Application of Information Technology in the Formation of the Concept of Functions

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Abstract: *This paper takes a practical problem as the starting point to make students understand the meaning of variables and constants. By analysing the particular problem, students investigated the changing process of functions, identified the relationship between two variables and approached the pattern of the changes.*

Students were encouraged to use graph calculators in the research. The relevant software in the graph calculators enabled students to tabulate and observe the pattern of data. It also enabled students to plot the set of data in order to identify the changing process. The combination of tables, images and formulas helped students to have a better understanding of the concept of functions. When analysing the characters of functions, information technology provided different methods of researching according to the preferences of each student. It also motivated the passion of students and showed their intelligence, as well as the beauty of Mathematics.

1. Introduction

The importance of information technology is felt everywhere in our daily life. Information technology is also used as a helpful tool for students to study. In the process of learning mathematics, building the cognition of concepts is crucial. When forming the concepts of mathematics, whether information technology is useful? How can it be used? In what ways can information technology be helpful? How to interpret the value of such uses? The series of questions has close relations with our cognition of the use value of information technology. In addition, it enables us to view students' activities of mathematics learning from a different perspective and to inspect the essence of mathematics.

Function is an important model in mathematics which describes the changes of the objective world. Because of its core position, the cognition of the concept of function has significant effects on students' understanding of other related mathematics concepts, knowledges and mathematics itself. As a result, the process of learning the key concepts in mathematics, such as 'function', should be designed and analysed carefully.

This paper will introduce a few sections in the beginning of the lecture 'The Concept of Function' and analyse the use of information technology in this process.

2. Unexpected Surprises and Thinkings

The original design of the beginning of the lecture was each student would be given a string of 40cm long and would be asked a question: how to maximise the area of the rectangle by using the string as the rectangle's perimeter.

Students answered immediately: 'the area is maximised when the rectangle happens to be square'. The following is the conversation between teacher and students:

Teacher: Why do you think the biggest area happens when it is square?
Students: It feels like that.

Teacher: Why do you feel like that?
Students: Square is a special rectangle.

Teacher: Why do you think the specialty of square results in the largest area to happen?
Students: I don't really know.

Analysis: The conversation implies that students understood square was a special kind of rectangle. However, the relationship between the specialty of square and the limit values of its area was not clearly analysed by students. This was not only the problem of students, but also the perfect starting point of the lecture which encouraged students to think deeply.

First of all, students were guided to express the relationship between values in the problem. Assuming that one side of the rectangle was x, hence the other side was $y = 20 - x$ and the area was $s = x(20 - x)$. Furthermore, when the value of x increased, the value of $20 - x$ decreased. Therefore, in the polynomial $x(20 - x)$, one factor increased while the other decreased, how would the product vary? Students realised that it needed further thinking.

Some students suggested that we should substitute some numbers into the formula as shown in Table 1 below.

Table 1

x	1	2	3	4	5
$y = 20 - x$	19	18	17	16	15
$s = x(20 - x)$	19	36	51	64	75

Through tabulating, students had a better understanding of the concept of constant and variable. When the value of x was determined, the value of the adjacent side y and the area s could also be determined.

Students were then guided to study the changing relationship between x and y. They selected the function ‘Statistics 2Var’ in graph calculator, as shown in Figure 1 and the input values were shown in Figure 2.

Figure 1

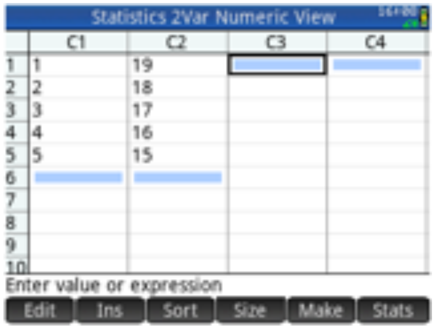


Figure 2

By using the graph calculator for plotting, shown in Figure 3, students observed the location characters of the scatters and guessed the pattern of distribution. A few students thought the scatters were spread in a straight line while others disagreed.

Students were then advised to use graph calculators for fitting and deciding whether the hypothesis was right. Interestingly, students saw different fitting figures, as shown in Figure 4, Figure 5 and Figure 6.

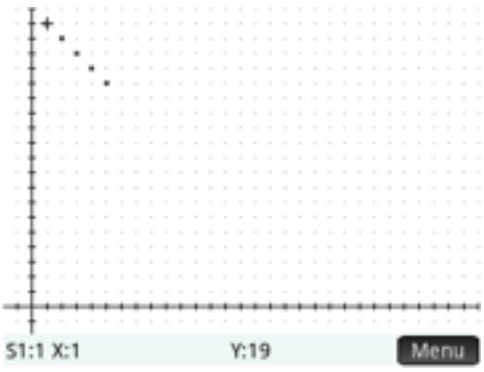


Figure 3

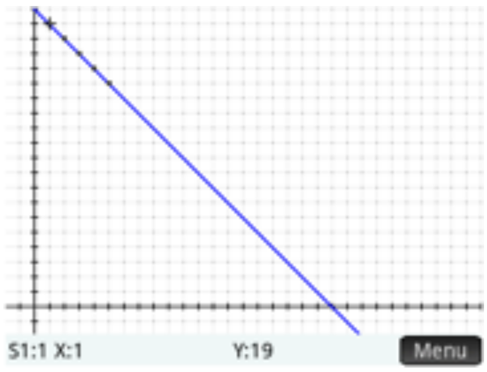


Figure 4

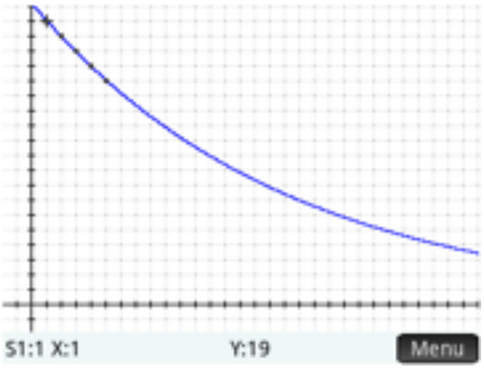


Figure 5

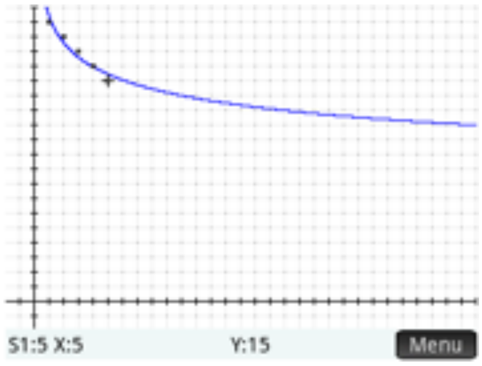


Figure 6

What just happened? Who was right? Students looked confused. Suddenly, some students realised that they should check the system setting of the graph calculator first, as shown in Figure 7. How could it be explained? Students realised that they lacked enough data, in other words, by using only five sets of data could not support the pattern of distribution of the scatters. So what to do next? Some students came up with the idea that more data could be listed, fitted and compared with the same type of curve, for example, linear, logarithmic, exponential and power (shown in Figure 8). Figure 9 to 16 showed that linear is the most accurate fitting type, namely, the changing relationship of variable x and variable y should be linear.

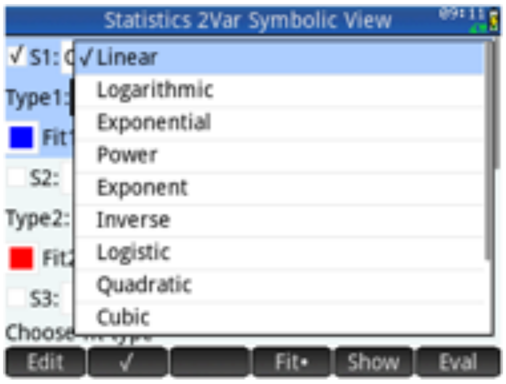


Figure 7

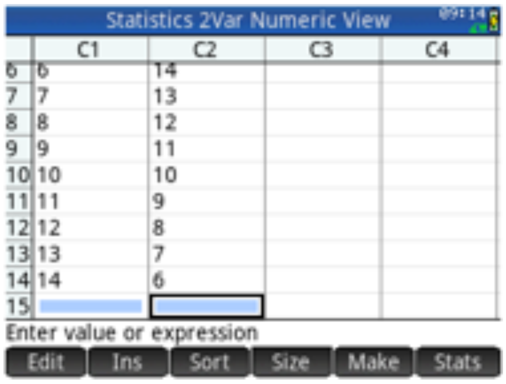
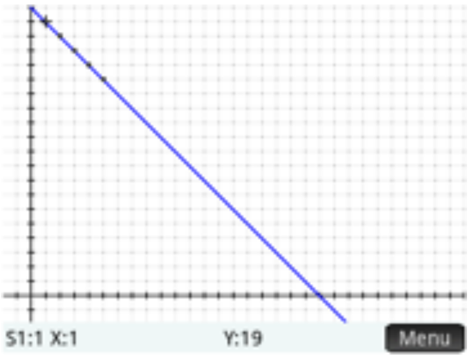
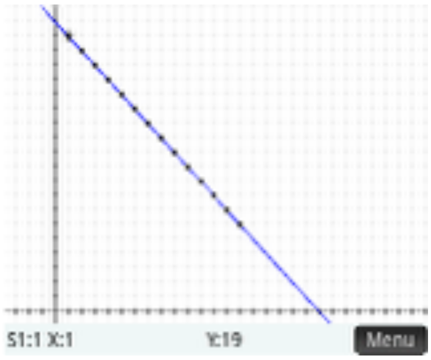
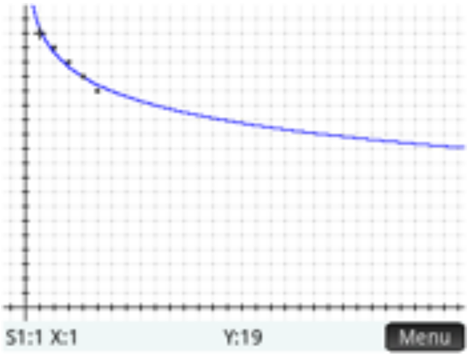
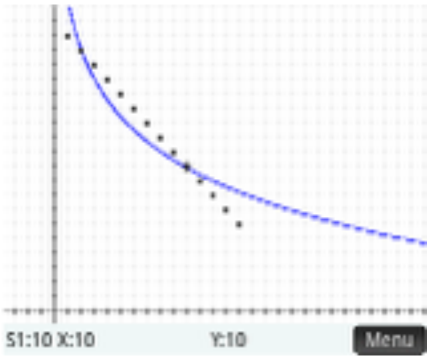
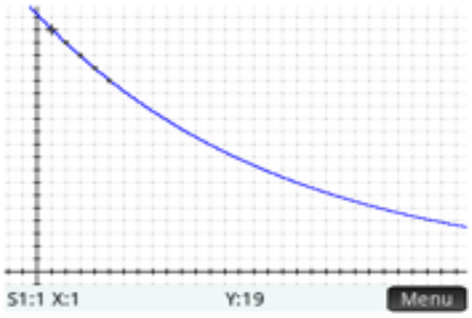
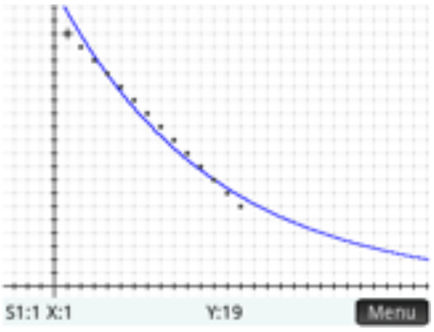
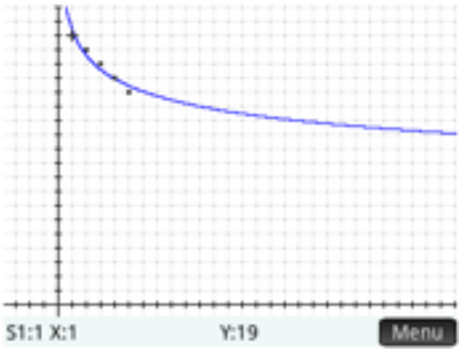
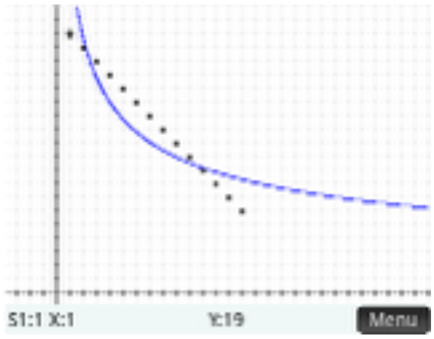


Figure 8

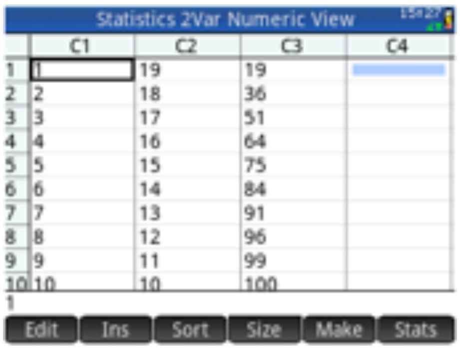
Fitting type	Fitting with 5 sets of data	Fitting with 14 sets of data
Linear	 Figure 9	 Figure 10
Logarithmic	 Figure 11	 Figure 12
Exponential	 Figure 13	 Figure 14

Power	 Figure 15	 Figure 16

The use of technology brought students confusions, but it ended up with helping students to solve the problem. Meeting with surprises, confusions, hypothesises, trials and verities enabled students to have a deeper understanding of the corresponding relationship between the two related variables.

3. Convenient Technology Platform Made Personalised Learning Possible

The previous research process made students feel much easier to explore the changing relationship between variable x and variable s (shown in Figure 17 and Figure 18). Students drew scatter plot (Figure 19) and used different types of functions for fitting (Figure 20 to Figure 25). Then, based on the analytic expression $s = x(20 - \mathrm{x})$, they drew the graph of function (Figure 26) to compare and discuss the rationality of the graph of the function. The researching process helped students to have a deeper understanding of the graph of the function.



Statistics 2Var Numeric View

	C1	C2	C3	C4
1	1	19	19	
2	2	18	36	
3	3	17	51	
4	4	16	64	
5	5	15	75	
6	6	14	84	
7	7	13	91	
8	8	12	96	
9	9	11	99	
10	10	10	100	

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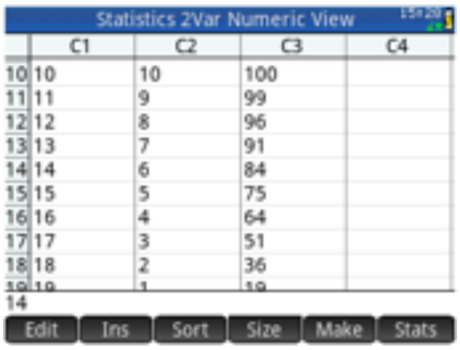
Figure 17



Statistics 2Var Numeric View

	C1	C2	C3	C4
10	10	10	100	
11	11	9	99	
12	12	8	96	
13	13	7	91	
14	14	6	84	
15	15	5	75	
16	16	4	64	
17	17	3	51	
18	18	2	36	
19	19	1	19	

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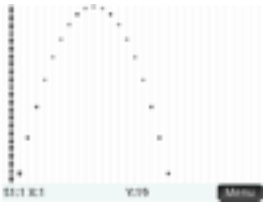
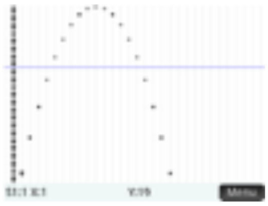
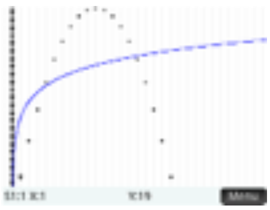
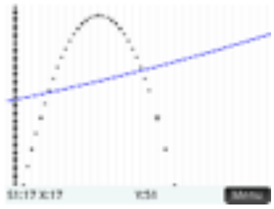
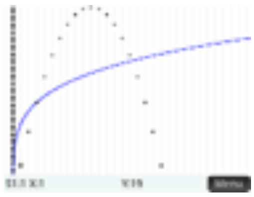
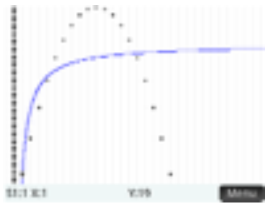
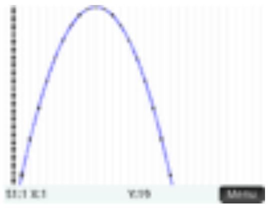
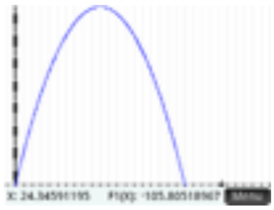
Figure 18

Scatter plot	Linear	Logarithmic	Exponential
 Figure 19	 Figure 20	 Figure 21	 Figure 22
Power	Inverse	Quadratic	Function $y = x(20 - x)$
 Figure 23	 Figure 24	 Figure 25	 Figure 26

Subsequently, according to the differences between students in terms of personal interests, focusing points, research directions and research results, we listed three main types:

Type1: Interested in investigating properties of functions

Through observing the graph of functions, some students concluded that the function $y = 20 - x$ was a straight line which went downwards from left to right while the function $s = x(20 - \mathrm{x})$ was a symmetric curve which went up first and then downwards. Was their conclusion correct? The students were not satisfied with the simple observations and hypothesises. They wanted to use the knowledge they learned to clarify the property of the functions $y = 20 - x$ and $s = x(20 - \mathrm{x})$.

The proving process of the property of the function $y = 20 - x$:

Assuming that $x_1 < x_2$, $\because y = 20 - x$, $\therefore y_1 = 20 - x_1$, $y_2 = 20 - x_2$, $y_1 - y_2 = 20 - x_1 - (20 - x_2)$, simplifying the above equation to $y_1 - y_2 = x_2 - x_1$. $\because x_1 < x_2$, $\therefore x_2 - x_1 > 0$, $\therefore y_1 > y_2$.

The conclusion is that the function $y = 20 - x$ has two properties:

a. When the independent variable increases, the value of the function decreases, so the function is monotonically decreasing.
b. The increment of the independent variable and the increment of the value of the function are opposite number with each other and hence this straight line has an angle of 45° with x axis.

The proving process of the property of the function $s = x(20 - \mathrm{x})$:

Assuming that $x_1 < x_2$, $s = x(20-\mathrm{x})$, $\therefore y_1 = x_1(20-x_1)$, $y_2 = x_2(20-x_2)$, $y_1 - y_2 = x_1(20-x_1) - x_2(20-x_2)$, simplifying the above equation to $y_1 - y_2 = x_2^{\,2} - x_1^{\,2} - 20(x_2 - x_1)$ and rearranging the equation to $y_1 - y_2 = (x_2 - x_1)(x_2 + x_1 - 20)$. $\because x_1 < x_2$, $\therefore x_2 - x_1 > 0$, if $x_2 < 10$, then $x_2 + x_1 < 20$, $\therefore y_1 < y_2$; if $x_1 > 10$, then $x_2 + x_1 > 20$, $\therefore y_1 > y_2$.
Therefore, it can be concluded that the function $s = x(20-\mathrm{x})$ has the following property:
When the independent variable is less than 10, the value of the function increases as the independent variable increases; when the independent variable is greater than 10, the value of the function decreases as the independent variable increases, in other words, the function does not have monotonicity.

Type 2: Interested in table analysis

From Figure 17 and Figure 18, students found that the columns C1 and C2 were arithmetic sequences, but the column C3 was not. They came up with a series of questions: why C2 was a arithmetic sequence? What was the changing pattern of C3? Why did C3 have such pattern? Students realised that because C1 was a arithmetic sequence, C2 was also a arithmetic sequence. They proved as follows:
Assuming that $x_1 < x_2 < x_3$, and $x_1 - x_2 = x_2 - x_3$, $\because y_1 = 20 - x_1$, $y_2 = 20 - x_2$, $y_3 = 20 - x_3$, $\therefore y_1 - y_2 = x_2 - x_1$, $y_2 - y_3 = x_2 - x_3$, $\therefore y_1 - y_2 = y_2 - y_3$, so sequence C2 was arithmetic sequence.

In the meantime, students found that sequence C3 was symmetric. The proving process is shown below:
Assuming that $\forall x_1 \in C_1,\ 0 < x_1 < 20$, then $y_1 = 20 - x_1,$ and $y_1 \in (0,20)$, $s_1 = x_1 y_1$; when $x_2 = 20 - x_1$, $y_2 = 20 - (20 - x_1) = x_1$, $s_2 = x_2 y_2$, $\because x_1 y_1 = x_2 y_2$, $s_1 = s_2$. Therefore, sequence C3 was symmetric and the graph was also symmetric.

Types 3: Interested in fitting graphs

This type of students showed great interest in observing Figure 21 to Figure 24 and wondered what types of functions they were. What were the characters of such analytic expressions? How would the graphs look like? What were the characters of the functions?

Students returned to the interface of fitting and selected their most interested function type (Figure 27 and Figure 28). In the process, students not only learned more about the different types of functions, but also experienced the colourfulness of the type of functions. More importantly, students realised that functions were models which describe the changes of the objective world.

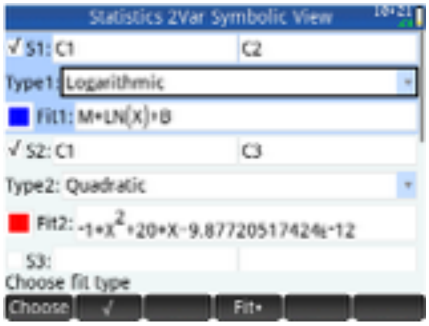


Figure 27

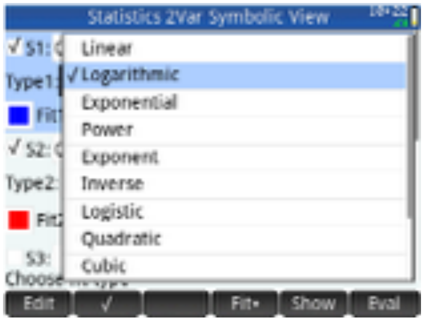


Figure 28

This lecture is about concept teaching and the concept of 'function' is very important in mathematics. After class we did some interviews with students and here are a few sections in the interview:

Teacher: Can you talk about what is your understanding of 'function'?
Student: Function is a relationship between two variables.

Teacher: Equation is also a kind of relationship. What is the difference between equation and function?
Student: Equation is used to describe the equality between two quantities while function is used to describe the quantitative relationship between two variables in the changing process.

Teacher: How can functions be expressed?
Student: Table, graph and analytical expression, three methods in total, which is special!

Teacher: Which one do you prefer to use?
Student: Graphs.

Teacher: Why?
Student: It is intuitive and looks interesting.

Teacher: How to interpret the 'interesting' you said?
Student: On the one hand graph is vivid. Sometimes it goes up and sometimes it goes down and even if it continues going up, the situations are still different; on the other hand, graph is not a simple geometric figure. We can analyse the changing relationship of the quantities within it.

Teacher: What do you think is the most special about this class?
Student: Freedom.

Teacher: Can you explain that in detail?
Student: We could explore, select data, operate and use graph calculator for experiment all by

ourselves. It was not the case that everybody had to follow the same steps, same direction and same contents. Because everybody was doing different experiments, the content of our discussions became more colourful and interesting!

Conclusion

In the process of exploring in mathematics, information technology can help us draw sophisticated graphs and complete complicated calculations. It also provides varies methods for students to build cognition of important mathematic concepts. Information technology not only encourages students to research and explore, but also shows their valuable ideas and creativity. More importantly, it shows the beauty of mathematics.

Reference

[1] Ji Ping. *What to Teach*. Educational Science Publishing House (China).

[2] *Mathematics Methodology*. Beijing Education Press.

[3] *Mathematics Standard Courses Clarifie*d. Beijing Normal University Press.

[4] *On Math Thinking*. Beijing Education Press.

[5] Xin Lei. *Mathematics Education Reform and the Research*.