The Hand-held Technology in Flipped Classrooms

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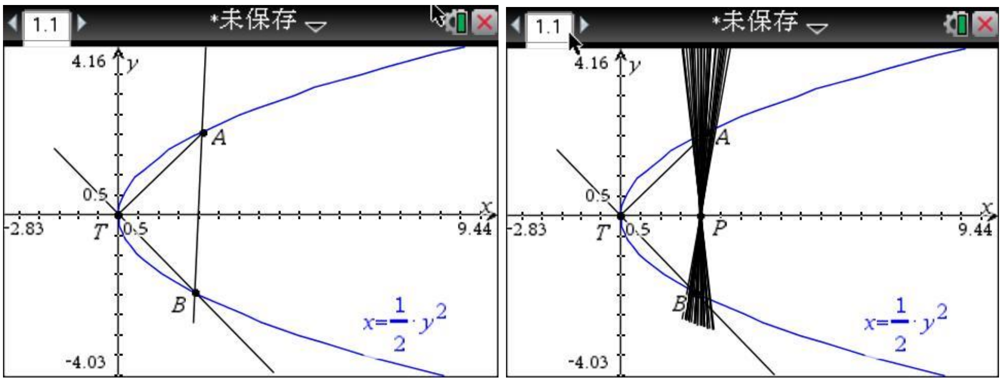
Abstract: *With the popularity of Khan Academy, flipped classroom and online micro-video lectures are becoming effective tools and facilitators in education, changing the way of both teaching and learning. Graphing calculator is widely applied in math class as well, serving as an innovative tool for students to further their learning and exploration. What will happen if we combine online micro-video lectures and graphing calculator together in everyday learning? Last semester, I made a micro-video to explain the problem of constant in conic section by using TI-nspire graphing calculator. With the aid of graphing calculator, students dived deep into a series of problems and discussions, drawing more conclusions than expected through cooperation and exploration. Their passion and curiosity for learning are enormously aroused, leading to efficient learning and discussion in class. Therefore, the class is successfully flipped.This paper provides a typical teaching case study of how online micro-video lectures and graphing calculator can be combined to improve everyday learning mode, thus offering practical help to the schools experimenting with flipped classroom.*

1. Use micro-video lecture to introduce mathematic problems

Not long ago, I filmed a five-minute online class for high school juniors discussing a conclusion in the topic of conic section using the Ti-nspire calculator. The initial aim of this class is simply to clarify a mathematical problem, yet the video sparked unexpectedly strong interests from the students. In this paper, I will briefly discuss the methodologies used in solving the problem, as well as the mindset behind filming this online class.

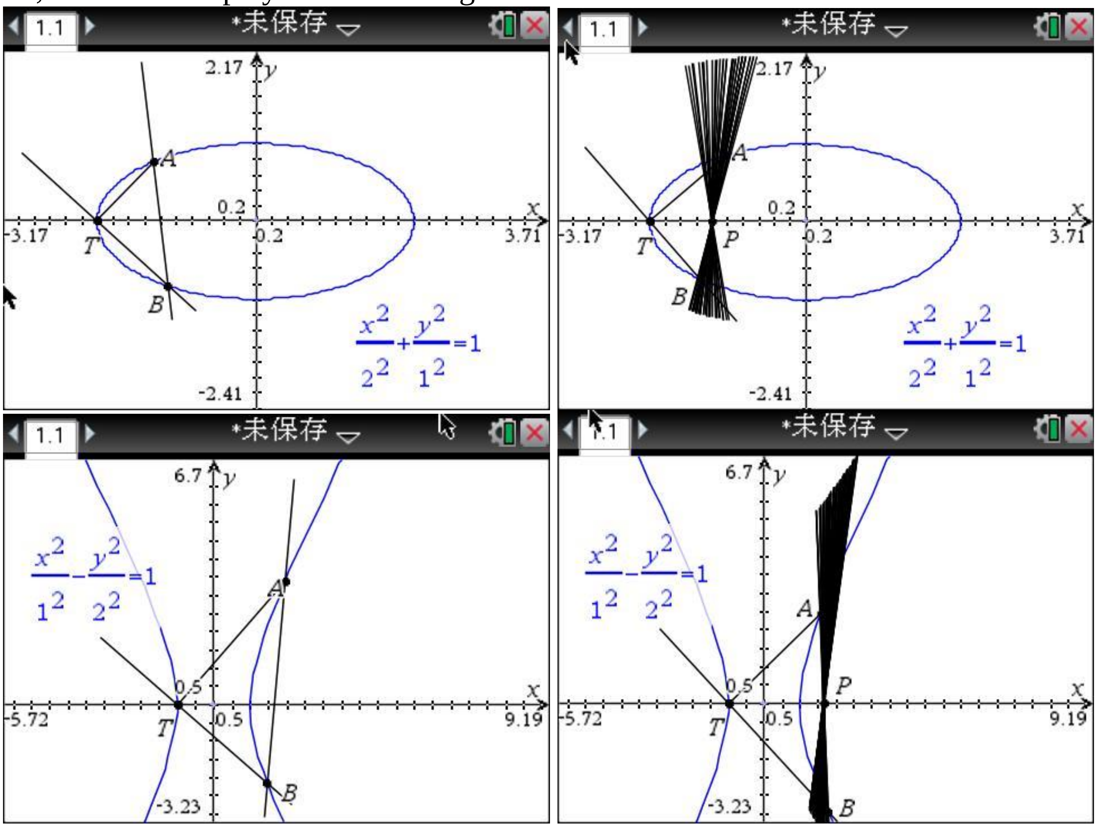
We first draw a parabola $y^2 = 2x$ using the Ti-nspire calculator. We set the vertex of the parabola to be T, then use the graphing calculator to draw two perpendicular strings TA, TB. Connect AB, then drag the two points A, B to move randomly, while letting TA, TB remain perpendicular to each other. Using the tracking function of the calculator, we are able to record the trajectory of line AB. It is very clear that line AB always goes through the fixed point P on the x-axis. This is an interesting finding. At the end of the short online class, I brought up two questions: first, can we discover similar theorems within the broader topic of conic section? Second, how do we prove the theorem we just found?



There are two reasons why I want to film this online class. The first is that the source of this problem is one of the exercises in the textbook, and with the help of the graphing calculator, we can display this theorem more vividly to the students and therefore spark their interests in mathematics. The second reason is that the conclusion in this particular problem can be expanded to other fields, for example, if we replace the parabola with an ellipse or a hyperbola, we will have similar findings. This provides a broader space for the students to explore.

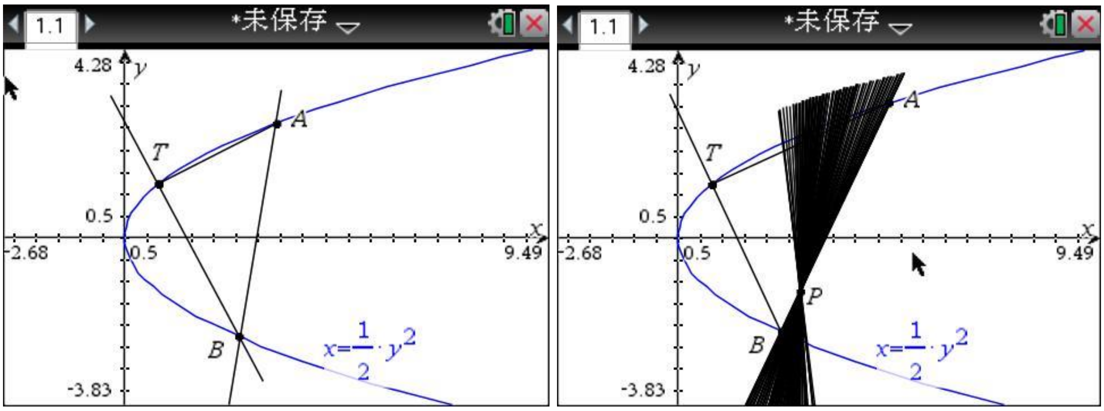
2. Students' inquiry and discovery

After sending this video to students, I received the expected initial results, and soon there were students who wanted to discuss the topic with me. They discovered similar theorems in ellipse and hyperbola, which are displayed as following:

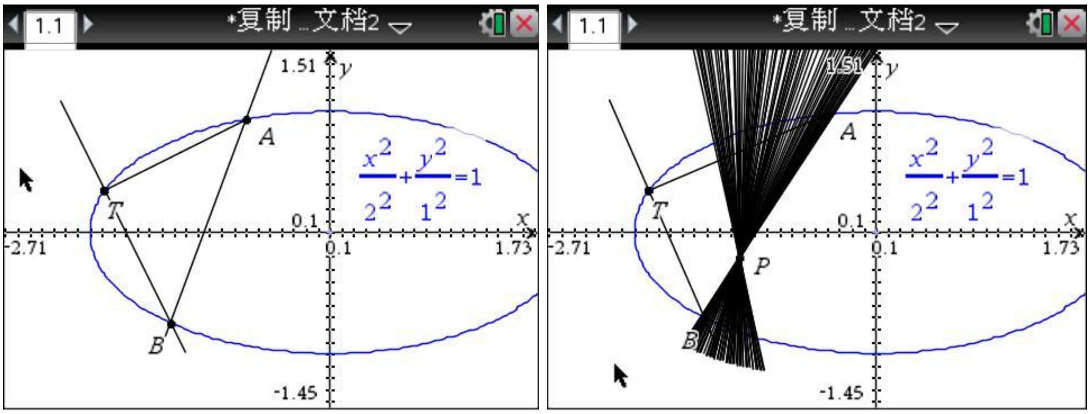


As we can see, if we draw two strings TA, TB that go through the vertex of the ellipse or hyperbola, line AB always goes through a fixed point on the x-axis.

Just when I thought the goal of this class was reached, there was another student who came to me with new findings. He moved the point T away from the vertex and along the parabola, while making sure that TA, TB are perpendicular to each other. He found that line AB still goes through a fixed point P only that this point is not necessarily on the x-axis. As the pictures below show, this means that when T is any point on the parabola, the theorem that line AB goes through a fixed point is likely to still hold.

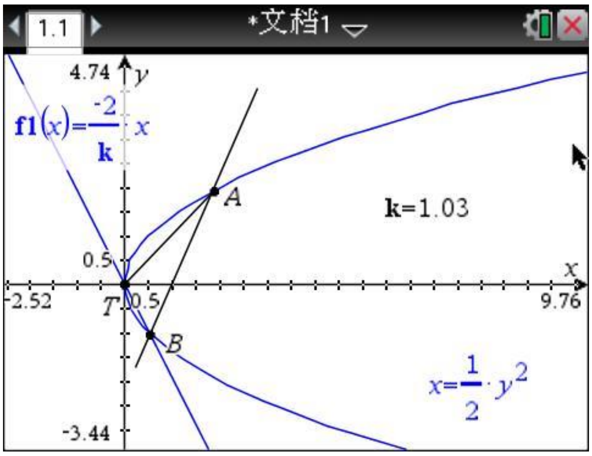


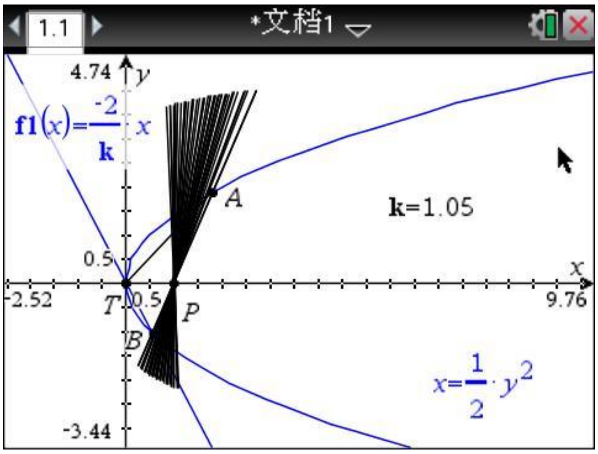
This student tested the theorem on ellipses and hyperbolas, and his findings are displayed as following:



At this point, the students have developed the initial conclusions through graphing calculators to the current one: through any point T on a conic section, draw two perpendicular strings TA, TB, connect AB, then line AB always goes through a fixed point.

This beautiful conclusion inspired more passionate discussions from the students. Considering the fact that $TA \perp TB$ means that the product of the slopes of the two lines TA, TB, $k_{TA} \cdot k_{TB} = -1$, what conclusions would we have if the product $k_{TA} \cdot k_{TB} = -2$? The students used the slope testing function of the graphing calculator to construct two strings TA, TB whose slopes' product is -2. Magically, line AB still goes through a fixed point. The results are displayed as following:



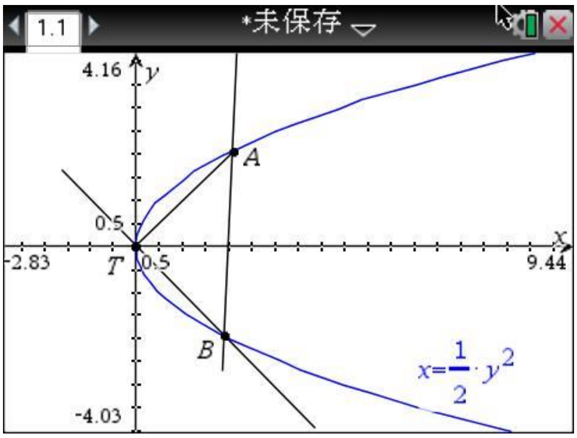


Until now, through making bold hypothesis and testing them using hand-held technology, the students have found a series of deeper and broader theorems from a simple conclusion in the online class. This has exceeded my expectations significantly. More importantly, all of this process happened outside the classroom, and the students have directly perceived the knowledge that used to be only available in class, which provides ample materials and space for further discussion within the classroom. As a result, students can participate in the class with the passion of finding out problems and solving them, which further generates more discussion and more in-depth teaching. In this way, the classroom is “flipped”.

3. Proof of Man-device integration

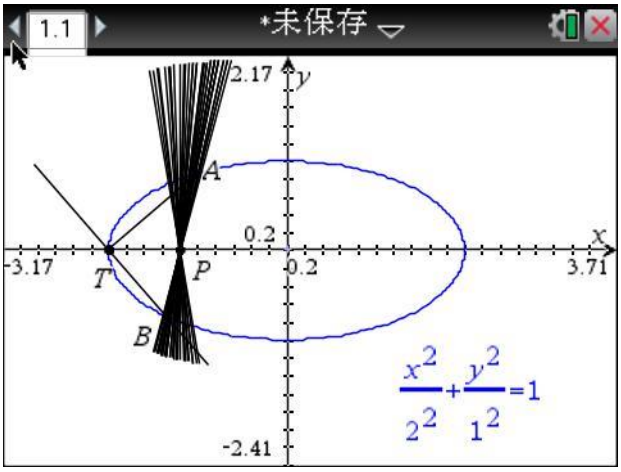
Perhaps it is not extremely difficult for the students to further explore these conclusions, which is why they do so with strong interests. However, the next question is much more challenging: how to prove the conclusions we arrived at. This, of course, is our task in analytic geometry: using algebra to solve geometry problems. This naturally became the emphasis of the forty-minute class session.

During the class session, I first let the students report their findings in groups and display their conclusions to the audience, helped them sort through their conclusions and form a more systematic one, then I began to tackle the most difficult part of this class: proving these conclusions. The initial theorem in the online video class is relatively easy to prove, and many students have done so after class. One of the solutions is displayed as following:



Since $TA \perp TB$, we set the coordinates of points A and B as $A(x_1, y_1), B(x_2, y_2)$, and we have $x_1x_2 + y_1y_2 = 0$①. Since $y_1^2 = 2x_1, y_2^2 = 2x_2$, substitute y into ① and we have $\frac{(y_1y_2)^2}{4} + y_1y_2 = 0$②; Set the equation of line AB as $x = ky + m$, then substitute x into the parabola $y^2 = 2x$, eliminate x and we have $y^2 - 2ky - 2m = 0$; from Vieta's Theorem, $y_1 \cdot y_2 = -2m$, then substitute y into ② and we have $m^2 - 2m = 0$; thus $m = 0$ or $m = 2$. $m = 0$ is dropped based on the question's pre-requisites, so when $m = 2$, line $AB: x = ky + 2$ always goes through the point $(2, 0)$.

Of course, here we only proved the theorem in a special case, which is when $y^2 = 2x$. Proving that the theorem holds to any parabola $y^2 = 2px$ and further expanding it to any conic section is much more complicated. Use ellipse as an example, draw two strings $TA \perp TB$ through the left vertex T of any ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. To prove that line AB always goes through a fixed point, we will do much more complicated calculations. One reason is that there are too many parameters involved, which increases the difficulty. It is quite unreasonable to guide the students to complete this proof within a forty minute class. Therefore it's time for the CAS (Computer Algebra System) of the Ti-nspire calculator to display its powers. With its help, I'm able to guide the students to utilize technology in solving mathematical problems.



Using the thought process of the parabola problem, we can solve the ellipse problem in similar ways. Set the coordinates of A and B as $A(x_1, y_1), B(x_2, y_2)$. $TA \perp TB$ means that $(x_1 + a)(x_2 + a) + y_1y_2 = 0$, and we also have general formulas of a line $AB: y = kx + m$ and an ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; substitute information on x_1, x_2, y_1, y_2 into the equation $(x_1 + a)(x_2 + a) + y_1y_2 = 0$ and we can obtain the relationship between k and m, which leads us to the solution of the equation. The thought process is not too complicated, but the calculations are. With the help of CAS, the following process becomes much simpler:

Step one: solve the equations simultaneously and calculate the two roots

$$\operatorname{solve}\left(\frac{x^2}{a^2}+\frac{(k\cdot x+m)^2}{b^2}=1,x\right)$$

$$x=\frac{a\cdot\left(\sqrt{a^2\cdot k^2+b^2-m^2}\cdot b-a\cdot k\cdot m\right)}{a^2\cdot k^2+b^2} \text{ or } x=\frac{^-a\cdot\left(\sqrt{a^2\cdot k^2+b^2-m^2}\cdot b+a\cdot k\cdot m\right)}{a^2\cdot k^2+b^2}$$

Step two: set the two roots as x_1, x_2

$$x1:=\frac{a\cdot\left(\sqrt{a^2\cdot k^2+b^2-m^2}\cdot b-a\cdot k\cdot m\right)}{a^2\cdot k^2+b^2} \qquad \frac{a\cdot\left(\sqrt{a^2\cdot k^2+b^2-m^2}\cdot b-a\cdot k\cdot m\right)}{a^2\cdot k^2+b^2}$$

$$x2:=\frac{^-a\cdot\left(\sqrt{a^2\cdot k^2+b^2-m^2}\cdot b+a\cdot k\cdot m\right)}{a^2\cdot k^2+b^2} \qquad \frac{^-a\cdot\left(\sqrt{a^2\cdot k^2+b^2-m^2}\cdot b+a\cdot k\cdot m\right)}{a^2\cdot k^2+b^2}$$

Step three: substitute the two roots above into the equation $(x_1+a)(x_2+a)+y_1y_2=0$

$$(x1+a)\cdot(x2+a)+(k\cdot x1+m)\cdot(k\cdot x2+m)=0$$

$$\frac{a^4\cdot k^2-2\cdot a^3\cdot k\cdot m-a^2\cdot\left(b^2\cdot k^2-m^2\right)+b^2\cdot m^2}{a^2\cdot k^2+b^2}=0$$

Step Four: factorize the above results

$$\operatorname{factor}\left(\frac{a^4\cdot k^2-2\cdot a^3\cdot k\cdot m-a^2\cdot\left(b^2\cdot k^2-m^2\right)+b^2\cdot m^2}{a^2\cdot k^2+b^2}=0\right)$$

$$\frac{(a\cdot k-m)\cdot\left(a^3\cdot k-a^2\cdot m-a\cdot b^2\cdot k-b^2\cdot m\right)}{a^2\cdot k^2+b^2}=0$$

It is obvious that $ak-m=0$ does not satisfy the pre-requisites of the question, therefore the only solution is $a^3k-a^2m-ab^2k-b^2m=0$, so $m=\frac{a\left(a^2-b^2\right)}{a^2+b^2}\cdot k$. Therefore $AB: y=kx+\frac{a\left(a^2-b^2\right)}{a^2+b^2}\cdot k$, and it always goes through the fixed point $\left(-\frac{a\left(a^2-b^2\right)}{a^2+b^2},0\right)$. The proof is completed.

With an example like this, students can fully cultivate their potential and solve other challenging problems with confidence. Ti-nspire highly improved the efficiency of this class and magnified its positive impacts.

4. Summary

This class can be regarded as an example of "flipped classroom". Students took the first steps in studies through short online video classes, then explore on their own, and eventually master the knowledge in the real classroom. A critical element during this process is modern technology, which is shown not only through filming videos and sharing them online, but also through using hand-held technology. Without the help of graphing calculators, there were a few challenges hard to overcome. Graphing calculators in this class have two main functions. The first is helping students to explore the problems before class, since without images it is hard to construct and test a hypothesis. The key point here is the straight-forwardness of images. The second function is helping students calculate during the proof process, and without the help of graphing calculators the calculations will be long and arduous. The key point here is the calculating powers. With all these key points, we are able to complete the process of constructing hypothesis intuitively→testing them with machines→proving them with the combination of brain power and machine power. This is also the general process of solving problems using computers in modern mathematical studies. Therefore, in the flipped classroom of mathematics, the proper usage of hand-held technology can motivate students to think on a broader spectrum and solve problems with higher efficiency. This also shows that within a flipped classroom, the revolution in the mindset of teaching lies in the in-depth merging of the subject itself as well as modern technology.

References

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