Data-based Introduction to Calculus

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Abstract

One course, one theorem! And latter is proved through data analysis of comics. It needs only 15min!

1. Introduction

Our working principle: Think Simple.

Rather than ask

To count 9 after 2

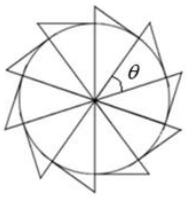
Better think

To count 2 after 9

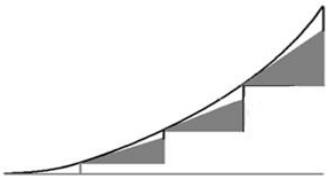
Minimalism: One Course, One Theorem!

$$\int_a^b f'(x)dx = f(b) - f(a) \quad \text{(Newton-Leibniz formula)}$$

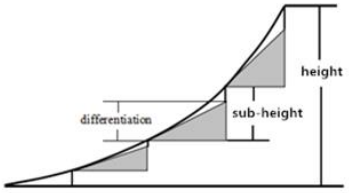
The course of calculus roots in this fundamental theorem. The original proof of this theorem, unfortunately, needs 80 class hours. It is proved now through the data analysis of comics



Circumference



Curve length



Curve height

$$\frac{\text{Arc length}}{\text{Tangent length}} = 0.999\cdots \qquad \frac{\text{Tangent length}}{\text{Sub}-\text{curve length}} = 0.999\cdots \Leftrightarrow \frac{\text{Differentiation}}{\text{Sub}-\text{height}} = 0.999\cdots$$

It needs only 15 mininutes.

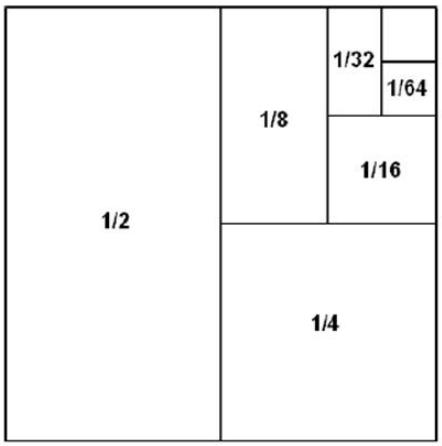
Speedway

Blitzkrieg

We follow the data-based introduction, just like the data-based news style recently adopted by CCTV.

2. Data-based Review of Zhuangzi's Saying

Great oaks from little acorns grow – calculus based on the concept of infinite.



https://en.wikipedia.org/wiki/1/2_%2B_1/4_%2B_1/8_%2B_1/16_%2B_…

Zhuangzi (~300 B.C.):

Take half from a foot long stick each day, you will never exhaust it in million years.

Tear 1 × 1 paper into 1/2, 1/4, 1/8, 1/16, ⋯ pieces.

Put together all the pieces,

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=1$$

This process brings about the infinite concept and infinite arithmetic.

Stepping from finite to infinite is a giant leap!

Data-based review of the process :Collect the data.

#Terms	Sum	
4	0. 9375	(one 9)
7	0. 9921875	(two 9's)
10	0. 9990234375	(three 9's)
14	0. 99993896484375	(four 9's)
17	0. 99999237060546875	(five 9's)
……	……	
34	0. 9999999994179233. . .	(ten 9's)

PS: Gold's purity is also through the data-based view, the percentage of 9, 99, 999, 9999. They are incredibly similar!

Concern: Would the process of adding 9 stop after a certain number of 9's?
Answer: No! If 9 does not keep being added, it could not reach 1. It must have infinite numbers of 9's.

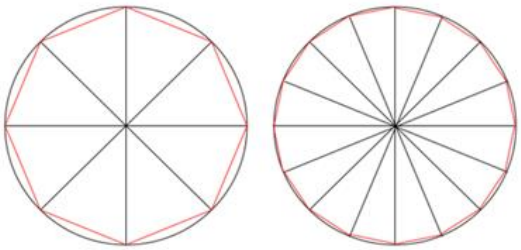
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=0.\dot{9}$$

$$0.\dot{9} = 1$$

The data-based principle can be a prerequisite for learning calculus.
Next, as we enter the core part of calculus, the models will become complex.

3. The First Battle: Circle and Polygon

Calculus originates from measuring the circumference.
Length could be measured by a ruler. How about the curve?
The circle drawn by computer, if magnified, becomes a regular polygon.



Secant line approximation
(Lower approximate)

This is the cyclotomic method of Archimedes and Liu Hui.
Observation: The chord length and the arc length get closer and closer.
Data-based review of the process: Collect the data.

N-Polygon	Perimeter	Percentage
6	6	0. 9549296586
7	6. 0743723476	0. 9667663853
8	6. 1229349178	0. 9744953584
12	6. 2116570824	0. 9886159295
14	6. 2305861508	0. 9916285843
16	6. 2428903046	0. 9935868512

Percentage:

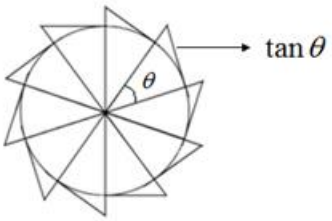
$$\frac{\text{chord length}}{\text{arc length}}\left(=\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}}\right)=0.999\cdots\longrightarrow\frac{\text{Sum of chord lengths}}{\text{Circumference}}=0.999\cdots$$

Infinite number of 9's will be added, as $0.\dot{9}(=1)$ or numerator = denominator.

Define

Integral of chord length = Circumference

An important step: simplify the secant line to tangent line (Only related to a single tangent point)



Tangent line approximation
(Upper approximate)

Observation: the tangent length and the arc length get closer and closer.

By similar process, collect data, then observe the percentage,

$$\frac{\text{Arc length}}{\text{Tangent length}}\left(=\frac{\theta}{\tan\theta}\right)=0.999\cdots\rightarrow\frac{\text{Circumference}}{\text{Sum of tagent lengths}}=0.999\cdots$$

Infinite number of 9's will be added, as $0.\dot{9}(=1)$ or denominator $=$ numerator.

Define

Integral of tangent length = Circumference

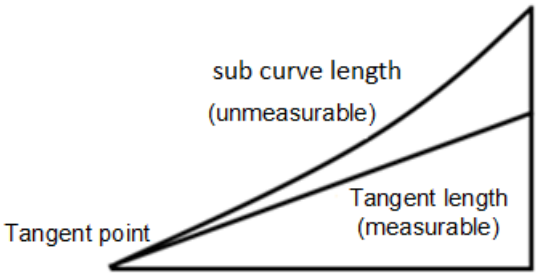
The above definitions cannot give the exact value, but always the approximate value. This has been a historical puzzle.

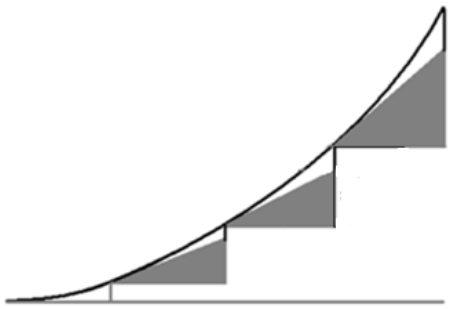
4. Curved Triangle

Let us extend from the circle to the general curve (e.g. hillside).

The curve length is unmeasurable. We may change it to straight line.

The curved triangle can be changed to straight triangle, e.g. the tangent triangle with hypotenuse tangent to the curve.





Similar as last lecture about circle's tangent (i.e. collect data, then observe percentage)

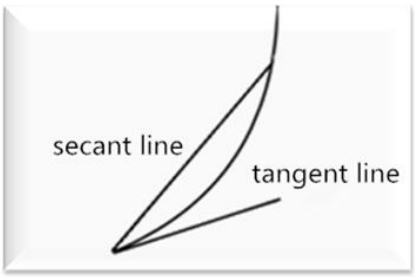
$$\frac{Tangent\ length}{Sub\ curve\ length}=0.999\cdots(\text{Assumption})\xrightarrow[by\ contradiction]{quick\ proof}\frac{Sum\ of\ tangent\ lengths}{Total\ curve\ length}=0.999\cdots$$

Infinite number of 9's will be added, as $0.\dot{9}(=1)$ or numerator = denominator.

Define Integral of tangent length= Curve length

This is an extension of circle's circumference.

PS: Why is the curve length calculated based on tangent line rather than secant line? Our daily experience suggests to use pace to measure, i.e. the secant length step by step. When the pace becomes shorter and shorter, the secant length would become identical to the tangent length. The tangent length can be measured from the tangent point, without need to consider the other end point.

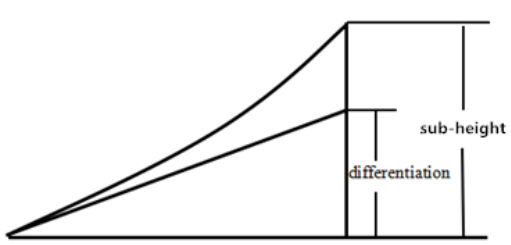


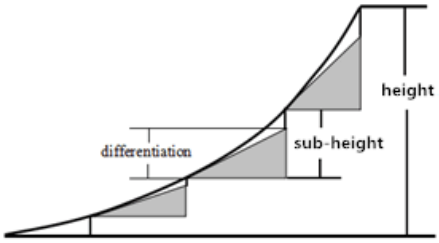
5. Calculate Height of Hillside

No need to start all over again. Simply take the isomorphism and change one to the other.

Calculate Length of Hillside $\overset{\text{Isomorphism}}{\longleftrightarrow}$ **Calculate height of Hillside:**

Convert the word "length" to "height"





Percentage:

$$\frac{\text{Differentiation}}{\text{Sub}-height} = 0.999\cdots(Definition) \xrightarrow[\text{by contradiction}]{\text{quick proof}} \frac{\text{Sum of differentiations}}{\text{Height of hillside}} = 0.999\cdots(\text{Integrability Theorem})$$

Data Collection When the hillside is a parabolic curve:

#Subintervals	2	4	10	20	100
$\frac{\text{Sum of differentiations}}{\text{Height of hillside}}$	0.5	0.75	0.90	0.95	0.99

The percentage also has one 9, two 9's, three 9's, four 9's …

Infinite number of 9's will be added, as $0.\dot{9}(=1)$ or numerator = denominator. Define

Integral of differentiations = Height of hillside

Here the integral is deterministic and finite. It could be exact value instead of approximate, thus solved the historical puzzle (e.g. circumstance). It is called the

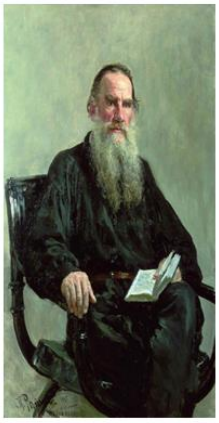
fundamental theorem of calculus.

It is enough to teach students the fundamental theorem of calculus, while leaving others (e.g. formula of circumference, Taylor's expansion) as exercises. Students when working on exercises will get to know the power of the fundamental theorem. Thus saves the class hours.

Teachers save even more time when the fundamental theorem of calculus is proved through data analysis of comics. It needs 15 minutes only, instead of 80 class hours. How efficient and effective!

6. Data-based Introduction of Literature and Art

The Mona Lisa by Leonardo Da Vinci is so famous for being associated with the golden ratio (0.618).
The War and Peace by Tolstoy used the calculus language.



"Only by taking infinitesimally small units for observation (the differential of history, that is, the individual tendencies of men) and attaining to the art of integrating them (that is, finding the sum of these infinitesimals) can we hope to arrive at the laws of history."

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