

Simulating the Spirograph Works by the Geometer's Sketchpad

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Abstract

The spirograph is one of our childhood toys in China. It can help drawing unpredictable and amazing patterns simply by the rotation of the gears. In this article we construct its mathematical model using the Geometer's Sketchpad. We analyze how the spirograph works and try to simulate the drawing process on the computer. We get some beautiful patterns by applying the idea of iteration, and on this basis, we create a Square Spirograph powered by the Geometer's Sketchpad. In this process we feel the charm of mathematics and the practicability of modern techniques.

Introduction

In our school life we often use *the Geometer's Sketchpad*. The tracing function (Fig1) reminds us of the childhood toy, the spirograph (Fig2). This toy arouses our interests to think why it can help drawing such beautiful patterns and what its working theory is. Could we explain it with the help of the modern software, *the Geometer's Sketchpad*? The following is our studying process.



1 Learn the working theory of the spirograph

The spirograph is one of our childhood toys in China. It can produce unpredictable and amazing patterns simply by the rotation of the gears. We learn how it works by searching information. It consists of the parent ruler (the light green one in Fig2) and the child ruler (Fig3). Common parent ruler has an inner ring gear; the child ruler has an outer ring gear with pores. When operating, the child ruler is put inside the parent ruler, embedding the gears. Then just draw according to the pattern repeatedly in circular motion. In the process, the gears should always remain embedded. After doing so for a while, the spirograph will leave an incredible beautiful flower. Note that

tiny displacement of the child ruler will cause great changes in the finished pattern. Fig4 shows some beautiful patterns made by the spirograph.



Fig3



Fig4

2 Construct mathematical model by the Geometer's Sketchpad

First of all we set up the mathematical model. View the two rulers of the spirograph as circles of different radii (apparently the parent one is bigger) and the pen point as a point. When the child ruler rotates in the parent ruler, they should always attach to each other. How can we simulate it on *the Geometer's Sketchpad*? Actually this exactly means that the two circles are internally tangent, i.e., the child one inscribes the parent one. Aha! It is just the relation of positions of two circles. We can make an orbit and let the small circle move on it, and then we are done!

3 Drawing geometric graphs by the mathematical model

3.1 Drawing simple graphs by the mathematical model

3.1.1 Make the parent ruler

Set up the plane coordinate system in the graphing option in the main menu (Fig5). Also in this option, choose to draw a point on the axes, and we can set a point, namely A , on the x -axis. Suppose A has coordinate $(10, 0)$ and O is the origin $(0, 0)$. Join OA and draw a circle centered at O with radius OA . Denote the circle by a and this is the geometric graph corresponding to the parent ruler (Fig6).

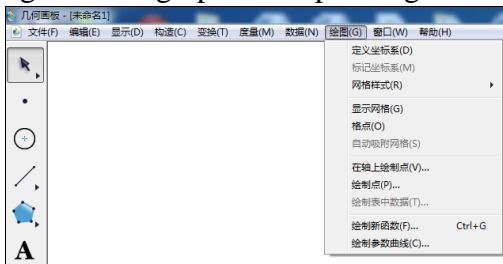


Fig5

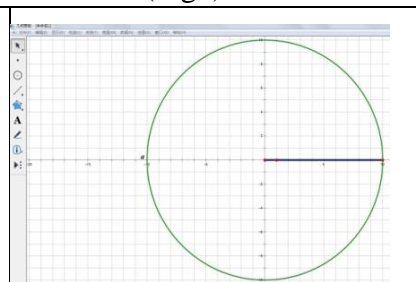


Fig6

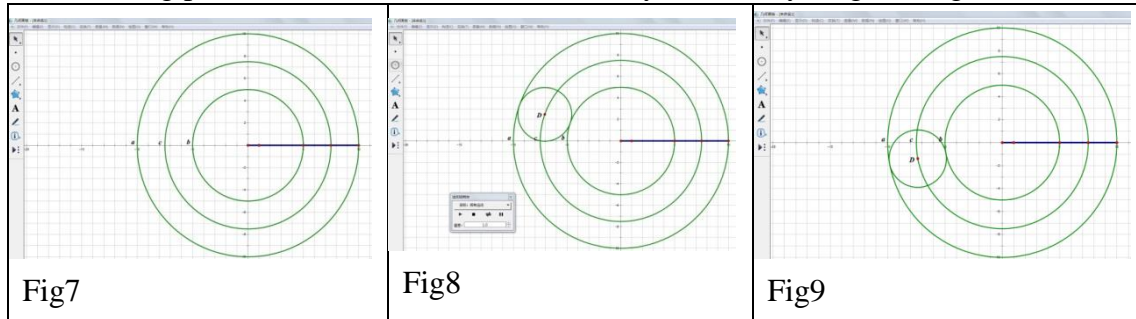
3.1.2 Make the child ruler

Let the smaller circle have radius 2.5. Now use the same function in the graphing option to get two points B and C , on the x -axis with coordinate $(5, 0)$ and $(7.5, 0)$, respectively. Join OB and OC , and again draw circles centered at O with radii OB and OC , and denoted by b and c , respectively (Fig7). Now pick a point D on the circle c , and draw another circle centered at D with radius 2.5 (Fig8). Denote it by d and it is the geometric graph

corresponding to the child ruler.

3.1.3 Simulating the moving of the child ruler

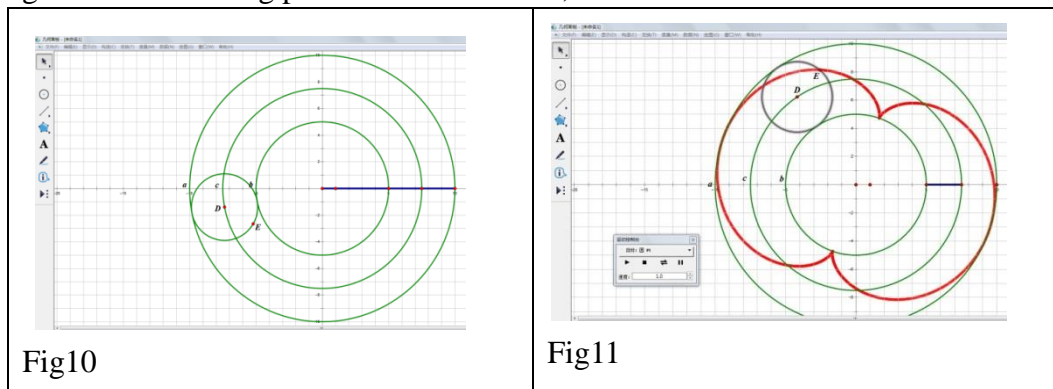
Select the circle d and we can generate its moving process by using the displaying function in the main menu. In this rotating process, the circles a and d are always internally tangent (Fig9).



We set the above numbers merely for simplicity. By no means are they the unique choices. In fact one can set the numbers as he or she likes, following our three-step drawing method.

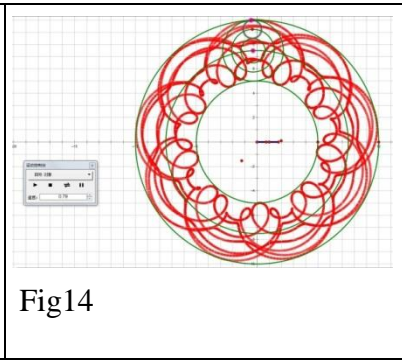
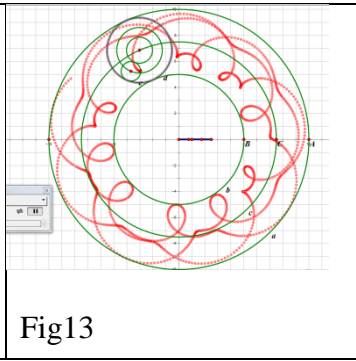
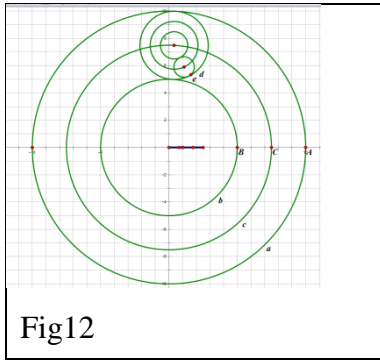
3.1.4 Drawing simple graphs by the model

Now keep the circle d static, and pick a point E on it. Select the point E (Fig10). We can trace it and generate its moving process by the two corresponding functions in the displaying function. Then we select the circle d and generate its moving process. After a while, we can observe the formation of some pattern (Fig11).

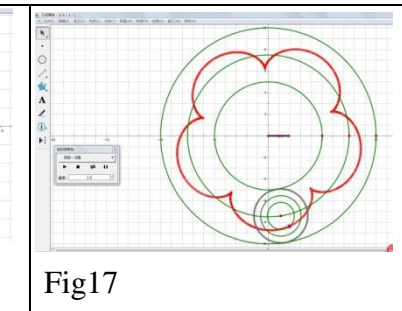
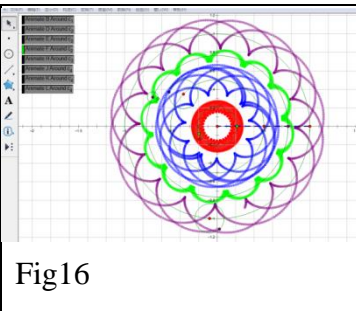
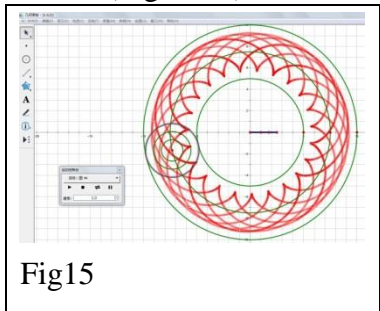


3.2 Drawing complex graphs by the mathematical model

The simple graphs can not satisfy our ambitions. We further hope to draw something beautiful like what is shown in Fig4. In order to add complexity, we think of using iteration. What will happen if we have another circle inscribe the circle d ? Can we get what we want? Following the idea of iteration, we draw another circle e inside the circle d (Fig12, with the same method before). Now the circle d is the bigger one and the circle e is the smaller one. Let them move and we see the pattern in Fig13 being generated. The final pattern is shown in Fig14.



Through iteration, we draw the following beautiful patterns by changing the number of circles and the point being traced (Fig15-17).



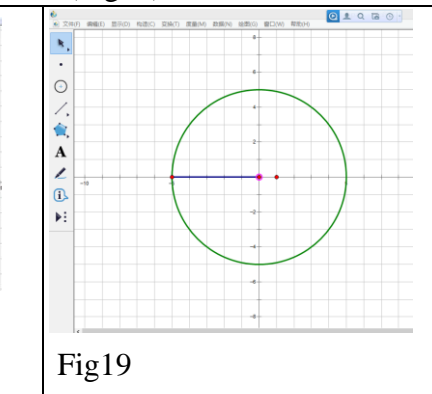
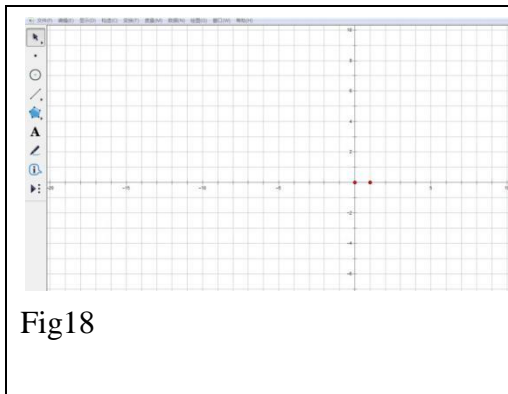
4 Creating new model: the *Square Spirograph*

In our preceding study, we find that the circles are of great importance. It reminds us to think what if we replace all the circles with squares. Therefore by modification we build a new model, called the *Square Spirograph*.

4.1 Make the *Square Spirograph*

4.1.1 Drawing the circumcircle of the parent ruler

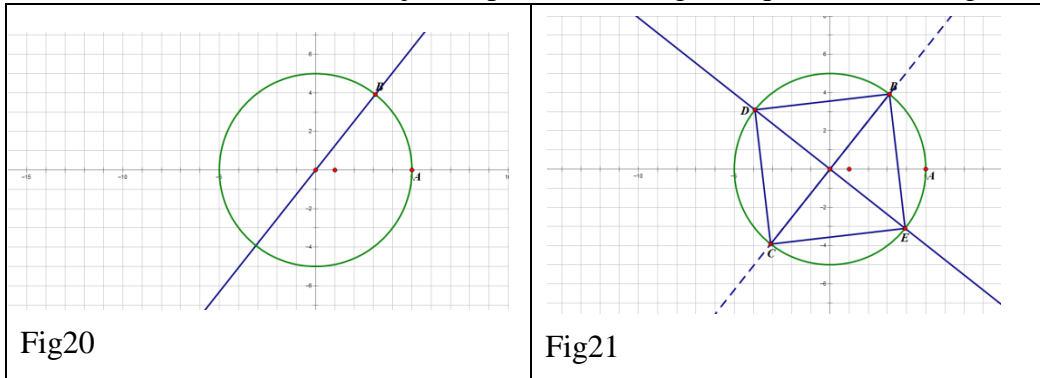
Set the coordinate system as before (Fig18) and pick a point A on the x -axis with coordinate $(-5, 0)$. Join OA and draw a circle centered at O with radius OA (Fig19).



4.1.2 Make the parent ruler of the *Square Spirograph*

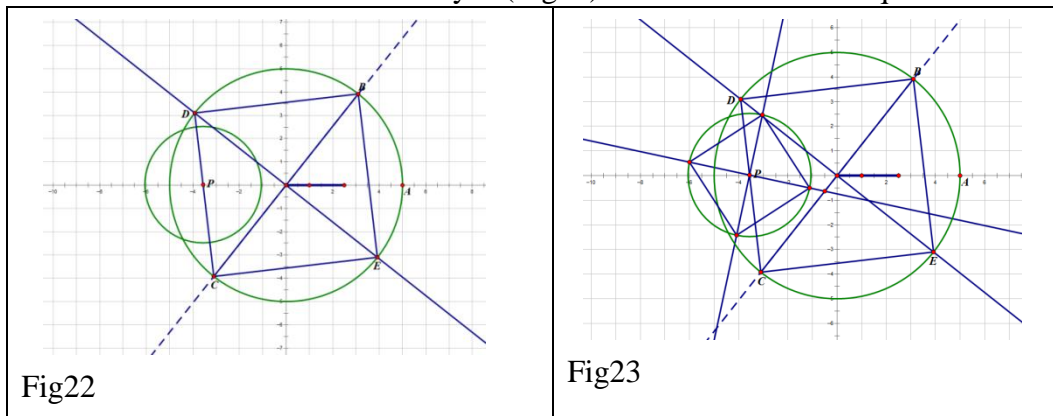
Select the circle and use the constructing function in the main menu to pick a point B on it. Then select O and B and make a line by the constructing function again (Fig20). Denote by C the other intersection of the line and the circle. Join BC and make the perpendicular line of it passing O . Denote by D and E the two intersections of the

line and the circle. Join the adjacent points and we get a square $BCDE$ (Fig21).



4.1.3 Make the child ruler of the *Square Spirograph*

Choose one side of the square and pick a point P on it by the constructing function. Draw a circle centered at P with radius 2.5 and denote it also by P (Fig22). Make an inscribed square of the circle P as before (Fig23).

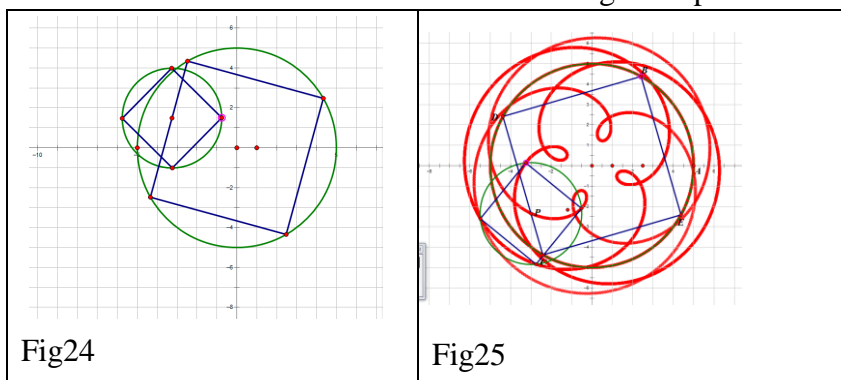


4.1.4 Hide the unnecessary lines

Selecting the objects that we want to hide and clicking the right-hand button, we can hide them to simplify the figure (Fig24).

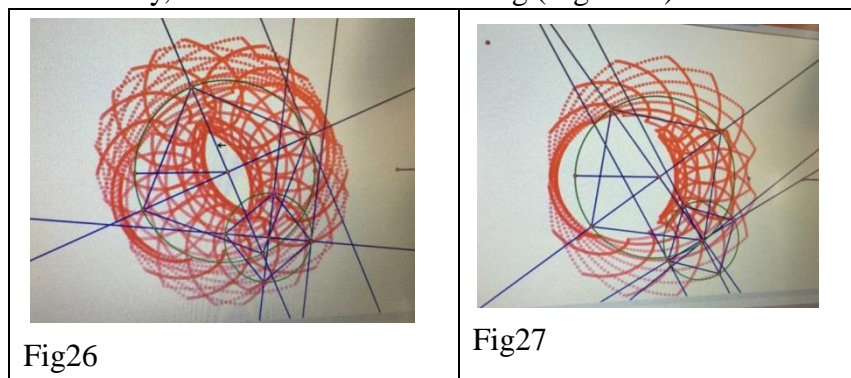
4.1.5 Draw graphs using the model

Select the two points on the circle and generate the moving process. We can observe that the two squares rotating in the circles. Trace the four vertices and we get the pattern shown in Fig25.



4.2 Draw other patterns by the *Square Spirograph*

In this way, we also draw the following (Fig26-27).



Results and reflection

In our study, our group has the following experience. We first learn how the spirograph works, and then build a mathematical model for it in *the Geometer's Sketchpad*. After that we use the model to draw patterns by the idea of iteration, and at last we create a new model. Here are our results.

First of all, we know more about the modern tool, *the Geometer's Sketchpad*, and gradually become more skilled when using the software. Moreover, it solves our doubts and questions in our life, giving us the opportunity to enjoy the study and our interests in mathematics. The most successful part of this research is that it inspires our group to think actively and view our childhood toys in a perspective of modern technology, successfully simulating how the spirograph works.

Secondly, this study on the spirograph works also gives us a better understanding of its working principle. The theory is quite simple: let a small inscribed circle rotate along a big circle, and then the trajectory of any fixed point on the small circle gives the simplest and most primitive patterns. Based on this principle, more complex patterns can be formed by adding smaller circles. This research gives us a more profound understanding to this seemingly simple toy.

In addition, this study also deepens our understanding of the circles. The key reason for the successful simulation of the spirograph works is that we are able to abstract such concrete issue and apply our knowledge of circle, such as the relation of the positions of two circles. Then the problem becomes simple and reachable so that we are able to solve it and write an article.

Finally, by simulating the spirograph, we also design our own version of spirograph, creating a new model. We make the *Square Spirograph*. Moreover, the spirograph is not limited to squares: we can also design pentagon spirograph, hexagon spirograph, and in fact n -polygon spirograph. If we go from finite to infinite, then the regular polygon goes to a circle, back to our primitive spirograph.

In the future our group would like to study further: we are considering applying the knowledge of ellipse to the issue, replacing the child ruler with an ellipse. We look forward to simulate more pretty patterns. We believe that we can move on in this topic through further learning.

References

- [1] Jiang, Y. (2011). *The Geometer's Sketchpad 5.0: From Primer to Master*. Sun Yat-Sen University Press
- [2] Wang, C. (2008). *A Course in the Geometer's Sketchpad*. Central China Normal University Press