A model for the educational role of calculators

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Abstract: *Calculators can be used effectively for mathematics education in a number of ways, although frequently they are regarded merely as devices for undertaking computations. In this analytic paper, we describe and illustrate a four-part model to understand more fully the potential role of calculators for learning mathematics. The four elements of the model include representation, computation, exploration and affirmation. Effective use of a calculator by students learning mathematics will often involve more than one of these four components. The model has been derived from analysis of educational materials developed to support rich calculator use.*

1. Introduction

Since their inception in the early 1970's hand-held calculators have been developed by manufacturers in many ways, so that it is unsurprising that they have been prominent in previous ATCM conferences. The first calculators were capable of handling only arithmetic, and early elaborations of these were known as 'scientific' calculators, presumably because of their capacity to undertake some computations of significance to scientists (and engineers). From the mid-1980s, calculator developments included a graphics screen, and the focus of usage of the devices shifted from the sciences to education. Since the mid 1990s, hand-held calculators have been developed with increasing mathematical capabilities to become powerful, all-purpose mathematical devices.

Alongside these technical developments, calculators have increasingly been designed to cater for educational purposes across a wide spectrum of ages, and have been heavily influenced by teachers and mathematics educators, rather than by scientists, engineers and mathematicians. Consequently, the suite of mathematical capabilities designed into modern calculators includes those regarded as most relevant to school mathematics. Similarly, changes to user interfaces and to screen displays have targeted the needs and interests of students of mathematics. In addition, calculators have been designed for students of particular ages to match their mathematical needs, with devices intended for primary school children correspondingly different from those for pupils in early secondary school, students in the senior school years and undergraduate students.

Despite these substantial developments, many mathematics teachers and mathematicians still seem to regard calculators as essentially devices for undertaking computations only, and consequently regard them as problematic for education in general and pre-university education in particular, believing that students will become reliant on their calculators for basic skills. This seems to be the case despite substantial and consistent evidence of the benefits for mathematics education. (e.g., [3], [13].) So, the main purpose of this paper is to highlight and clarify the potential of hand-held calculators for education, extending well beyond computation. In this paper, we elaborate this position and outline a theoretical model to help explain the significance and potential of calculators for mathematics education.

While such a case is of importance generally, since calculators are universally available across countries, calculators are important especially for developing countries, as they are more likely to be affordable to students, schools and school systems than alternative, more expensive, forms of technology. [7] For many of the students in countries associated with the ATCM, the calculator still offers the best prospect for engaging students with learning mathematics with the help of technology, so that an analysis of the potentials is of extra significance for this conference.

2. Background

Many studies concerned with investigating effective learning of mathematics and statistics with the use of ICT have been undertaken since the mid 1990s, (see for example: [3], [4], [5], [12]). These have been concerned with the use of scientific and graphics calculators as well as computers, connected to the Internet, in the classroom at all learning levels from primary to tertiary levels. Studies have used a range of methods and results, for both developed and underdeveloped countries. Factors examined include engagement, motivation, time spent and improvement of learning as well as the practical challenges of using calculators and IT devices in classrooms.

While early studies gave positive results regarding the use of scientific calculators and so educators encouraged schools to use them, initially there was a limited adoption of learning mathematics with calculators. For example, in the 1990s a large US study of 15,000 eighth graders indicated that only about 50% of the students used calculators and only 10% used computers [5]. The lack of use was attributed in some studies to the cost of the calculators [12] but it seems just as likely that the teachers had insufficient professional development to be able to make efficient use of them in their classrooms. Comprehensive professional development plays an essential part in developing competent teachers and integrating the technology into teaching.

Since 2000 there has been serious interest and discussion about the use of ICT in the mathematics classroom and consideration about the associated advantages and disadvantages. Significant research summaries and syntheses (such as [1], [2], [3], [13] and [14]) have highlighted the positive effects of calculator use in particular. The relevance of such work for the developing world in particular was elaborated in [7].

It is hardly surprising that educators and teachers have found some challenges in incorporating calculators into the curriculum [8]. The practical challenges include the cost of calculators for each student to have one to work with, and insufficient knowledge by the teachers who may have some bias from many years of teaching without calculators. In developed countries students are generally now expected to provide themselves with a calculator, but the provision of the hardware does not necessarily mean that it will be used effectively. Another challenge is that in some countries graphics calculators and devices that can connect to the Internet are not allowed in mathematics and statistics examinations. Some authorities require that the memories be deleted; indeed this continues into some university exams too. Such practices can generate a discontinuity between learning and examinations that distorts the learning process.

An earlier paper [6] proposed a framework with three different roles for technology, described as computational, experiential and curriculum influence. The present paper modifies and extends that work to consider the variety of roles adopted by calculators in supporting mathematics learning, in order to understand more thoroughly how they might be effectively used by teachers.

3. A four-part model

Some recent work by the authors, developing educational resources for mathematics teachers, has focused on exploiting educational potentials of some scientific and graphics calculators. [9], [11] These resources have been organized according to the mathematical concepts involved (such as numbers, functions, trigonometry, calculus), and include extensive examples across many areas. The present paper draws on this body of work to highlight four different kinds of activity that are involved, in order to understand better the significance of these and differences among them. In this section, examples are offered to highlight the distinctive contributions to student thinking and learning involved. The four parts of the model are not entirely distinct, and it is often possible to view a student's activity with a calculator as related to more than one part at a time. However, the four parts are sufficiently different to warrant considering them separately, as we do in this paper.

3.1 Representation

A calculator allows students to represent some mathematical concepts on the screen. In recent times, the representations involved have been increasingly similar to those that appear in other forms of communicating mathematics, such as textbooks and whiteboards, in contrast to the earliest representations, which looked rather different from conventional notations. At the simplest level of representation, calculator numerals have changed from the original 7-segment versions shown in Figure 1 at left to those on modern calculators, shown on the right.

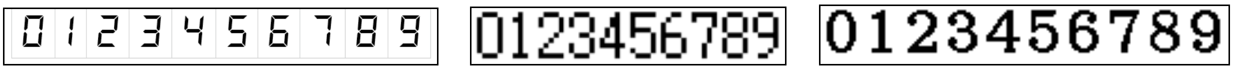

Figure 1. Developments in representing numerals on calculators

These changes are a result of increasing screen resolution, but they also mean that calculators show numbers on screens in the same way as in textbooks, of significance for young children in particular. The importance of representation goes well beyond the numerals themselves, however. The two pairs of screens from a CASIO fx-991ES PLUS calculator in Figure 2 show how representation of fractions and powers has improved from earlier to later calculators.

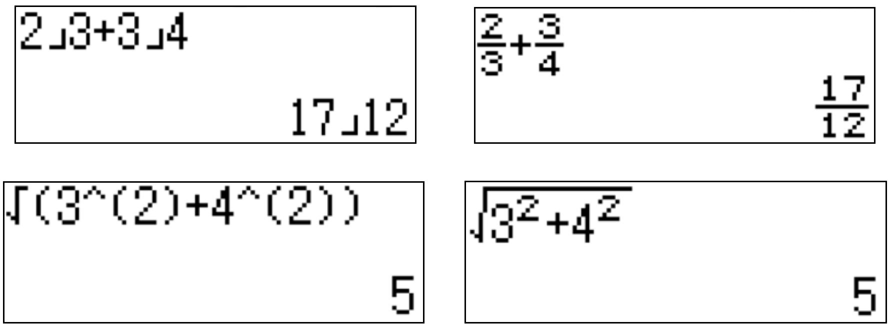

Figure 2: Developments in representing fractions and powers on scientific calculators

In effect, modern calculators have been developed to display conventional mathematical representations, including those of more sophisticated concepts than fractions and radicals, as the two screens from a CASIO fx-CG 20 indicate in Figure 3.

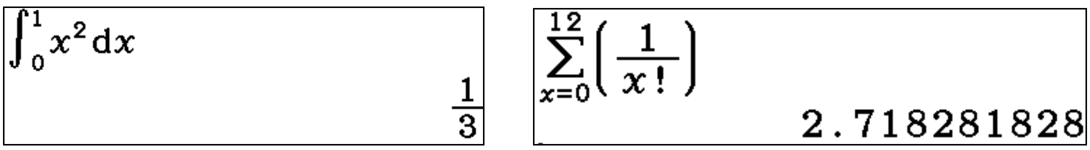

Figure 3: Representing sophisticated mathematical expressions on a modern graphics calculator

While screens of these kinds indicate that calculators represent various mathematical objects in the familiar and conventional notation, there is a much more important meaning of ‘representation’ than reference to the display. In the screens above, a calculator is being used to re-present a mathematical expression in a different form. These examples might be regarded as a form of calculation, in fact, even though they are in effect transforming an object from one form to another.

Calculators offer more powerful forms of re-presentation, helping students to see that two different forms are in some sense equivalent. Figure 4 shows some examples of this for the case of fractions and decimals on a CASIO fx-991ES PLUS. From such screens students can appreciate some profound aspects of real numbers, such as that fractions and decimals are merely different representations of the same number, that there are many different fractional representations of any particular number and that fractions and division of integers are closely related.

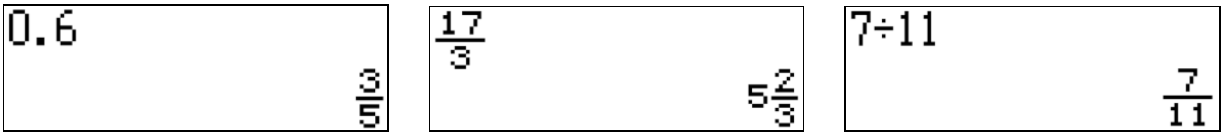

Figure 4: Representations showing equivalence of fractions and decimals

In some cases, different representations arise naturally on the calculator, while in others a keystroke (e.g. a fractions to decimals key) brings this about. Automatic calculator representations can provoke students into wondering how expressions can be transformed (that is, changed from one form to another) and considering which representation is preferred for some reason. Three examples are shown in Figure 5 from primary school, lower secondary school and senior secondary school respectively. In each case, to regard such representations as merely calculations is to misunderstand their significance for appreciating the mathematical ideas underpinning them. [10]

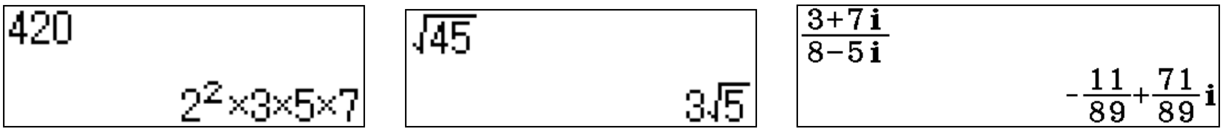

Figure 5: Calculator representations showing important equivalences

An important example of multiple representation is the so-called ‘rule of three’ on graphics calculators, which can represent a function as a formula, as a table of values or as a graph. For the linear function shown in Figure 6 on a CASIO fx-CG 20, the formula makes explicit the two parameters for slope and intercept, the table permits students to see how the function values change steadily with the independent variable, while the graph shows the linear nature of the relationship.

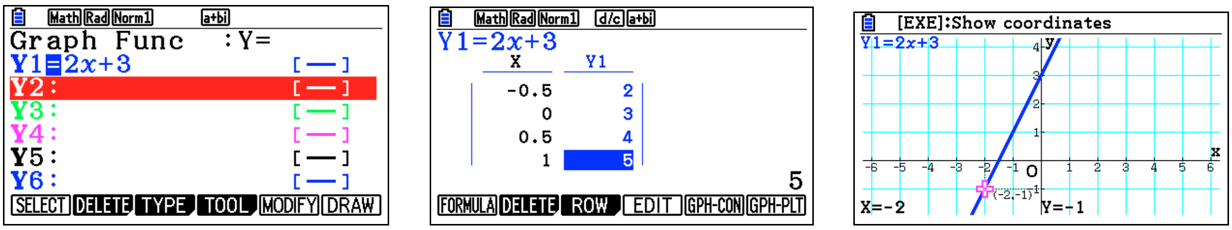

Figure 6: Representing a function symbolically, numerically and graphically

The capacity of students to move freely from one representation to another offers opportunities for learning about functions that were not present before such calculators appeared some thirty years ago. In this case, we are likely to regard students as having a good understanding of the concept of a linear function only when they can comfortably move between the three different representations. The calculator offers strong opportunities to develop precisely that kind of understanding. [10].

3.2 Computation

While we suggest that regarding a calculator solely as a device for numerical computation is to misunderstand it, it is certainly the case that the capacity of calculators to undertake calculations is of importance to mathematics education. Many practical questions can be resolved by undertaking a calculation of some sort, leading to the common view that mathematics is a useful endeavor, not just of theoretical interest, and increasing the extent to which mathematical modeling is regarded as part of the school mathematics curriculum.

When scientific calculators were first developed, their distinctive contribution to computation was to avoid the need to use mathematical table books, which had previously been the only way in which trigonometric, logarithmic and exponential values could be efficiently accessed. Modern educational calculators, except for those designed for primary school children, still include access to mathematical functions of these kinds, because it is too time-consuming to undertake arithmetic involving these without the aid of a calculator.

A consequence of having such functional values, together with arithmetic operations, on a calculator is that students are equipped to handle any computational task involving actual measurements, including importantly measurements that they have made themselves. Without access to a calculator assumed, textbooks and the mathematics curriculum are obliged to rely on artificial tasks (such as finding the volume of a sphere or radius 10 cm, rather than 10.16 cm) or theoretical treatments (such as finding the volume $4\pi x^3/3$ of a sphere of radius x cm), neither of which will help students to see that mathematics can be applied to real situations.

As calculators have developed to match the school curriculum better, however, their capacities to undertake computation have increased markedly beyond the original inclusion of tabular functions. More significant computations can now be undertaken on calculators. For example, Figure 7 shows on a CASIO fx-CG 20 graphics calculator the numerical solution of a system of linear equations. While students can solve such systems without using a calculator, the work is mostly tedious and comprises following a strict algorithm, adding little to their insights.

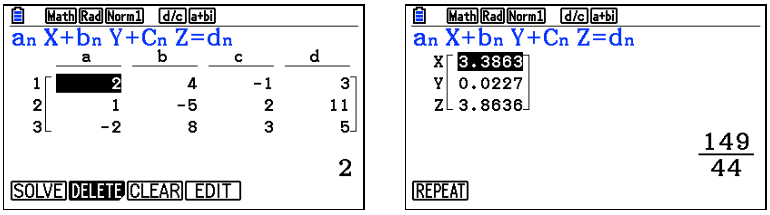

Figure 7: Routine computation of the solution of a system of linear equations

Calculators can also be used to solve equations without an alternative form of solution, as shown in Figure 8. In this case, the calculator quickly finds the solution $x \approx 1.756$ and verifies that, within the accuracy limitation of the screen, such a value for x renders the two sides of the equation the same. As well as providing a practical method for solving the equation, this aspect of the calculator suggest that reconsideration of the balance of time spent on equations may be warranted.

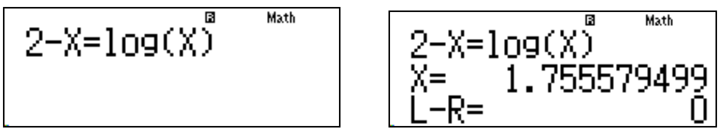

Figure 8: Numerical solution of an equation on a CASIO fx-991ES PLUS

Computation is especially important in statistics. A calculator can efficiently find descriptive statistics (such as measures of central tendency and spread), as well as more sophisticated calculations such as finding a line of best fit, a curvilinear regression, calculating residuals from a model or undertaking a statistical test of significance. Using a calculator, students can analyse actual data, rather than artificial data of the kind previously used in textbooks. Other kinds of tedious calculations are routinely available on calculators. Two examples are shown in Figure 9:

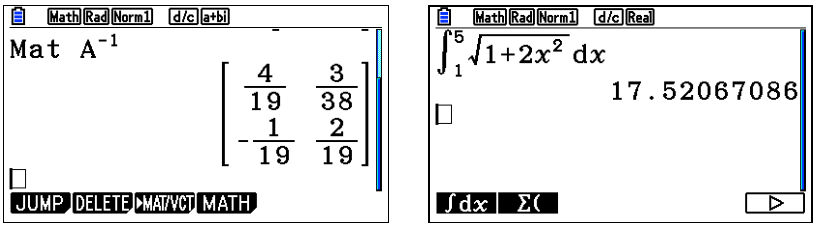

Figure 9: Examples of tedious computations completed on a graphics calculator

When students have access to calculators with significant computational capabilities like these, they can devote their time to important tasks like constructing equations, setting up matrices and representing situations with integrals, rather than becoming highly skilled with undertaking the necessary computations by hand, previously unavoidable. Such observations suggest that fresh balances between hand computation and machine computation might be sought, to allow emphasis on the most fundamental mathematical ideas, rather than an excessive amount of time spent developing manual computational expertise. For example, access to numerical integration and numerical equation solution raises issues on the appropriate emphasis in the mathematics curriculum to methods of integration and approaches to solving equations respectively.

3.3 Exploration

A significant capability afforded by a personal tool such as a hand-held calculator involves students exploring ideas for themselves, a form of experimental mathematics. Thus, rather than restricting attention to tasks set by a teacher or a textbook, students can undertake explorations for themselves, possibly with some guidance from a teacher. A good deal of educational literature has suggested that students can learn a great deal through their own thoughtful activity, including learning by discovery, in contrast to learning directly from others, such as a teacher.

To illustrate this concept, consider the linear function described in Section 3.1 and shown in each of the three representations of a symbolic expression, a table of values and a graph. With encouragement, students can modify the function by changing its slope or its y-intercept, and then observing the effects on the table of values and the graph. Through such activity, they can see the effects and hence the meaning of each of these two parameters through their own actions, and begin to understand their significance more deeply than would be the case otherwise.

Equivalent fractions can be explored on a scientific calculator, on which students can see that many different fractions are represented by the same decimal, by using the calculator capacity to switch between fractions and decimals. Students can be asked to find further examples of these, in order to understand better the key idea that many fractions can be written to represent the same number (in this case, the decimal number 0.3), as shown in Figure 10.

Figure 10: Exploring different representations of 0.3 on a CASIO fx-991ES PLUS calculator

Such explorations can be relatively open-ended, and directed by students themselves, which gives rise to some of the educational power involved. In a more sophisticated way, the screens below show a function and its derivative defined and then graphed simultaneously.

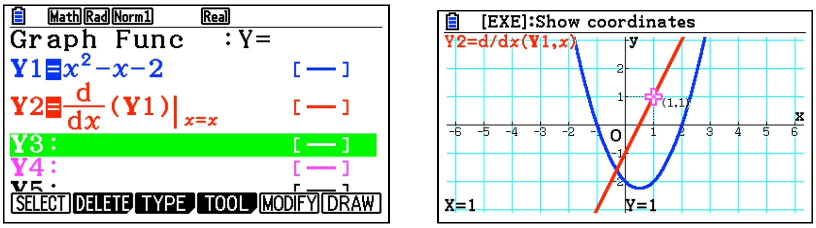

Figure 11: Exploring a quadratic function and its first derivative on a CASIO fx-CG 20

The graphs show that the function is quadratic and the derivative function is linear; students can learn much about the nature of derivatives by discussing it with other students. For example, the zero of the derivative occurs at the turning point of the quadratic function, the parabola is increasing to the right of that point and decreasing to the left. However, the value of the calculator arises from the possibility of exploring this situation actively. For example, by changing the parabola slightly and studying the consequences, students might be lead to discover for themselves that a vertical transformation of the function does not change the derivative, as shown in Figure 12.

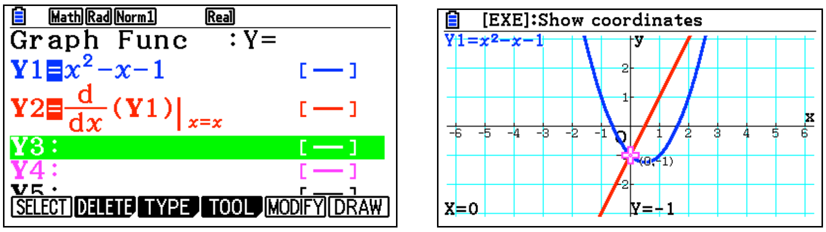

Figure 12: Exploring a modified quadratic function and its first derivative on a CASIO fx-CG 20

Alternatively, if the leading coefficient of the parabola is changed to be negative, the derivative function will be seen to be a line with negative slope, as shown n Figure 13.

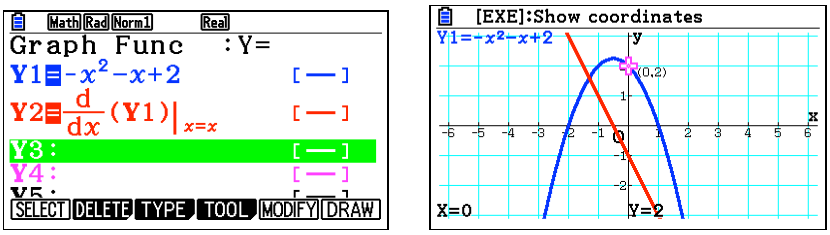

Figure 13: Further exploration of a quadratic function and its first derivative

Changing the function to a cubic offers opportunity for fresh insights. Exploratory work of these kinds can be discussed with other pupils or with the entire class, using a shared screen.

As a final example, modern calculators have inbuilt functions that have been specifically designed for exploration. A good example of this is the Probability Simulation applet shown in Figure 14 from the CASIO fx-CG 20. In using this applet, students choose a probabilistic experiment with which to engage. The screens show a choice of a dice rolling experiment. In this case, a simulation of 200 rolls of a pair of six-sided dice has been shown as a bar chart. Students will obtain a different chart each time, and different students will have (slightly) different results. Students using this application can see the effects of increasing the number of rolls, with increasing stability and symmetry of results. Work of this kind is very helpful to develop good intuitions about probability and an appreciation of the idea of long-run relative frequency.

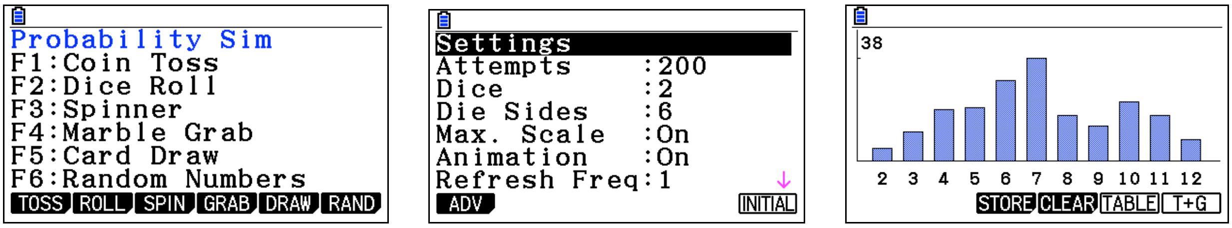

Figure 14: Using simulation to explore random phenomena on a graphics calculator

Calculators can be used to engage in exploratory work in many areas of school mathematics, not only in the few illustrative cases chosen here. There are many other examples of exploratory activities in the two books [9] and [11], with a suite of these associated with every module.

3.4 Affirmation

The fourth aspect of activity made available by a calculator involves affirmation. By this, we mean that students use the calculator to reassure themselves about the quality of their mathematical thinking in some sense. While it might be argued that sound use of a calculator should always involve some intuitions and informed hunches about a likely result of some operations, which might be affirmed or otherwise in practice, affirmation can be more explicitly involved as well.

An early use of affirmation involves students completing calculations mentally or by hand on paper and then using the calculator to check their result. Indeed, when teachers are concerned about students developing manual expertise with computation, a familiar educational task involves completing exercises without a calculator and then using the calculator to affirm them. While this is a relatively unsophisticated use of a powerful tool, it might still have a place in school. Although it is of dubious value for students to use as an alternative to recall of tables (such as 9 x 7), it would be more defensible to use in practicing mental methods of computation and approximation (such as 9.38 x 6.8). There are more sophisticated uses of verification, however. Some calculators have a Verify mode, explicitly designed for students to check their thinking. Figure 15 shows some examples, where a response of ‘True’ or ‘False’ is shown for the given inputs on the first line.

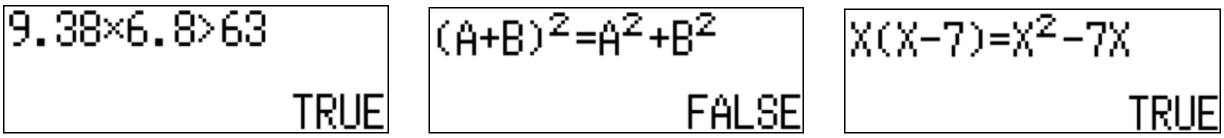

Figure 15: Using Verify mode on a CASIO fx-82AU PLUSII calculator to check thinking

The student is checking their thinking in these examples, not relying on the calculator to generate the responses. The first example above illustrates how this facility might be used to hone mental approximation skills. In a limited way, as the two examples on the right demonstrate, students can use this facility to explore the nature of identities and equations, key aspects of elementary algebra, making use of the idea of a variable.

Even when a Verify mode is not available in a calculator, the idea of affirmation is still educationally sound. A good example concerns identities, which can be explored through tables and through graphs on a graphics calculator. In the example in Figure 16, both the table and the graph reflect the fact that, for all values of the variable, the two expressions have the same value. This is precisely the meaning of an identity, in this case $\sin 2x = 2 \sin x \cos x$.

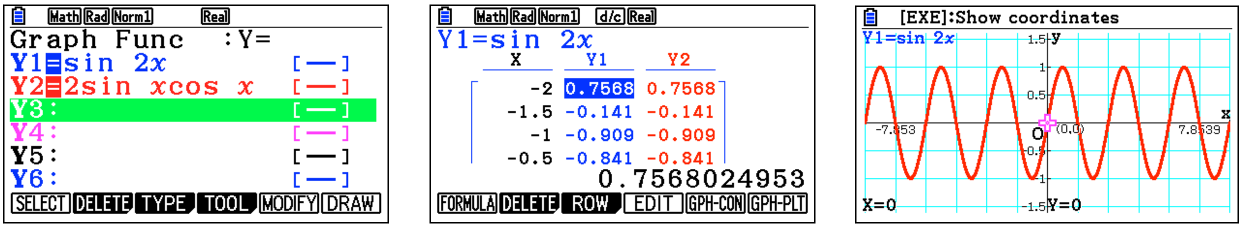

Figure 16: Exploring an identity on a CASIO fx-CG 20 graphics calculator

Each of the two table columns has the same values and the two graphs are overlapping, making visually clear that there is only one graph showing. So, the calculator has here been used to affirm the relationship. Of course, a mathematical argument is necessary to prove that it is an identity: the calculator can merely affirm that it seems to be so; in doing so, it potentially leads to and motivates the concept of a mathematical proof.

4. Implications of the model

The four-part model proposed is not intended to suggest that students use their calculators in only one of the four ways in a particular activity. Indeed, it is quite likely that sound use of a calculator will often involve three or even four parts of the activities described in the model at the same time.

The model suggests that educational discussions and policies regarding the use of calculators ought to be carefully focused, recognizing that different ways of using calculators may be regarded as of differential value in school mathematics. It seems especially important that both teachers and curriculum developers not regard calculators as beneficial only to undertake arithmetic computations. Materials used to support the work of teachers, as well as textbooks intended for students, will be improved if explicit attention is paid to the different ways in which calculators can be used, and advice given on using calculators for purposes additional to computation. While this is clearly a focus of materials produced to support the educational use of calculators, it seems important for it to be a focus of materials developed for general use as well, such as textbooks.

The model also invites a reconsideration of the relationship between examinations and calculators. When calculators are banned from use in examinations because of their computational capabilities, a typical response is to not use calculators in classrooms. In practice, a flow-on effect of prohibition of calculators in assessment is that students are denied access to the other aspects of calculator use, related to representation, exploration and affirmation. Better solutions to concerns about computation are needed, such as conducting some examinations without calculators, while allowing their use in other examinations, as happens in several different ways in many countries, including Australia.

The educational significance of calculator use seems to have been undermined by narrow views of their role, focusing on computation. In this paper, we have described and illustrated three other ways in which calculators might contribute to mathematics education. To increase the likelihood that calculators are used effectively, it is important to recognize that alternative roles to computation have an important part to play in learning mathematics. The theoretical model outlined here can be used both to interrogate the use of calculators in present practice and to improve it.

References

[1] Burrill, G. et al (2002) *Handheld graphing technology in secondary mathematics: Research findings and implications for classroom practice.* Michigan State University: Texas Instruments.

[2] Drijvers, P. & Weigand, H.-G. (2010) The role of handheld technology in the mathematics classroom. *ZDM Mathematics Education*, 42, 665-666.

[3] Ellington, A.J. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes. *Journal for Research in Mathematics Education.* 34(*5*), 433-463.

[4] Forster, P.A. & Taylor, P.C. (2003). An investigation of communicative competence in an upper-secondary class where using graphics calculators was routine. *Educational Studies in Mathematics*, 52, 57-77.

[5] Huang, S.-Y. L. & Waxman, H. (1996) Classroom observations of middle school students' technology use in mathematics. *School Science and Mathematics*. 96(*1*), 28.

[6] Kissane, B. (2002). Three roles for technology: Towards a humanistic renaissance in mathematics education. In A. Rogerson (Ed.) *The Humanistic Renaissance in Mathematics Education: Proceedings of the International Conference* (pp 191-199), Palermo, Sicily: The Mathematics Education into the 21st Century Project.

[7] Kissane, B. & Kemp, M. (2012) The place of calculators in mathematics education in developing countries. *Journal of Science and Mathematics Education in Southeast Asia*. 35(*2*), 102-118.

[8] Kissane, B. & Kemp, M. (2012). Calculators and the mathematics curriculum. In W.-C. Yang, M. Majewski, T. de Alwis & K. Khairiree (Eds.) *Creative and Critical Thinking in Mathematics Through Technology: Proceedings of the 17th Annual Conference of the Asian Technology Conference on Mathematics.* (pp 178-187) Bangkok, Thailand: ATCM Inc.

[9] Kissane, B. & Kemp, M. (2013) *Learning Mathematics with ES PLUS Series Scientific Calculator.* Tokyo: CASIO, Inc. Retrieved 18 August 2014 from http://edu.casio.com/education/activity/

[10] Kissane, B. & Kemp, M. (2013) Conceptual development and the modern scientific calculator: using a forgotten technology. In W.-C. Yang, M. Majewski, T. de Alwis & I. K. Rana (Eds.) *Technology Integration into Mathematics Curriculum: Proceedings of the 18th Annual Conference of the Asian Technology Conference on Mathematics.* (pp 178-187) Mumbai, India: Mathematics and Technology LLC.

[11] Kissane, B. & Kemp, M. (2014) *Learning Mathematics with Graphics Calculators.* (In press, at http://edu.casio.com/education/activity/)

[12] Odera, F. & Ochanda, O. (2011) Challenges in the use of scientific calculators in the teaching and learning mathematics in secondary school education. *International Journal of Information and Communication Technology Research*, 1(*5*), 230-238.

[13] Ronau, R., Rakes, C., Bush, S., Driskell, S., Niess, M. & Pugalee, D. (2011). *NCTM Research Brief: Using calculators for learning and teaching mathematics*. Retrieved 29 July 2012 from http://www.nctm.org/news/content.aspx?id=31192

[14] Ruthven, K. (1996) Calculators in the mathematics curriculum: The scope of personal computational technology. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.) *International handbook of mathematics education.* (pp 435-468). Dordrecht: Kluwer.