 1° (without any decimal places). Then click "OK". The slider will show on the screen.

Step 4: Select the Angle tool and click on the Angle with Given Size. Then change the value of angle to symbol, α and click "OK". The value of angle will be shown according to the slider.

Step 5: Rename the angle α. Then, try to move the slider and see how the angle changed its value.

Step 6: Construct two segments by selecting the Segment tool. When finish constructing the two segments, hide the label of segment by right click.

Step 7: Select the Text tool, type the name of angle into the Edit column. Then, the whole text will completely show on the screen.

Through this example, GeoGebra allows students to understand how an angle is constructed, how to name an angle and how an angle is measured by dragging the slider. (see Fig. 2, 3, 4, & 5)

Example 3: Exploring Interior Angles of Regular Polygon

A polygon is a two dimensional shape with straight sides. A regular polygon is defined as one having all sides equal and all angles equal. The interior angles of any polygon always add up to a constant value and it depends only on the number of sides. To prove that sum of interior angles in any polygon always add up to a constant value, GeoGebra allows pupils to construct and use the tool to measure the interior angle and see how the interior angles are formed and make a clear generalization from the construction. Here are some simple steps to construct the regular polygon and how an interior angle is measured:

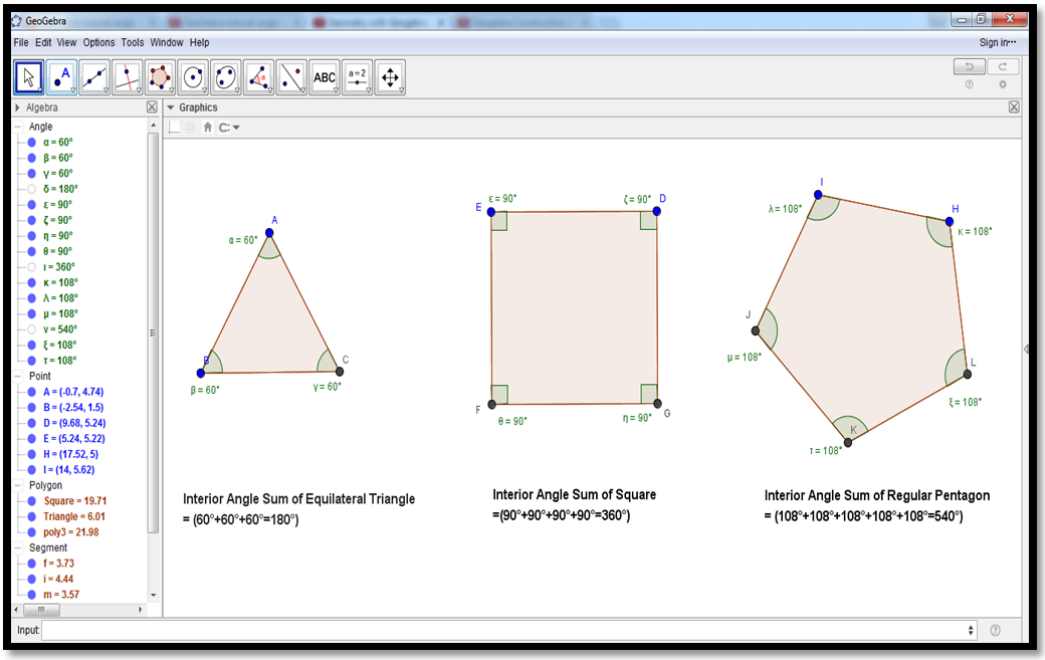

Figure 5.6 The Interior Angles Sum of Regular Polygon

Open a New GeoGebra File

Construction process:

Step 1: Select to hide the axes from graphics.

Step 2: Select the Regular Polygon from the toolbar.

Step 3: Click any two point on the screen. It comes out with a command. Type the number of vertices into the command. If we want to construct a regular pentagon (which has 5 vertices), we type 5 into the command.

Step 4: The shape is formed. We rename the shape by right click.

Step 5: To measure the interior angles of a regular pentagon, select the Angle tool and click on the two adjacent sides, the interior angle is shown.

Step 6: To get the interior angle sum of regular pentagon, type the formula into Input (placed at the bottom of GeoGebra window). For example, type (κ+λ+μ+ξ+τ), then enter, it comes out as a new symbol, ν to indicate that the interior angle sum is equal to 540°.

Step 7: Select the Text tool, type the formula (same as above) into the Edit column. Then, the whole text will completely show on the screen.

This simple example has shown how an equilateral triangle, a square and a regular pentagon is formed using GeoGebra. GeoGebra allows pupils to demonstrate what they can construct by using the angle tool method. It is easy and quick for pupils to learn the concept of angle. For teachers, they can use the above examples as a teaching guided practice or modify it to create a simple worksheet for their pupils to explore how an interior angle is formed and measured.

6. Experience with Teaching and Learning Activities

Two weeks of lessons with total of 12 hours which contained six main GeoGebra activities about the angle of Geometry were planned according to the latest mathematics curriculum in Malaysia. These six main instructional activities were planned as follows in the mathematics curriculum:

(a) Constructing and measuring the angle in a triangle.
(b) Constructing Acute angle
(c) Constructing Obtuse angle
(d) Constructing Right angle
(e) Constructing Straight angle
(f) Constructing and measuring the interior angle of regular polygon (ex: Quadrilateral and Pentagon).

The aim of these activities is to encourage students to use GeoGebra to learn the abstract concept more easily, in a more dynamic and visual way. The instructional activities were conducted in August 2016. A group of Year Six pupils from a primary school was selected with total of 22 female and 28 male to participate in this study. Before starting the activities, the use of GeoGebra software was introduced in the introductory lesson. Through the introductory lesson, students learned sample activities from the GeoGebra file and shared their works both with dynamic and visual features. During the lesson, students also learned the examples, drawings and exercises from the textbooks constructed with GeoGebra.

Instructional activities were given after the students had mastered the basic skills of using GeoGebra. Students must complete each instructional activity and submit them before the next lesson. Each instructional activity required students to apply higher order thinking skills to solve the questions. For example, in the activity, students were required to explore the construction of the interior angle of a regular polygon (pentagon). Next, they must write it out the command on the answer sheet given. After the exploration using GeoGebra, they had to describe what happened to the calculation of a regular polygon and how it relates to the solution given.

7. Evaluation

The activity sheet was distributed before the end of each lesson. The sheet contained 4 objective questions related to what they have done in the lesson. The related question would require students to do further exploration by themselves and discuss with friends in a group. The students were given 2 hours' time to complete each activity. A test was conducted two weeks after the last lesson. Questions were set up based on the textbook and students must answer on the question sheet. They were not allowed to refer to any textbook or notes. The test aimed at testing students' concept of angles that have achieved and also the skills that they acquired in using GeoGebra software. In the test, part I involved writing out the commands using GeoGebra and the possible outcome of that action while part II required students to solve mathematical problems according to the textbook syllabus.

8. Survey Summary and Analysis

In the final lesson, a survey was given out to the students. There were a total number of 40 pupils in this study. The survey questions given to students were:

1. Did you had a good understanding of the concept of angle before taking the lesson?
2. Did you have a high level of interest of using GeoGebra software before taking the lesson?
3. Did you have a high anticipation to learn the concept of angle with the aid of GeoGebra before taking the lesson?
4. Did you feel that learning and using GeoGebra in learning the concept of angle could be useful before taking the lesson?
5. Did you appreciate the usage of GeoGebra in learning the concept of angle after taking the lesson?
6. Has your understanding about the concept of angle improved after taking the lesson?
7. Has the GeoGebra enhanced your understanding of the topics in mathematics explored after taking the lesson?
8. Has your experience working with GeoGebra during the past two weeks given you an insight into the importance of integrating technology into the classroom after taking the lesson?

Table 1. Percentage respondents to GeoGebra survey

Item	1(%)	2(%)	3(%)	4(%)	5(%)	6(%)	7(%)	8(%)
SA/Agree	80.00	77.50	82.50	75.00	92.50	90.00	90.00	95.00
Not Sure	10.00	15.00	22.50	22.50	7.50	10.00	10.00	5.00
SD/Disagree	10.00	7.50	0.00	2.50	0.00	0.00	0.00	0.00

Based on the results shown in Table 1, the pupils' understanding on the topic of Geometry was good in the beginning before going through the exploration of GeoGebra. After using GeoGebra for the past two weeks, 90.0% of pupils agreed that they gained better understanding about the concept of angle and also enhanced their understanding through exploration. Furthermore, 92.5% of pupils appreciated the usage of GeoGebra and 95% realized the importance of integrating technology into the classroom after using GeoGebra.

Other important views are:

- Pupils felt that the exploration with GeoGebra gave them a chance to construct the geometric shapes and enabled them to check and prove all the features with the program itself.
- Pupils felt that GeoGebra can give them opportunity to prove and observe each construction conditions of geometric features for each activity.
- Pupils enjoyed to learn and explore the concept of angles with using the GeoGebra after they mastered the basic skills of GeoGebra.
- Pupils gained better understanding on the concept of angle after they were able to visualize the geometric shapes effectively.
- Pupils felt that GeoGebra software is useful in the teaching and learning process as it helps to promote the understanding of abstract concepts.
- Pupils are excited to learn all topics of mathematics using GeoGebra in their own classroom as it attracts them to do further exploration by themselves.
- Pupils felt that it was fun in learning mathematics by this technology in the classroom and requested to have more activities or practice on other mathematics topics in future.

9. Summary and Conclusion

Most of the pupils gave comment after they survey about they felt 2 weeks were not enough for them. They wished to have more time allocated to use GeoGebra to construct and develop further knowledge about the topic they learned. However, pupils were told that they could visit the GeoGebra website as this software can be downloaded for free. Therefore, students could use it at home to do their own exploration and also can share their works on the website. Through these instructional activities, researchers found out that pupils now liked the GeoGebra to learn mathematics and they had more understanding on the concept of angle. Furthermore, the use of GeoGebra helps pupils to think higher, explain about how to do and know why they need to do to arrive at the mathematical solution. Based on the survey, pupils showed positive feedback about using the GeoGebra to learn the concept of angle in Geometry.

In conclusion, we had discussed how to use GeoGebra to teach the concept of angle in Geometry. To enhance the teaching and learning process, teachers should always find time to explore the construction of each regular or irregular polygon and write down the steps so that their students can follow using their tools. The GeoGebra website also provides teachers with GeoGebra files. Teachers are encouraged to open the GeoGebra files, like “Hexagon Construction.ggb”, “Triangle Construction.ggb” or “Square Construction.ggb” to gain some ideas on how to use GeoGebra as teaching tools in mathematics. These GeoGebra files show all the detailed steps of construction and are very useful for teachers to use and display the simple construction steps for students to learn and follow.

GeoGebra brings a new dimension into the teaching of primary school geometry in Malaysia. By using dynamic software such as GeoGebra in the teaching and learning process, the concept of angles can be developed more effectively. Besides that, the graphical representation of angles can be constructed more attractively and creatively compared to the traditional method using paper-pencil only. Therefore, constructing angles using GeoGebra is a very important part of developing the concept of Geometry among the primary school students.

References

[1] Preiner, J. (2008). Introducing dynamic mathematics software to mathematics teachers: The case of GeoGebra.na.

[2] Laborde, C. (2014). Interactivity and flexibility exemplified with Cabri. In Plenary lecture given at the Asian Technology Conference in Mathematics.

[3] Pannen, P. (2014). Integrating technology in teaching and learning mathematics. In Electronic Proceedings of the 19th Asian Technology Conference in Mathematics. Yogyakarta: Indonesia.

[4] NCTM. (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

[5] Hohenwarter, J., Hohenwarter, M., & Lavicza, Z. (2009). Introducing Dynamic Mathematics Software to Secondary School Teachers: The Case of GeoGebra. Journal of Computers in Mathematics and Science Teaching, 28(2), 135-146.

[6] Budai, L. (2011). GeoGebra in fifth grade elementary mathematics at rural schools. In Annales Mathematicae et Informaticae (Vol. 38, pp. 129-136).

[7] Hohenwarter, M., Hohenwarter, J., Kreis, Y., & Lavicza, Z. (2008). Teaching and calculus with free dynamic mathematics software GeoGebra. In 11th International Congress on Mathematical Education.

[8] Domènech, N. I. (2009). Influence of dynamic geometry software on plane geometry problem solving strategies (Doctoral dissertation, Universitat Autònoma de Barcelona).

[9] Furner, J. M., & Marinas, C. A. (2012). Connecting Geometry, Measurement, And Algebra Using Geogebra For The Elementary Grades. In Twenty-fourth Annual International Conference on Technology in Collegiate Mathematics (pp. 63-72).

[10] Dogan, M., & İçel, R. (2011). The role of dynamic geometry software in the process of learning: GeoGebra example about triangles. Journal of Human Sciences, 8(1), 1441-1458.

[11] Saha, R. A., Ayub, A. F. M., & Tarmizi, R. A. (2010). The effects of GeoGebra on Mathematics achievement: Enlightening coordinate geometry learning. Procedia-Social and Behavioral Sciences, 8, 686-693.

[12] Idris, N. (2006). Teaching and Learning of Mathematics, Making Sense and Developing Cognitive Ability. Kuala Lumpur: Utusan Publications & Distributors.

[13] Battisa, M. (1999). Geometry Results From The Third International Mathematics And Science Study. Teaching Children Mathematics, 5(6), (pp. 367-373): Reston,VA: NCTM.

[14] Shadaan, P., & Leong, K. E. (2013). Effectiveness of Using GeoGebra on Students' Understanding in Learning Circles. Malaysian Online Journal of Educational Technology, 1(4), 1-11.