

A Practical Case for e-Mathematical Experiment with “Geometry Apps” from HP Prime

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Abstract: *Since 2008, The MCL (see (1)) project has initiated a preliminary inquiry of mathematical experiments (see [2]). The standard of setting up a mathematical laboratory for primary and secondary schools has also been a concern in the public. One objective of setting up a mathematical laboratory, which provides e-math services, is that it analyzes inquiry activity about analytic geometry. On one hand, it emphasizes providing opportunities for students to experience the whole process of mathematical inquiry; while on the other hand, it analyzes specific mathematical applications (especially the dynamic geometrical tools) which supports mathematical experimentation and includes “moving point trace function”, “point line tool” and “slider tool”. This type of e-mathematical laboratory program is likely to enrich students’ mathematical experience.*

1. Introduction

In recent years, math courses have been reformed in Chinese mainland schools, resulting in more emphasis on training the mathematical ability of students. This can be seen in the specific student’s math ability required by the high school course standards, which state to “train spatial skills, abstract epitome ability, reasoning ability, calculating and solving ability, data handling capacity, application sense and consciousness of innovation” (see [1]).

Due to the mandate by the government, some teachers start paying more attention to allow students experience the inquiry process of math as well as the process of mathematical experimentation. This deeper transformation could inspire math teachers to design new mathematical activities for students, especially on those challenging mathematical problems. The aim is to let students meet the challenges and experience the whole process of solving math problems and the process of recreating or discovering mathematics. The change also reflects on the design of exam questions.

On the other hand, many teachers are very aware of the ICT (see (3)) and those mathematical applications. Teachers realize that these mathematical applications are not only presentation tools which benefit teachers, but also beneficial resources to students. It can support students to experience the entire process of mathematical inquiry by supporting those students to properly pose problems, structure hypothesis, verify the hypothesis and the process of their mathematical reasoning.

MCL prioritizes setting up e-mathematical laboratories and participates in setting the standard for mathematical laboratory for primary and secondary schools. Therefore, it needs to explore practical mathematical models in a mathematical laboratory, which focuses on the aim, content, process, evaluation and teachers' suggestions on mathematical experiments. As an activity, e-mathematical laboratory, should especially focus on the function of mathematical apps and the proper method of using these apps.

2. Experience the Whole Process of Mathematical Creation

We believe that we should support more opportunities for students to experience the entire process of mathematical inquiry at the high school level. The situation of e-mathematical experiment holds a more prominent role. In this essay, the authors regard the inquiry of the relationship between straight line and ellipse as a carrier of problems and a case (see [4]) for an e-mathematical experiment.

Problem Given point B on the circle $O: x^2 + y^2 = 2$. We construct the line l perpendicular to x axis through point B , and intersect at point D with x -axis. We pick the point E to be on the line l , and satisfying $\overline{BD} = \sqrt{2}\overline{ED}$, when point B moving on the circle, we let the locus of point E to be curve Γ . We consider the followings:

(a) Find the equation of Γ .

(b) The line through $H(1,0)$ cuts circle O at I . Making a line K perpendicular to IH passing through I . Prove that line K is tangent to curve Γ .

(c) Given line m is a line on the same plane of the circle O . Make a line K perpendicular to line m through the point $H(1,0)$, and the foot point is P , then connect the point O and P (see Fig.1). According to the length of OP , please discuss the quantities of common points for Line m and curve Γ . (Only a conclusion is needed)

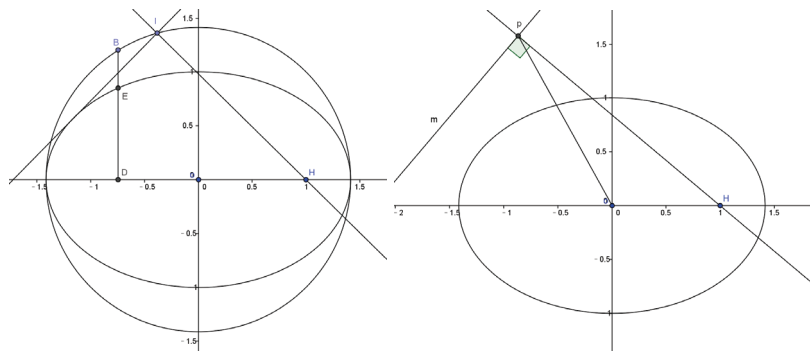


Figure 1

The design rationale of mathematical inquiry activity is: “it requires itself to find problems from research, the transfer of practical problems into mathematical problems, and learning to solve practical problems by using mathematical logic and methods”. “We research connotation and the essence of this conclusion and put the proposition into use”. “The process of solving problems goes through the abstract summary, reasoning and demonstration, plausible reasoning, critical proof, and from perceptual knowledge to rational knowledge.”(See [2]) It follows that this inquiry activity is challenging for high school students. It is typical that they will experience the entire process of inquiry so that they can solve this problem and be motivated to use several mathematical abilities.

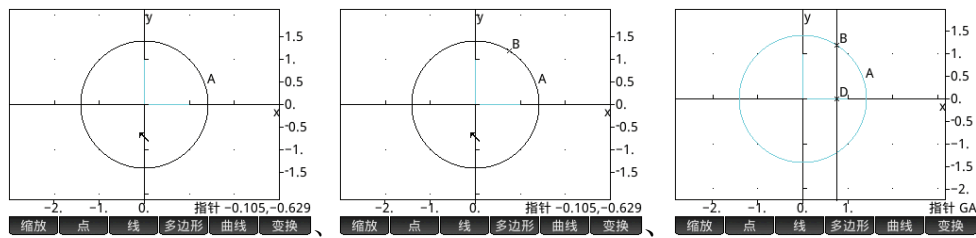
Using traditional methods to solve this problem, students should use “coordinate spot” to evaluate locus, know the method of problem, which straight line is tangent to ellipse by using “the thought of function and equation”, and master “guess and demonstration thinking” to conclude the condition for the relationship between straight line and ellipse.

However, with the introduction of this technology, the whole process of inquiry for the same problem will have changed. “Correlation spot” method is replaced by “moving point trace function”, “point and line” tool takes the place of “tangent line problem”. Guess and demonstration could go on speculating visually by the “slider tool”. These new methods inspire students’ imagination and will bring them wonderful enjoy.

What will students do when they are in the situation of an e-mathematical experiment? In other words, what dynamic geometry app can bring to students? In the followings, we use dynamic apps within HP Prime to demonstrate how the activities may assist students’ exploring mathematics. There are certain dynamic geometry apps which are familiar to us and also some new functions, but the issue is what can teachers provide to students? And how can teachers make use of them?

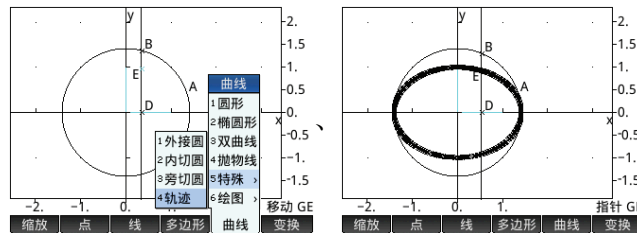
2.1. Functionality of trace

Example 1. In geometric symbol view, we input elliptic equation $x^2 + y^2 = 2$ (see Fig.2), choose any point B (see (5) and Fig.3) on the circle. We construct a perpendicular line through point B with the x -axis, and intersect the x -axis at point D . (see (6) and Fig.4).



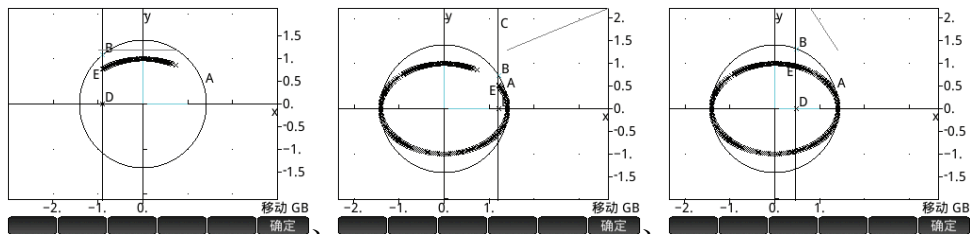
Figures 2-4

Choose point E on BD satisfying $\overline{BD} = \sqrt{2}\overline{ED}$ (see (7)). We refer the result of the point E to Figure 5.



Figures 5-7

There are two ways of obtaining the locus of E . One way is to use “locus function” of “drawing area” and draw locus of E directly (see (9) and Fig.6-7). The other is to use “moving point trace function” (10) to draw. First, we choose the second moving point E , then move the main moving point B by “moving point trace function”. During movement of the point, the system draws the locus of second moving E automatically (see Fig.8-10).



Figures 8-10

Discussions: In this problem, the traditional solution is to find the coordinate relationship between

the main and the second moving point $\begin{cases} x_E = x_B \\ y_E = \frac{\sqrt{2}}{2} y_B \end{cases}$, then through eliminating the coordinate for the

main moving point B , we get the relationship between the x-axis and y-axis of the second moving point E . We make a further observation that the locus of E is an ellipse. However, the situation will be different if we make an inquiry with “moving point trace function” and “solving locus function”. If using “moving point trace function”, it only needs to set a main moving point and move it, then it could draw the locus of second moving point. The process of dynamic showing is quite simple and vivid. If using “solving locus function”, it only needs to confirm the main moving point and the second moving point; they could draw the locus of second moving point. This method omits forming the process of animation, but it is efficient and intuitive. According to this situation,

if we continue our research, some students may ask when the followings will be true.

$$\overline{BD} = \sqrt{3}\overline{ED}, \overline{BD} = \sqrt{5}\overline{ED}, \dots, \overline{BD} = \lambda\overline{ED} (\lambda > 1).$$

some may ask if $\overline{BD} = \frac{1}{2}\overline{ED}, \overline{BD} = \frac{1}{3}\overline{ED}, \dots, \overline{BD} = \lambda\overline{ED} (0 < \lambda < 1)$, what's the locus equation of E ?

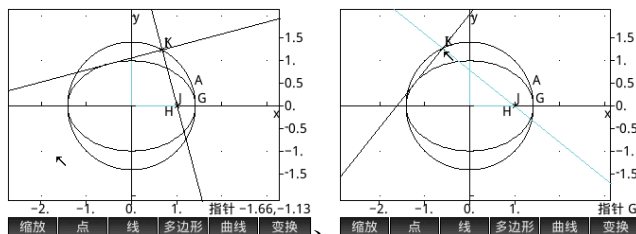
In addition, if we change the circle into ellipse, what will the result will be? Inquiry is the most significant part of “experiencing the whole process”, so the purpose of the “geometry app” is served. On one hand, students can take this problem to continue derivative inquiry, while on the other hand, we can “let students experience the real math and train their attitude and consciousness of scientific research step by step.”(See [3])

2.2 Point and line tool

Example 2. Choose a fixed point $H(1,0)$ (see Fig.11) at coordinate planes and moving point I (see Fig.12) on the circle. We connect HI (see Fig.13) and make line K to be perpendicular to line HI through point I (see Fig.14). We move the point I and observe the positional relationship between line K and the ellipse. (See Fig.15).



Figures 11-13



Figures 14-15

Obviously, the answer is that the line is tangent to ellipse.

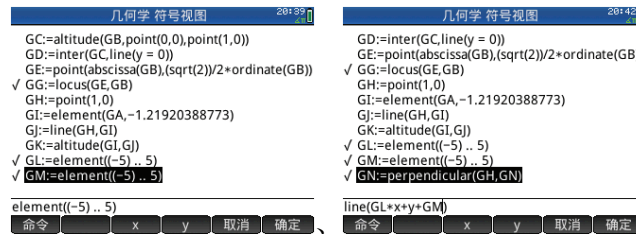
Discussions: There are abundant tools for users to explore when using HP Prime, such as midpoint, symmetry points, point of intersection, segment, perpendicular and parallel tangent line, and etc. There are new methods of inquiry for geometrical property. Perpendicular function can get the line which is tangent to line HI . We can use the point of intersection functionality to get the intersecting coordinates of straight line and circle (or ellipse). The length function can assist us to obtain the value of segment AI , and so on. With the help of these tools, we also can analyze whether the geometric length is proportional or not, what the areas and angles are, and what the position of the line and curve are. The “Point and line tool” plays a role for data processing during “the whole process”, which is accurate, fast, and reliable. Compared with using a traditional calculator on paper, the problems that students inquire have become deeper, wider and more challenging.

Students are more open-minded during the inquiry process, they could find and solve problems, bring up and verify questions. Furthermore, students are able to find and seek for patterns. For students, inquiring mathematical problems process has been rewarding and interesting. At the same time, “training students’ abilities of finding problems and inquiring solutions are an important goal of mathematical education.”(See [4])

2.3. Slider function

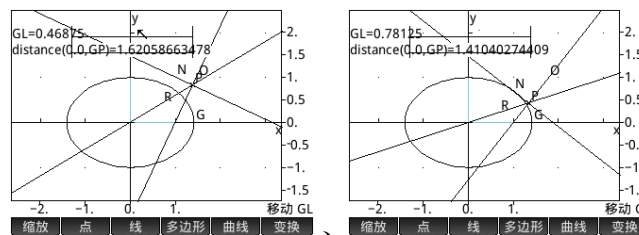
Example 3. Draw a dynamic line m (The line is GN in this example). Here we use the functionality of the slider to make the slopes of GL as variables and the range of slopes is $-5 \leq GL \leq 5$ (see (11)) and GM as the y -intercept ($-5 \leq GM \leq 5$). We note this is the equation of the line:

$$(GL) \cdot x + y + GM = 0 \text{ (See Fig.16-17)}$$

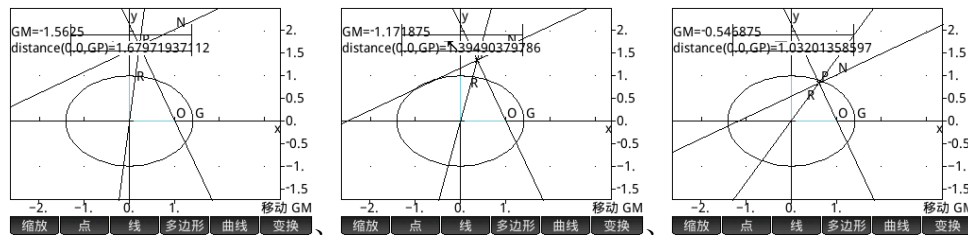


Figures 16-17

Through $H(1,0)$, we make the vertical line m and intersect at point P . We next connect line OP and find the length of line OP (see (12)). Fix the y -intercept GM , and change the value of slope GL and observe the condition of intersection and the length of OP (see Fig.18-19). We next fix the slope GL , change the value of y -intercept GM and observe the condition of intersection and the length of OP (see Fig.20-22).



Figures 18-19



Figures 20-22

Obviously when $OP = \sqrt{2}$, there is only one common point. There is no intersection point when $OP > \sqrt{2}$. Finally, when $0 \leq OP < \sqrt{2}$, there are two intersection points.

Discussions: The main effect of “slider function” is to intuitively show the complicated and abstract forming process of dynamic geometry. For example, when inquiring the position between straight line and ellipse, we set up a “slider” (element (a..b), similar to slider block and slider bar in geometer sketchpad) and regard “slider” as slope of straight line. Once we move “slider” (change numerical value), there is a dynamic line whose slope changes continually. “Slider” is an important tool during “experiencing the whole process”. With this tool, we can inquire into some dynamic geometry problems with parameters, so it goes without saying the importance of “slider”. “The inquiry situation, which is provided from slider, can make close integration of operating, observing and thinking, and also can provide a stage for students to gain knowledge.” (See [5]) “In traditional mathematical inquiry teaching, it is difficult to present dynamic geometric image, during the change of graph, the characteristic of quantity changing can be intuitively represented to students. We could use graph to teach and solve graph with algebra. Meanwhile, students can understand mathematical thoughts and methods deeply.”(See [6])

3. Conclusions

With the support of technology and dynamic geometric apps, especially the core technology for moving point trace function, point and line function and slider function, it provides abundant opportunities for students to experience the whole process of inquiry. Through observing, students go on judging, guessing, demonstrating and making conclusion. During concluding and rethinking, they put forward new questions, then judge and guess them, “students know well mathematical thoughts and gain abilities which is asked from new curriculum standard. It also means that mathematical activities will be more abundant in the future; students will be more confident and have the courage to challenge more complicated problems. They enjoy and dare to inquire into math. The authors hope that people can develop more mathematical inquiry issue and excellent issue like this. “Using *GC* (13) to study math, inquire into applying it, students can acquire basic experience of e math”. “Besides, it is helpful for students to experience the original condition of studying and creating math, for developing students to understand mathematical concepts in multiple ways and for setting up mathematical cognitive structure and enrich their mathematical experience.”(See [7]) At the same time, it also allows the chance for setting up mathematical laboratories for primary and secondary schools.

Supplementary glossary notes:

(1)MCL: Mobile Calculating Laboratory, which is the abbreviation for the cooperation program, which is named hand-held technology and high school mathematical new curriculum integration by HP with Laboratory of Mathematics and Complex Systems (Beijing Normal University), Ministry of Education.

(2) e math: e-math education for all, that is to say, nowadays, schools should offer opportunities for all students to use mathematics digital technology, study math, explore math and apply math.

(3) ICT: Information Communication Technology.

(4) The 19th question in science quality testing, Fujian Province, 2014.

(5) Element (GA , 1) means to choose a moving point on GA , the beginning coordinate of this point is 1.

(6) Altitude (GB , point $(0,0)$) means the line which goes through B and is tangent to $(0,0)$ and $(1,0)$, inter (GC , line $(y = 0)$) means to solve the crossing point between Line GC and Line $y = 0$.

$$(7) \text{ Use } \begin{cases} x_D - x_B = \sqrt{2}(x_D - x_E) \\ y_D - y_B = \sqrt{2}(y_D - y_E) \end{cases} \Rightarrow \begin{cases} x_E = x_B \\ y_E = \frac{\sqrt{2}}{2}y_B \end{cases} \text{ to solve coordinate of } E.$$

(8) Abscissa (GB) means abscissa value of midpoint B on Layer G . Ordinate (BG) means ordinate value of midpoint B on Layer G .

(9) Locus (GE , GB) is track command. GE represents for point E on Layer G , which is the second moving point. GB represents for main moving point. When the main moving point GB is moving, please solve the locus graph for second moving point GE .

(10) Trace () means moving point trace function.

(11) Element (a , b) means to set up a “slider” (parameter), whose range is between a and b .

(12) In “geometric symbol view”, input command distance (A , B); calculate the distance between A and B .

(13) GC : Graphing Calculator.

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