Exploring Derivative Functions using HP Prime

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Abstract

Derivative is one of the essential topics in calculus, which is an important basic skill should be mastered before integration comes in. Learning derivative in higher education can be achieved by using interesting graphing-calculator based learning. In this paper, activities using HP Prime to explore derivative function are highlighted. The usage of Function and Statistics 2Var applets are emphasized and used in these activities. Through these activities, students learn how to sketch the graph of a derivative from its original function, through observation on the changes of slopes, and tangent lines in HP Prime graphing calculator.

1. Introduction

Calculus is a compulsory subject for engineering and mathematics students in higher learning institution. Derivative is one of the essential topics in Calculus. Students in Malaysia learn about the basic of derivative when they are taking additional mathematics subject in their secondary level of education. Nevertheless, the topic could be rather abstract and dry without graphic demonstration. Hence, we would like to share our activity in class to get the students engaged in learning about derivative.

According to a famous Chinese proverb "*I hear and I forget, I see and I remember, I do and I understand*", getting students involved in the learning process is important for them to understand the important concepts [1, 2]. As Edward Bolles ever mentioned that "*We remember what we understand; we understand only what we pay attention to; we pay attention to what we want.*" Once the concept is understood, it will be easier to be memorized. Since Calculus is an accumulative subject, basic knowledge is needed to continue the learning in this subject. If the students are able to memorize the concept, then they would face less difficulties in future learning in Calculus.

Quesada et al. [3] reported the success in teaching pre-calculus students using graphing calculator. The students learning with graphing calculators generally performed better than those who were using scientific calculators. In another study, students with graphing calculators outperformed their peers with traditional learning method in algebra [4]. In addition to that, graphing calculator had been reported to assist in the understanding of the concept of derivatives [5] and probability [6]. In this paper, we suggested the using of graphing calculator in learning calculus.

In a usual practice, students learn about derivative using the concepts of limits first. After that, they will be asked to memorize the derivative of certain functions. Here, a graphing calculator-assisted activity can be added before the learning of derivative. This activity requires the students to sketch the graph of derivative, follows by the identification of the derivative function. With graphing calculator, students can visualize the graphs from the functions, and by that, enhance the memorization of derivative functions. In this paper, Section 2 describes the activities done. Section 3 gives the suggestions on the activity and conclusion comes in the last section. HP Prime Graphing Calculator was used in the activity and Geogebra was used in producing the worksheet.

2. Design of Activity

The activity started with (1) the revision of the previous topic about slope, (2) finding slopes at different points and sketch the graph of derivative, and (3) summary of the process and enhancement of the learning.

2.1 Reviewing Previous Topic about Slope

In the beginning, students were shown a graph of polynomial, without telling them the function of the graph (Figure 2.1). Then, they were asked to identify the type of the polynomial. Later, they were required to determine the part(s) of the graph that produce positive, zero and negative slopes.

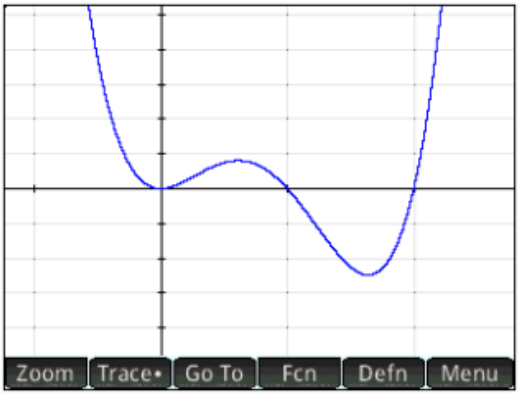

Figure 2.1 The first graph (polynomial) shown to students.

2.2 Finding Slopes and Derivative Graph Sketching

After that, a tangent line were shown with the slope value (Figure 2.2). The cursor was moved left and right to get the students to observe the changes of slope graphically and numerically.

Later, the equation of the graph was revealed to the students, and they were required to key in the function into F1(X) in Function aplet. Next, they were asked to find other slopes at different values of x using their graphing calculator. Then, they were required to record the slopes in the graphing calculator (Figure 2.3).

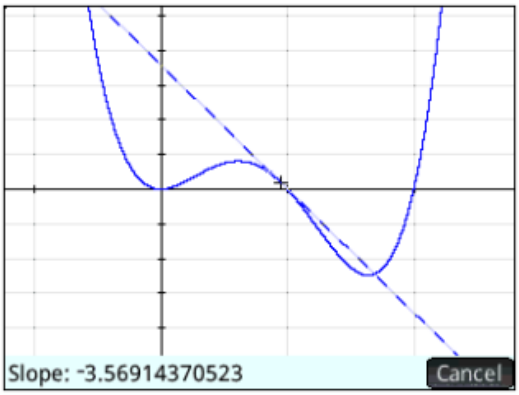

Figure 2.2 The slope is shown at the bottom left corner of the screen.

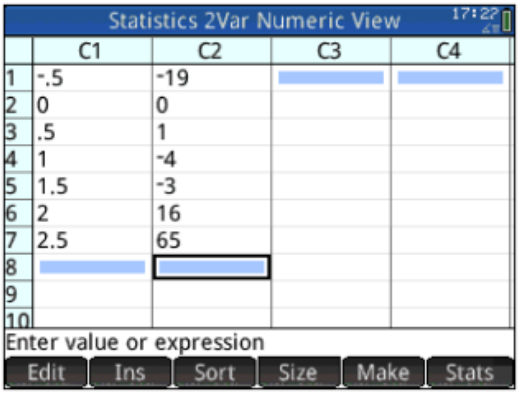

Figure 2.3 The points were keyed in taking column C1 for x and C2 for y'.

Afterwards, students were asked to plot the points of (x, y') as a scatter plot (Figure 2.4). From the scatter plot, they were first asked to indentify the type of polynomial the graph y'. Then, Fit key (Figure 2.5) was activated to connect the dots. As the default best fit option is usually the linear function, students have to choose the suitable type of best fit in the Symbolic View (Figure 2.6), and activate the Fit key in Plot View again to view the best fit (Figure 2.5).

Then, they were asked to copy the best fit function found in Statistics 2Var aplet into F2(X) in Function aplet (Figure 2.7). F2(X) was then plotted together with F1(X). From the graph of F2(X), the x-coordinate of the extremum of y (F1(X)) can be found by searching the x-intercept of y' (F2(X)).

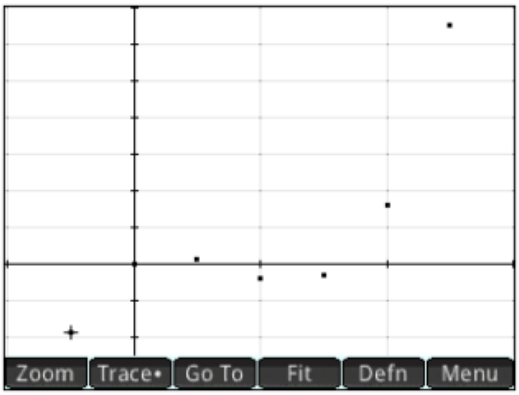

Figure 2.4 Scatter plot of (x, y').

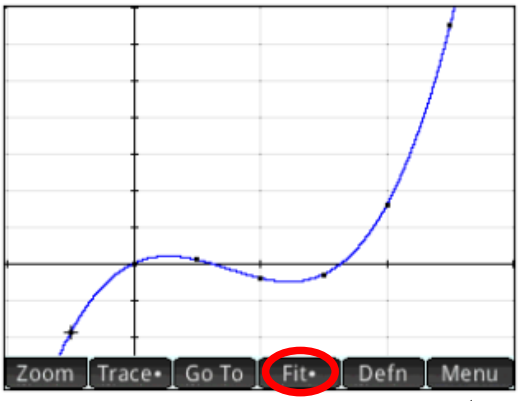

Figure 2.5 Best fit of the scatter plot of (x, y').

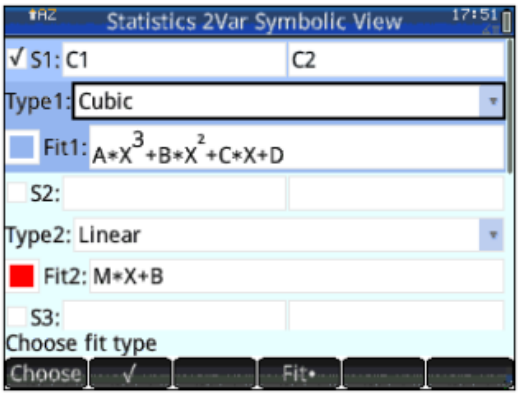

Figure 2.6 Best fit type for scatter plot of (x, y').

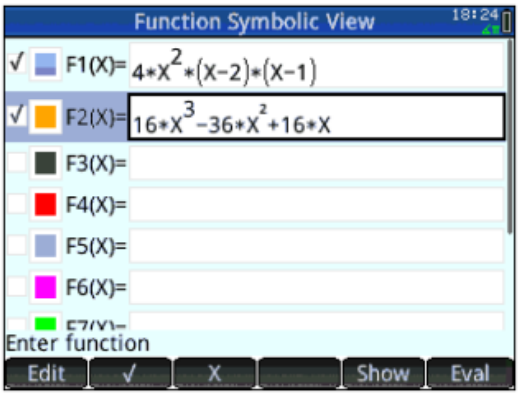

Figure 2.7 Best fit function for scatter plot of (x, y') was keyed in F2(X).

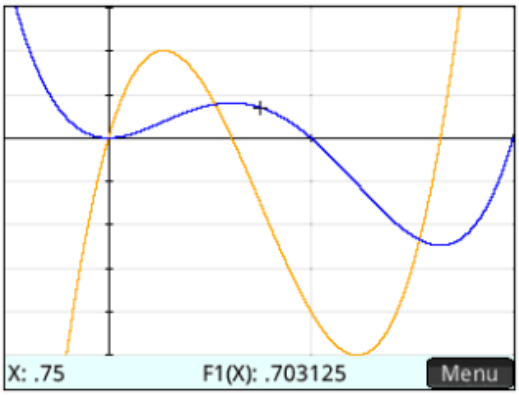

Figure 2.8 The graphs of y (blue) and y' (yellow).

2.3 Summarize and Consolidate the Knowledge

At the end of the class, the whole process was summarized using the screens in Figure 2.9 by moving the cursor along the graph of $f(x)$ and asking the students to observe the changes of (a) the slope of y, and (b) the value of y' simultaneously.

To consolidate the concept, a worksheet with another type of function was given to the students, and they were required to repeat what they had done before this. In order to strengthen their findings, analysis questions were asked. Then, they were required to sketch a graph of derivative on the worksheet paper, without knowing the function of the given graph. Lastly, a bonus question was given for the students to sketch the graph of $f(x)$ through observation on the graph of $f'(x)$.

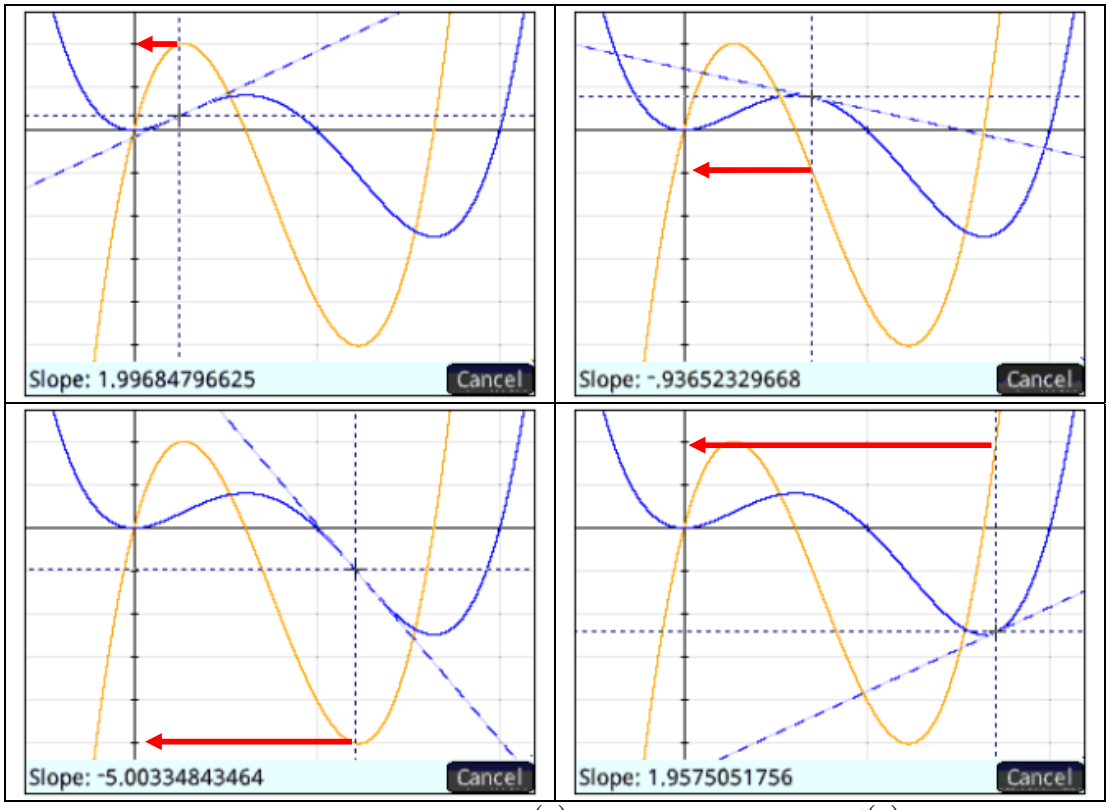

Figure 2.9 Screenshots of graphs of $y = f(x)$ (blue curve) and $y = f'(x)$ (yellow curve).

3. Recommendation and Suggestion

Students may need 15 minutes or more to complete the worksheet, thus it is suggested to allow the students to do it as a homework.

For the best learning process, this activity shall be conducted between the lessons teaching the concept of limit and derivative. Alternatively, this activity can be conducted after the students have learnt about the derivative and its application. This activity can be flexibly done in two sections as well, i.e. exploration (Questions 1 – 2) before teaching derivative, whilst analysis (Questions 3 – 4), after learning about application of derivative.

Different functions can be given in the worksheet (but limited to what the students have previously learnt), to prepare the students to learn about different derivative functions later. The sketching of derivative graphs will help them to memorize the derivative functions, e.g. the trigonometry functions which the students may confused by the negative sign. The bonus question can be given as preparation in teaching integration.

4. Conclusion

This activity is used to help the students to build up the ability to visualize the derivative of a graph and hence memorize the derivative more effectively. It also helps the students in graph

interpretation. With this ability, students are able to generate derivative graph from any given 2-D graph and also to analyze and interpret different type of 2-D graphs. This will lay a firm foundation when the students encounter topics of derivatives in higher dimension in the future.

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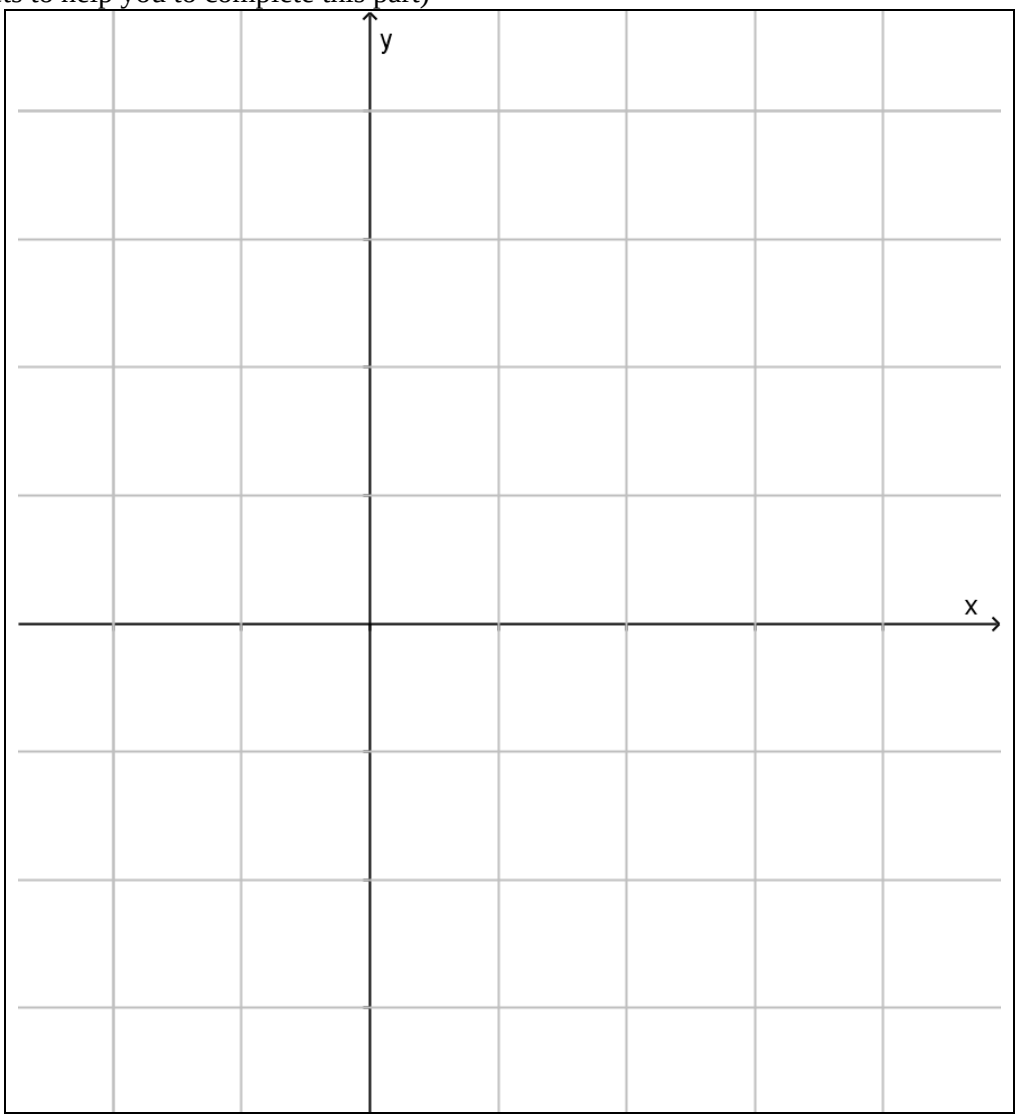
Worksheet

Given a function $f(x) = 2\sin x$.

1) Find the slope of the line of tangent at the following point. Record each the values of each row in your HP Prime using Statistics 2Var. Record the data using 3 columns: C1, C2 and C3.

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$f(x)$							
$f'(x)$							

2) Plot the points $(x, f(x))$ with 'o' and $(x, f'(x))$ with 'x' in the graph below. Then, connect the respective points with a smooth curve. (Hints: You may use both Function and Statistics 2Var aplets to help you to complete this part)



3) Fill in the blanks below:

a) As the slope of $f(x)$ increases, the graph of $f'(x)$ _____.

b) $f(x)$ has its extremum point at x = _____ and x = _____, where $f'(x)$ has its _____.

c) As the slope of $f(x)$ decreases, the graph of $f'(x)$ _____.

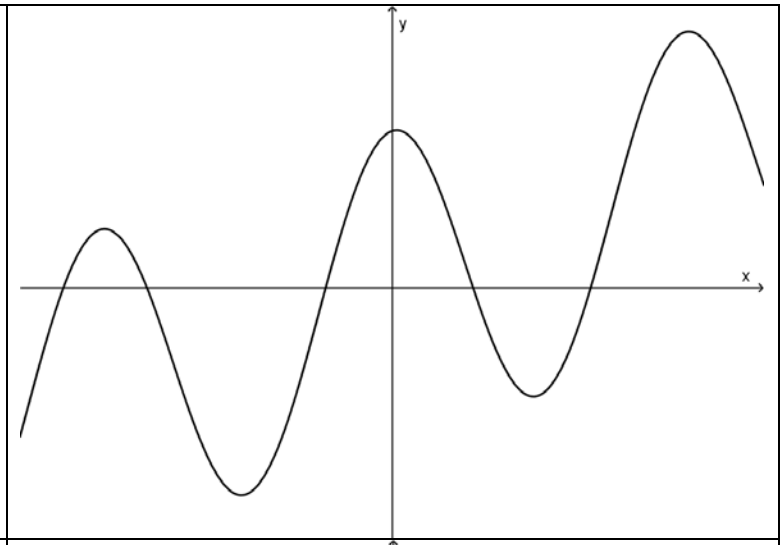
d) When $f(x)$ is a concave, the value of $f'(x)$ _____ as x increases.

e) When $f(x)$ is a convex, the value of $f'(x)$ _____ as x increases.

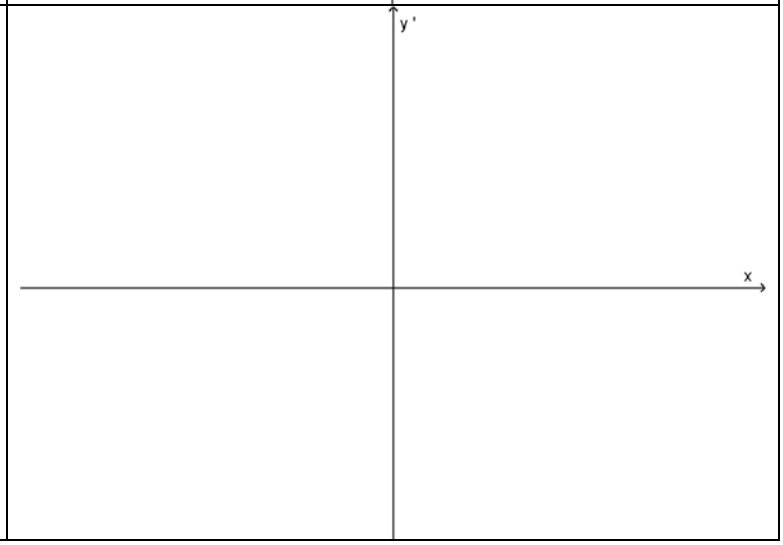
f) If $f'(x)$ reaches its extremum at $x = a$, $f(x)$ reaches its point of _____.

4) Given a new function as drawn at the *xy*-plane in part a). Sketch the graph of its derivative at the *xy*'-plane in part b).

a)



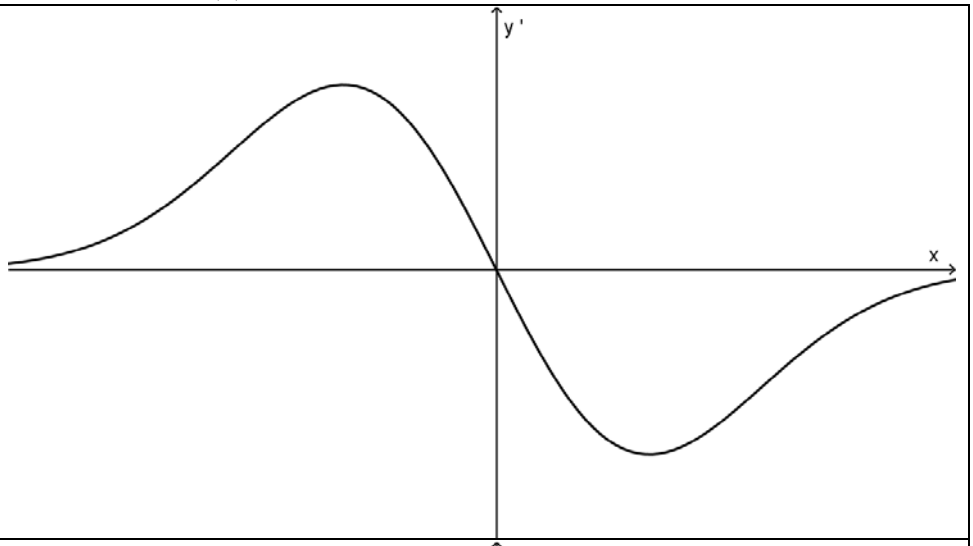
b)



Bonus Question

Given a derivative function $y' = f'(x)$ as drawn at the *xy*'-plane in part a). Sketch the graph of its antiderivative function $y = f(x)$ at the *xy*-plane in part b).

a)



b)