Figure Drawing using KETCindy and its Application to Mathematics Education
– Practical example of application of mathematics to mathematics –

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Abstract

Geometric construction normally means generation of a gure suited for given conditions using rulers and a pair of compasses only for a nite number of times. Hereinafter, this is referred to simply as geometric construction. This paper presents a discussion of, in addition to geometric construction, the practice of drawing gures by adding mathematical contents.

When mathematical material is added to geometric construction using rulers and compasses, the use of dynamic geometry (DG) software is one option, whereas KETCindy is used for this study because KETCindy is equipped with DG′s Cinderella as GUI and can be used for drawing gures by Script as CUI. Therefore, mathematically precise gures can be drawn with ease, producing beautiful results. This paper explains gure drawing while the quadratic curve concept is added to geometric construction. The author considers that gure drawing by Script is extremely useful for mathematics education from the viewpoints of application of mathematics to mathematics. This point will be discussed hereinafter.

1 Introduction

Quite a few teachers use LaTeX for the production of handouts to be used for math classes and test questions. The author encountered teachers who were struggling to nd and use a system that can output precise and good-looking gures and graphs. The author tested several software packages and now uses KETCindy because it can output precise and good-looking gures and graphs by simple manipulations.

One bene t of KETCindy is that it uses dynamic geometry software Cinderella as the GUI, thereby allowing visual operation. Regarding GUI operation, although operating environments are almost identical to those of other software, an important bene t of Cinderella is that the generation of gures and graphs is made possible by a Character User Interface (CUI).

An additional bene t is that necessary commands and function formulae can be added by programming[1]. Figures in Japanese mathematics[2], which might be drawn only slightly, can be drawn now to a greater degree by the addition of mathematical contents such as quadratic curves.

In addition, with KETCindy, the quality of gures and graphs is equal to or greater than those depicted in the textbook. Furthermore, gures are generated precisely by simple manipulation. Results thus obtained can be output beautifully by LaTeX. Thanks to this system, the quality of gures and graphs used in the data for author′s class and academic papers were improved remarkably. The prime reason why the author uses KETCindy in LaTeX lies here.

All gures shown in Chapter 2 are drawn by KETCindy. The high quality of these gures is readily apparent: they are equal to or greater in terms of quality than those depicted in the textbook. Figure drawing in which Script is used concomitantly is explained taking Japanese mathematics problem as an example. At the same time, the bene ts of rendering gures using LaTeX and KETCindy are reported. In Chapter 3, future tasks of mathematics education [1] and fostering of mathematics teacher will be discussed based on the contents of Chapter 2 [2]. In the Appendix, the contents of the Encyclopedia of Geometric Solution [3], which presented the problem consciousness for this paper, are cited.

2 Quality gures drawn using Script and KETCindy

Wasan dealt with numerous problems related to gures [4]. If a drawing meeting with the conditions is given, then the solution itself leading to an answer is not so di cult.

However, in some cases, the generation of a gure is di cult even if the conditions are being given. Here, example problems 1, 2 and 3 shown below are addressed [5]. In fact, the center of circle P can be drawn by adding a quadratic curve concept to gure drawing using rulers and compasses. This illustration presents a quadratic curve drawn using Script. Beautiful drawings can be output by KETCindy.

Ex 1: Find the length of one side of a square when the diameter of circle P is 1.

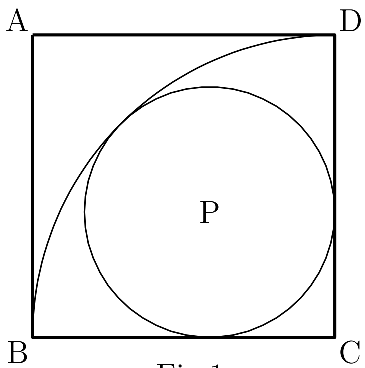

Fig.1.

Ex 2: Find the length of one side of a square when the diameter of circle P is 3.

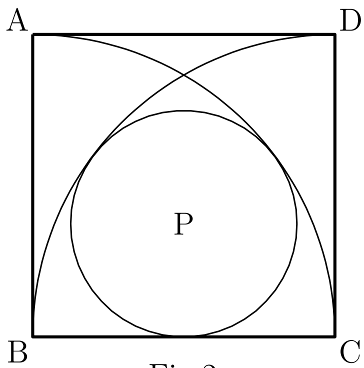

Fig.2.

Ex 3: Two large circles Q and two small circles P are shown in outer circle O. When the diameter of the small circle P is 2, nd the diameter of the large circle.

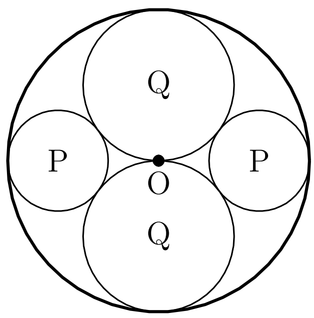

Fig.3.

[1] Function formula capable of creating parabola, ellipsoid, hyperbolic curve, and symbols showing that the lengths of line segments are equal are added by the program.

[2] Japanese mathematics: Wasan which was developed during the Edo Period (1603 – 1867).

2.1 Figure drawing of Example 1: Utilization of a parabola

Example 1 : As illustrated at right, in square ABCD, circle P is inscribed in quadrant C having its center at C, and inscribed on side BC and side CD.
Find the length of one side of the square when the circle diameter is 1.

Answer : The length of one side of the square is $\frac{1+\sqrt{2}}{2}$.

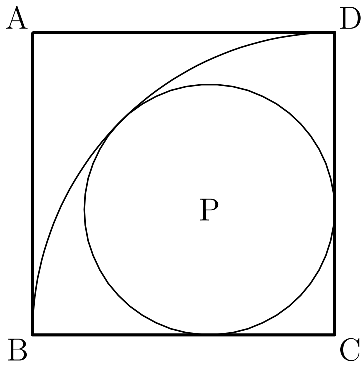

Fig.1.

2.1.1 Procedures for drawing circle P

Procedures for drawing circle P shown in Example 1 are the following.

1. Center P of the circle passes through intersection E of quadrant C and diagonal line AC.
2. Center P of the circle draws a parabola while E is the focal point and side BC is the directrix.
3. Intersection of a parabola and diagonal line AC is the center P of the circle to be obtained.

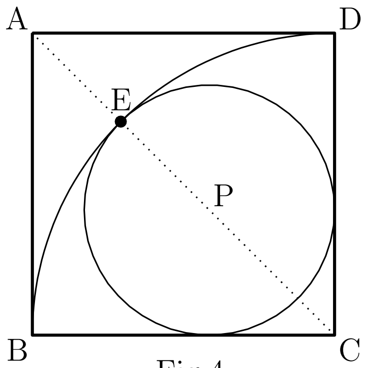

Fig.4.

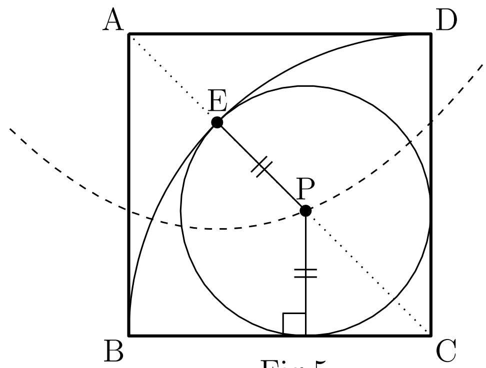

Fig.5.

2.1.2 Example of Script of Example 1

Here, the example of Script of Fig.5 is presented.

1:Fhead="Parabola.tex";	File name: Parabola.tex
2:Ketinit();	Initialization of KETCindy
3:Addax(0);	Coordinate axes are not drawn
4:Listplot([A,B,C,D,A],["dr,2"]);	Square ABCD is drawn by line 2
5:Listplot([A,C],["do,1"]);	Diagonal AC is drawn by dotted line
6:Circledata([C,B],["dr,1","Rng=[pi/2,pi]"]);	Quadrant C, the part of circle C from $\pi/2$ to π
7:Parabolaplot("1",[E,B,C],["da,1"]);	Parabola with focal point E and directrix BC
8:Putintersect("P","rt1para","sgAC");	Intersection P of parabola and diagonal line
9:Circledata([P,E],["dr,1"]);	Circle passing through center P and point E
10:Pointdata("1",E,["size=7"]);	Point E is shown by size 7
11:Pointdata("2",P,["size=7"]);	Point P is shown by size 7
12:Letter([A,"nw","A",B,"sw","B",C,"se", "C",D,"ne","D",E,"n2","E",P,"n2","P"]);	Points A, B, C, D, E, P are shown
13:Windispg();	Displayed on the display

Notes : The le name written on the rst line and the script shown above are written between the 2nd line and the 13th line. Then the gure can be output by KETCindy on Cinderella, which is the GUI. Furthermore, actual gures can be con rmed by PDF. For the alteration of symbols in the gure and the vertex, the gure might be changed directly returning to Cinderella or changed by Script. At the same time, a tpic le which can be inserted by LATEX can be generated by the name of the rst line. If the le name in the rst line is inserted under the layer environment, then a gure can be inserted at the desired point[3]. KETCindy includes expressions of various kinds used in the textbook from mathematics education viewpoints. For example, for the line type, line, dashed line, and dotted line available, each is designated by "dr,n", "da,n" and "do,n" and the thickness can be designated too. The script in the fourth line means a line and thickness 2. The default of the line type and thickness is "dr,1". Function formula Parabolaplot in the seventh line for drawing parabola is added by the program. With KETCindy, function formula and symbol can add functions to be generated as necessary. Those added as necessary are explained hereinafter in each case.

2.1.3 Bene ts of using KETCindy

As described previously, the author uses KETCindy to employ mathematically precise gures which are also quality printing materials for mathematics education. Quality printing materials can then be inserted into LATEX. Comparison of quality of the drawing between DG (a gure of Cinderella is used here) and a gure by KETCindy reveals the di erence between the two at a glance.

Furthermore, it might be cited that with KETCindy, the description of Script is mathematical and brief. For example, as represented by the seventh line of parabola Script in the statement above, the focal point and directrix are simply designated. At designation, the focal point and directrix are simply designated by symbols referring to the gure of Cinderella.

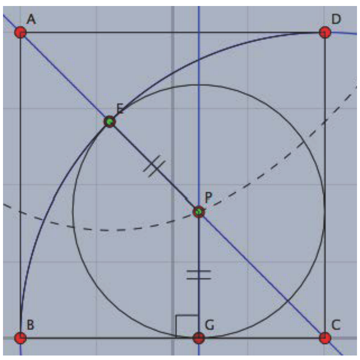

Fig.6.

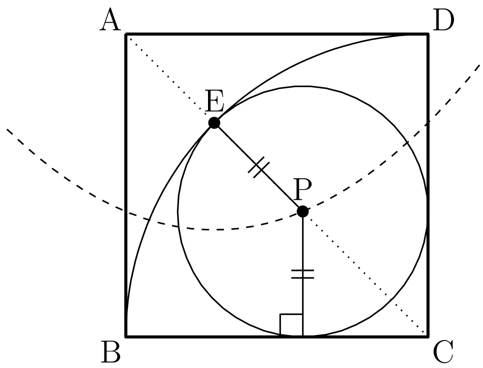

Fig.5.

The following points can be cited as bene ts of KETCindy:

1. Cinderella is useful as DG.
2. Precise and quality gures are drawn by DG and might be presented to students as printing materials.
3. Alteration of drawn gures is simple.
4. Program of KETCindy is mathematical and easy to understand.
5. It is freeware and can be introduced easily into school education.

[3] Details are shown in the Appendix provided at the end of this paper.

2.2 Figure drawing of Example 2: Utilization of ellipsoid

Example 2 : As illustrated at right, circle P is in square ABCD and is tangent to two quadrants B and C having its center at point B and point C and to side BC. When the length of the diameter of the circle is 3, nd the length of one side of the square.

Answer : The length of one side of the square is 4.

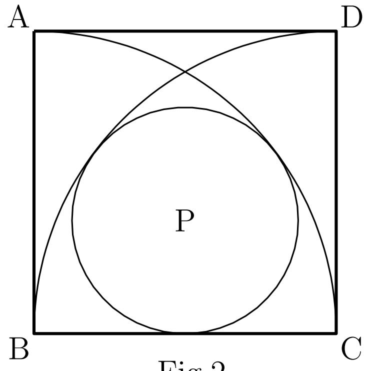

Fig.2.

2.2.1 Procedures for drawing circle P

Procedures for drawing circle P shown in Example 2 are the following.

1. Circle P passes through midpoint E of side BC.
2. Circle P is inscribed to quadrant B at point Y.
3. BP + EP = BP + PY = BY = BC (constant).
4. Circle P passes through point E inside of circle B (quadrant B). Therefore, point P is on the ellipsoid having focal point on two points B and E.
5. Center P of the circle is also on the vertical bisector of side BC.
6. Therefore, center P of the circle to be obtained is the intersection of the ellipsoid and vertical bisector.

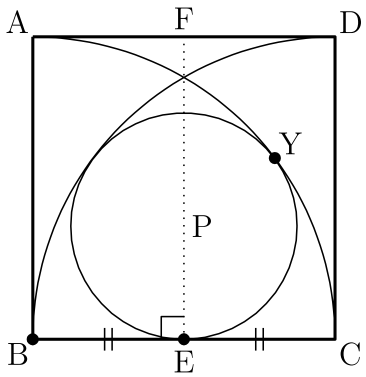

Fig.7.

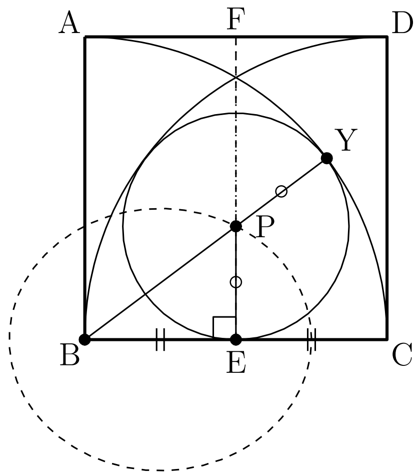

Fig.8.

2.2.2 Example of Script of Example 2

Here, the example of Script of Fig.8 is presented.

4:Listplot([A, B, C, D, A],["dr,2"]);	Square ABCD is drawn by line 2
5:Circledata([B,C],["dr,1","Rng=[0,pi/2]"]);	Quadrant B, the part of circle B from 0 to $\pi/2$
6:Circledata([C,B],["dr,1","Rng=[pi/2,pi]"]);	Quadrant C, the part of circle C from $\pi/2$ to π
7:Listplot([E,F],["do,1"]);	Line segment EF: Vertical bisector EF
8:Ellipseplot("1",[B,E,|B C|],["da,1"]);	Ellipsoid having two focal points B and E
9:Putintersect("P","rt1elp","sgEF");	Intersection P of the parabola and segment EF
10:Circledata([P,E],["dr,1"]);	Circle passing through center P and point E
13:Pointdata("1",P,["size=7"]);	Point P is shown by size 7
15:Drawsegmark("1",[B,E],["Type=2"]);	Mark showing equivalent line segment
16:Drawsegmark("2",[C,E],["Type=2"]);	Mark showing equivalent line segment

Notes : In Example 1, all statements of Script are shown. In Example 2, only major Scripts are shown. Therefore, some line numbers are missing. Drawsegmark shown in the 15th line and 16th line means that two line segments are equal as shown in Fig.7 and Fig.8. Namely, BE = CE. It is visually understood by this symbol that point E is the midpoint of side BC. This expression is used frequently in Japanese mathematics textbooks. In Fig.8, if results show that two points B and E are of focal points, then point P to be obtained should satisfy BP + EP = $2a$ ($2a$ is the sum of distance) as characteristics of the ellipsoid. It is important to recognize that it is the radius of quadrant B. If this is found, |B C| is simply designated in KETCindy. This is similar to vector notation and is mathematically simple.

2.3 Figure drawing of Example 3: Utilization of hyperbola

Example 3 : As illustrated at right, two large circles Q and two small circles P are shown in outer circle O.
When the diameter of small circle P is 2, nd the diameter of the large circle.

Answer : The diameter of the large circle is 3.

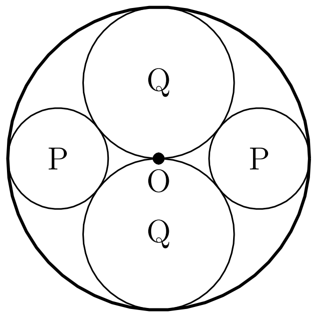

Fig.3.

2.3.1 Procedures for drawing circle P

Procedures for drawing circle P in Example 3 are the following. Two large circles Q are designated as Q1 and Q2. Two small circles P are designated as P1 and P2. Locations of the centers of two large circles Q1 and Q2 might be readily apparent. Procedures drawing center P1 and P2 of two small circles are explained hereunder.

1. Points of tangency of the circle P and circle O are designated respectively as A and B. The line segment AB is a diameter of circle O.
2. Circle P1 and circle Q1 are circumscribed at point Y.
3. Point P1 is outside of circle Q1.
4. |Q1P1 AP1| = |Q1P1 P1Y| = |Q1Y| = |Q1O| (constant).
5. Therefore, P1 is a hyperbola having two focal points of A and Q1.
6. The intersection of hyperbola and diameter AB is center P1 to be obtained.

Similarly, drawing of the center of small circle P2 is obtainable by the hyperbola.

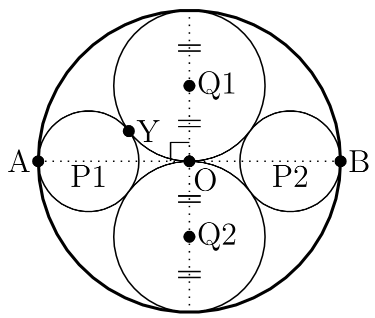

Fig.9.

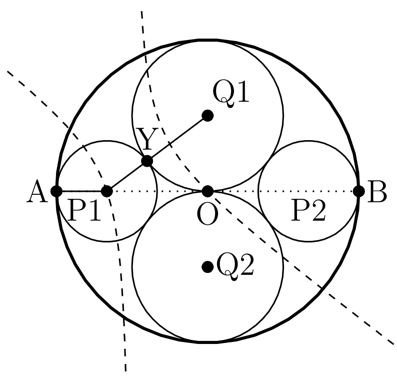

Fig.10.

2.3.2 Example of Script for Example 3

Here, the example of the Script for Fig.10 is presented.

4:Circledata([O,A],["dr,1"]);	Line type 1 passing through A at center O
5:Circledata([Q1,O]);	Circle having its center at Q1 and passing through O
6:Circledata([Q2,O]);	Circle having its center at Q2 and passing through O
7:Listplot([A,B],["do,1"]);	Line segment AB is shown by dotted line
8:Hyperbolaplot("1",[A, Q1, |Q1 O|]);	Hyperbola having focal points A and Q1
9:Putintersect("P1","rt1hyp","sgAB");	Intersection P1 of hyperbola and line segment AB
10:Hyperbolaplot("2",[B, Q1, |Q1 O|] ,["notex"]);	Hyperbola of focal points B and Q1 which is not shown in TEX
11:Putintersect("P2","rt2hyp","sgAB");	Intersection P2 of hyperbola and line segment AB
12:Circledata([P1,A]);	Circle having its center at P1 and passing through A
13:Circledata([P2,B]);	Circle having its center at P2 and passing through B
17:Pointdata("1",P1,["size=7"]);	Point P1 is shown by size 7

Notes : Only the major Script relating to Example 3 are shown. Therefore, some line numbers are missing. Regarding the drawing of the hyperbola shown in the eighth line, if results show that in Fig.10 two points of A and Q1 are focal points similarly to Example 2, then the point to be obtained should satisfy $|\text{AP1} \quad \text{Q1P1}| = 2a$ (where $2a$ is the di erence of distance) as characteristics of the hyperbola. An important matter is recognition that it is the radius of circle Q1. If this is found, then it is designated simply as $|\text{Q1} \quad \text{O}|$ in KETCindy.

For drawing of P2, a hyperbola having two focal points of B and Q1 shown in the tenth line should be generated. However, this will complicate Fig.10. Then, "notex" is included, which means that no drawing is provided at the end of the tenth line by the designated option.

3 Summary and Future Tasks

The problems of gure drawing shown in Examples 1, 2 and 3 taken up here are resolved by KETCindy, in which contents of the quadratic curve are added to geometric construction using rulers and a pair of compasses.

Generating a drawing using rulers and a pair of compasses only for a nite number of times is normally designated as geometric construction in the mathematical eld. According to the author, generation of a gure by adding mathematical contents to said gure drawing is extended geometric construction. When extended geometric construction is used, a regular heptagon that is impossible to draw accurately can be drawn with extended geometric construction manner by solving high-degree equations. Such educational material is regarded as e ective from the perspective of application of mathematics to mathematics. Extended geometric construction is to apply mathematics itself to mathematics and the author considers that from viewpoints of utilization of the learned contents; it is appropriate content for challenge learning in high school and for application of mathematics and mathematical exploration currently proposed.

Next, as represented by KETCindy, a system that can output generated results beautifully as the printed educational material is conducive to mathematics education for pupils and students. Simultaneously, it might be used as a means for research announcements by pupils engaged in

SSH research[4]. The author considers that the application of mathematics to problem-solving is urgent to meet the demands of society as well as curriculum guidelines. Therefore, the author intends to report research outcomes at international conferences in the form of an academic paper under the title of "Application of Mathematics to Mathematics" [1].

With the program used here for gure drawing, parabolas, ellipsoids, and hyperbolas can be generated by designating a focal point and a directrix in a similar manner as the de nition of quadratic curves. As a future task, such a program will need Script. To do this, some programming knowledge is necessary for mathematics teachers in secondary school so that they can solve their own problems as well as ICT applications. To that end, the ICT training system will become increasingly important.

Finally, to draw gures meeting the conditions shown in this paper mathematically precisely and beautifully, problems listed in the Encyclopedia of Geometric Solution (1959) [3] should be studied. Then, gures can be drawn easily and mathematically precisely. The author wonders why, although tips for extended geometric construction were listed in the literature published a half century ago, no extended geometric construction idea as introduced by the author has been devised. The reasons for this might be that teaching of quadratic curves was insu cient and that e ective application of ICT to mathematics education was not attempted actively. The author feels that ICT is not used extensively in present-day mathematics education. As a future task, we mathematical teachers should apply ICT aggressively. In this case, hardware and software should be modi ed so that our students might become familiar with them. Teachers should use them to a greater degree. It is desirable that they are able to perform basic mathematical programming of Cinderella and KETCindy introduced in this report. To do so, the author intends to develop and distribute valuable educational materials for mathematics using KETCindy.

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Appendixes

1. For gure drawing by Script in Examples 1, 2 and 3 in the Encyclopedia of Geometric Solutions [3], reference is made to Item 4 Trajectory of quadratic curve, Section 4 Problems relating to circles, Chapter 5 Trajectory. When results of this problem are used, circle P can be drawn by adding the quadratic curve concept to geometric construction by rulers and a pair of compasses as stated in this paper. Problem numbers 1440, 1444, and 1443

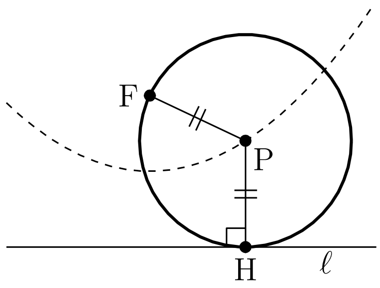
[4] Under the SSH program, in collaboration with universities and other institutions, high schools focus on science and mathematics in conducting experiential learning, research projects and curriculum development. The program aims to foster students with a high level of creativity and a passion for science and technology. [6]

in Encyclopedia of Geometric Solution referred to in this paper are incorporated here as references[5].

Problems from Encyclopedia of geometric solution

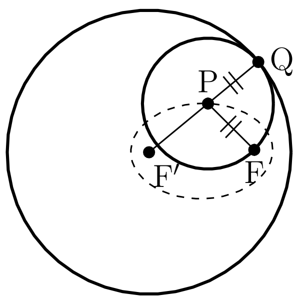
1440. Fixed straight line ℓ and xed point F outside this straight line are given. Obtain the trajectory of center P of the circle which passes through F and is tangent to ℓ.
1444. Obtain trajectory of center of circle P which passes through xed point F inside xed circle F$'$ and is tangent to xed circle F$'$.
1443. Obtain the trajectory of center of circle P which passes through xed point F outside xed circle F$'$ and which is tangent to xed circle F$'$.

The trajectory of circle P is represented by a parabola having focal point F and directrix ℓ, ellipsoid having focal points F$'$ and F, and hyperbola, respectively. These are drawn by KETCindy.



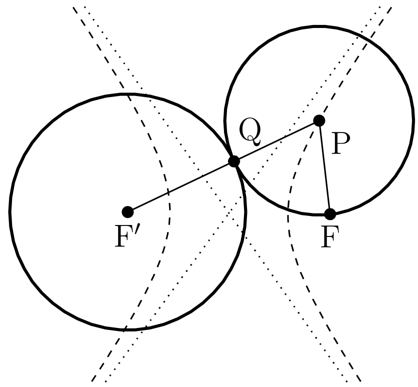
1440.

Point H is the point of contact of circle F and ℓ



1444.

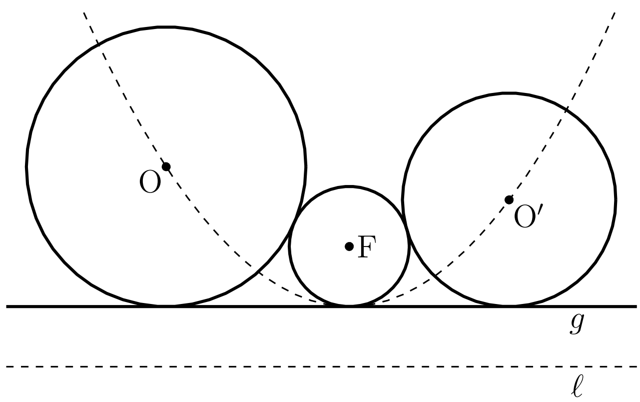
Point Q is the point of contact of circle F$'$ and circle F



1443.

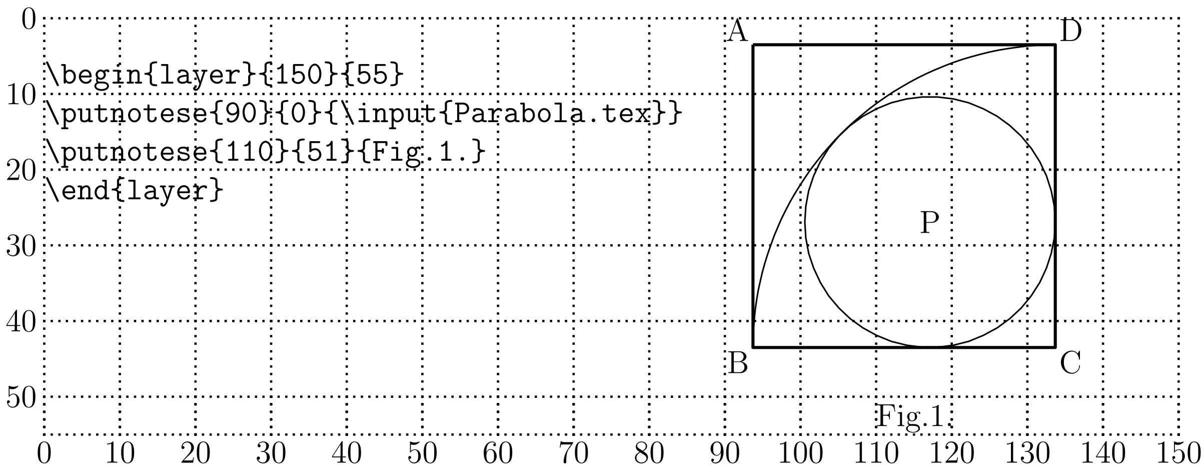
Point Q is the point of contact of circle F$'$ and circle F

2. The following illustration is used frequently in Wasan. Circle O and circle O$'$ tangent to common line of tangency g which is tangent to circle F can be drawn. However, point F is the focal point of the parabola shown by dashed line; ℓ shown by dashed line is directrix.



[5] Although these problems ask a student to obtain a trajectory, these are modi ed to meet the contents of this paper. Figures are of extended geometric construction by Cinderella and ketcindy, and vertex and points are uni ed to those used in this paper.

3. The author shows how to insert the tpic le in the LaTeX under the layer environment. It is possible to determine the position of the gure as follows.



The rst line means that you display the plane in a grid pattern from point $(0, 0)$ to point $(150, 55)$ by x, y coordinates. The second line means that the le of "Parabola.tex"– Fig.1′s upper-left corner – is placed on the southeast of point $(90, 0)$. When you are satis ed with the position of the gure, you should turn o the grid as below.

```
\begin{layer}{150}{0}*
\putnotese{90}{0}{\input{Parabola.tex}}
\putnotese{110}{51}{Fig.1.}
\end{layer}
```

Therefore, it is possible to obtain the gure of Example 1.

References

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