The impact of a technology-rich intervention on grade 7 students' skills in initial algebra

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Abstract: *This paper describes a classroom experiment on the use of digital technology in initial algebra. Indonesian grade seven students of 12-13 year-old took part in a four session teaching sequence on beginning algebra enriched with digital technology, and in particular applets embedded in the Digital Mathematics Environment. The intervention aimed to improve students' conceptual understanding and procedural skills in the domain of equations in one variable. The qualitative analysis of written and digital student work, backed up with video observations during the experiment, reveal that the use of digital technology affects student thinking and strategies dealing with equations and with related word problems. Practical and theoretical consequences of the results are discussed.*

1. Introduction

Mastering algebra, a core topic in secondary school mathematics as a gateway to either advanced study or professional work ([14]), is crucial for students' future all over the world. Indonesia is not an exception to this view. The 2011 Trends in International Mathematics and Science Study (TIMSS), however, shows that Indonesian students have low performances in algebra: ranked in 38th position out of 42 countries ([18]). This raises the question of how to improve Indonesian student performance, and of finding possible approaches to enhancing student conceptual understanding, and skills in algebra.

Nowadays, Information and Communication Technology (ICT) plays an increasingly important role in daily life, mathematics education, and algebra education in particular. The National Council of Teachers of Mathematics, for instance, claimed that "technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology" (see [19], p.1). Several studies reveal that the use of ICT in general can have positive effects on student mathematics achievement ([16]) and on students' perception towards mathematics ([2]), and can attract students in doing mathematical explorations ([10]). In algebra education, ICT use contributes significantly to its learning and teaching ([21]). For example, the use of digital tools in algebra education can promote students' development of both symbol sense and procedural skills ([5]), and may foster the development of the notion of the function concept ([8]). Furthermore, the use of a digital environment can support students' mathematical problem solving in algebra ([26]).

In response to this worldwide use of technology, the Indonesian Ministry of National Education set up a policy that introduced ICT as a new subject for secondary schools ([7]), and suggests to integrate the use of ICT in all school subjects, including in mathematics. However, this integration is relatively new, the mathematics teachers' training on using ICT is still limited ([20]). As a result, the potential of ICT for enhancing the quality of mathematics and algebra education is still unexploited.

Taking the above into account, we have an ample reason to set up an ICT-rich teaching experiment to investigate the impact of the ICT use on student conceptual understanding and procedural skills in algebra. In this paper, we report on the results of this enterprise.

2. Theoretical framework and research question

This section addresses the theoretical framework, including difficulties in initial algebra and the didactical functions of ICT in algebra education, and the research question of this study.

2.1 Difficulties in initial algebra

In an earlier study, we have identified five categories of difficulties in initial algebra ([12]). First, difficulties in applying arithmetical operations and properties in numerical and algebraic expressions include difficulties in adding or subtracting like terms (e.g., [11]; [17]); in applying associative, commutative, distributive, and inverses properties; and in applying priority rules of arithmetical operations (e.g., [6]; [27]). Second, difficulties dealing with the variable concern understanding it as a placeholder, a generalized number, an unknown, or a varying quantity ([6]; [11]). Third, the difficulties in understanding algebraic expressions include the parsing obstacle, the expected answer obstacle, the lack of closure obstacle, and the gestalt view of algebraic expressions ([1]; [23]). Fourth, the difficulties in understanding the different meanings of the equal sign concern difficulties in dealing with the equal sign, which usually invites a calculation in arithmetic, while it is a sign of equivalence in algebra ([11]; [15]). Fifth and final, the category of mathematization concerns the difficulty of transforming the problem situation to the world of mathematics and vice versa, and to reorganize the symbolic world of mathematics ([24]; [25]).

These five categories serve as a point of departure for analyzing observable student difficulties in learning of algebra. To better understand the background of these difficulties, we use the lens of operational and structural views on algebraic activity ([13]). This lens originates from Sfard's theory of reification, that is, a transformation of a process performed on an accepted mathematical object to become a new object. According to [22], an abstract notion, such as an algebraic expression, can be perceived in two different complementary ways: operationally as a process and structurally as an object.

2.2 Didactical functions of ICT in the learning and teaching of algebra

In [9], Drijvers, Boon and Van Reeuwijk distinguish three didactical functions of technology in algebra education: a tool for doing mathematics, an environment for practicing skills, and an environment for developing algebraic concepts. In the first function, technology acts as an assistant to carry out algebraic routine procedures, such as expanding algebraic expressions and drawing graphs, and the user does not necessarily know and understand how the technology produces the outcomes. In the second function, technology may offer feedback to students' responses ([4]); a variation of tasks to avoid repetition ([5]); and a compatibility of the technological and the paper-and-pencil environment to guarantee transfer of notation ([5]). In the third function, technology is aimed at guiding the development of algebraic thinking. According to Beeson's glass box principle ([3]), the transparency of the representations and techniques of the ICT environment are crucial for fostering conceptual understanding because it provides an opportunity for students to perceive how the technology produces mathematical outcomes.

For the purpose of this study, the use of technology is devoted to the second and the third function. Ideally speaking, these two functions go hand in hand and are supported by ICT in an integrated way: conceptual understanding underpins the acquisition of skills, and the mastery of procedural skills, in turn, may strengthen the conceptual understanding.

2.3 Research question

The research question of the present study is: *What is the impact of the use of digital technology to students' conceptual understanding and skills in initial algebra?*

The domain of initial algebra in this study includes equations in one variable and the related word problems, which in the Indonesian mathematics curriculum is intended for grade VII students (12-13 year-old). The digital technology is the Digital Mathematics Environment (DME), developed at the Freudenthal Institute, Utrecht University, The Netherlands, and four applets in particular, called Algebra Arrows, Cover-up Strategy, Balance Model, and Balance Strategy.

3. Methods

To investigate the impact of the ICT-rich intervention on conceptual understanding and skills in initial algebra, 139 grade seven Indonesian students (12-13 year-old) from four classes of two schools involved in the teaching experiments. The intervention included: (i) an individual paper-and-pencil pretest; (ii) four 80-minutes lessons, partly in whole-class teaching and partly in groups (of 3-5 students) in which the students work on the designed tasks making use of the four applets: Algebra Arrows, Cover-up Strategy, Balance Model and the Balance Strategy applets; and (iii) an individual paper-and-pencil posttest, similar to the pretest. Each of the four lessons after each activity with the applet ended with a daily intermediate paper-and-pencil assessment.

The activity in each of the four lessons consisted of three parts: paper-and-pencil activity, digital activity, and daily intermediate assessment as well as reflection. The paper-and-pencil activity included posing problems and whole-class discussion. The digital activity consisted of a whole-class demonstration of how to work with applets, student group digital work and discussion—in which each group was given a unique account to access the DME. Next, students were requested to individually do the paper-and-pencil daily intermediate assessment tasks. Finally, the teacher guided students to reflect upon the lesson. During the intervention lessons, the researcher, while video-taping one group of students in each class, helped these students as to act as a substitute teacher, while the teacher took care of the other groups of students. The group that was video-taped in each class was based on the teacher's recommendation, i.e., consisted of mixed ability students and were communicative in front of a camera.

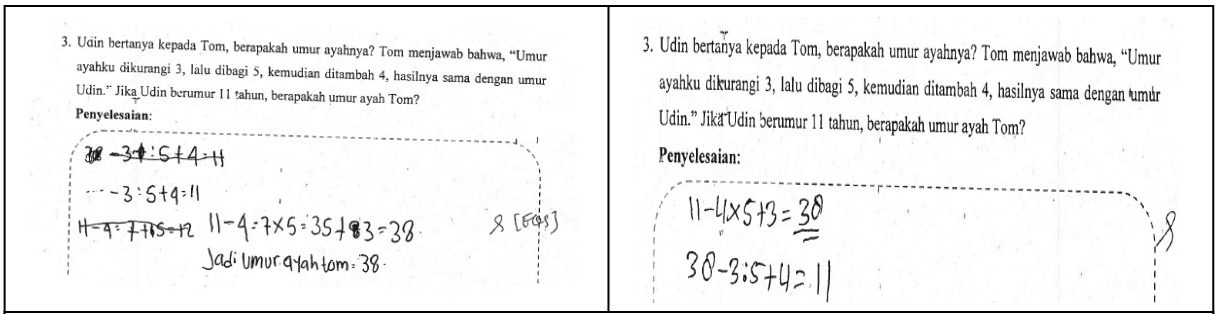
Data consisted of individual written student work of pre-and-posttest, video registrations of one group from each class, student digital work stored in the DME, student written work from four daily intermediate assessments, and field notes. An integrative qualitative analysis on these data, with the help of Atlas.ti software, was carried out to analyze the impact of the intervention on student conceptual understanding and skills.

4. Results and discussion

In the present paper, due to space limitation, we report the results of one group of students during the four lessons. This group consists of five 12-13 year-old male students with mixed ability and is considered to be representative of other groups. For each lesson, we start the description with a typical task of the lesson taken from the daily intermediate assessment and the student results. We interpret and discuss these results in terms of the theoretical framework, and if necessary back this up with appropriate evidence from the lesson observation.

4.1 Lesson 1: Algebra Arrows activity

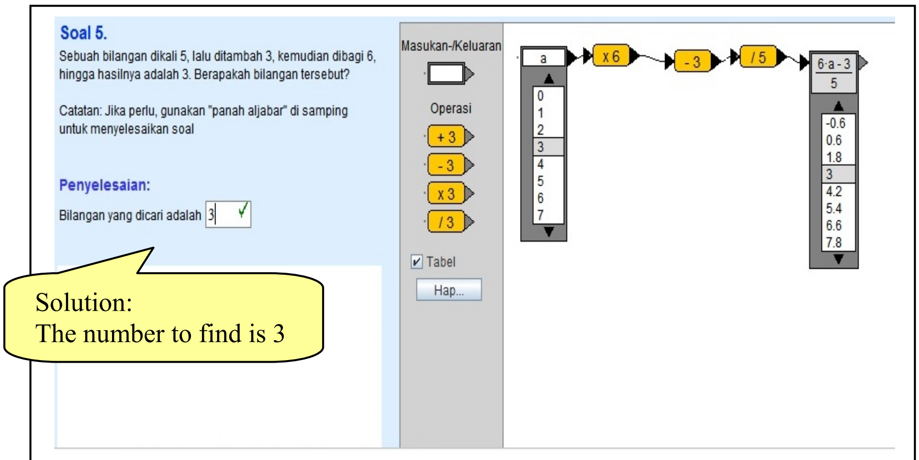
Figure 1 shows two examples of written student work on task 3 of the daily intermediate assessment after the Algebra Arrows activity. All five students solved this task correctly, all of them by using the reverse strategy.



Task 3. Udin asks Tom how old is his father's age? Tom replies that, "My father's age subtracted by 3, divided by 5, next added to 4, the result is equal to your age." If Udin is 11 year-old, how old is Tom's father?

Figure 1: Representative sample of written student work on task 3 of Lesson 1

In Figure 1 (left screen), the student first transformed the word problem into an informal equation: $\ldots - 3 : 5 + 4 = 11$. Next he solved the equation using the reverse strategy as shown in the line below. Even if the final answer is correct, the student made a notational mistake on the use of the equal sign, i.e., rather than to write $11 - 4 = 7$; $7 \times 5 = 35$; and $35 + 3 = 38$, the student wrote $11 - 4 = 7 \times 5 = 35 + 3 = 38$. In Figure 1 (right screen), rather than to first transform the word problem into an equation, the student directly used the reverse strategy to solve it, i.e., $11 - 4 \times 5 + 3 = 38$. Next, he checked the answer by substituting it into the equation: $\ldots - 3 : 5 + 4 = 11$, that is, by replacing the dots with the answer. Even if the answer is correct, the written notation is not appropriate as this violates the priority rules of arithmetical operations. The direct use of the reverse strategy for solving the word problem was probably a direct consequence of the learning process in the digital activity in which this group used the same strategy directly as, for instance, shown in Figure 2 and described in the excerpt below.



Task 5. A number is multiplied by 5, next added to 3, then divided by 6, the final result is 3. What was the number?

Figure 2: An example of student digital work in the Lesson 1

Observer: Here, the unknown number to find is not given yet. So, you should determine by yourself, with for instance *a*, *b*, *c* or *n*. This task is similar to task 1, is not it? [The students choose *a* as the variable. Interesting to note is that rather than to create an equation to represent the word problem, students directly apply the reverse strategy to solve the problem.]

Danang: [Puts a into the input box, clicks 3 from the table.] This [3] should be multiplied by 6, next subtracted by 3.

Saiful: [Then] divide by 5.

Danang: [He does the solution process in the computer to find the unknown number, that is, 3. He puts this into the answer box and presses enter. The answer is correct as shown in Figure 2.]

From the results described above, we conclude that even if the Algebra Arrows' notation did not emerge in written student work while solving problems, the two types of reverse strategies used by students in the daily intermediate assessment seem to follow from the use of the applet.

4.2 Lesson 2: Cover-up Strategy activity

We consider task 3 of the daily intermediate assessment Lesson 2—i.e., solve for positive a: $\frac{24}{(a+2)^2-1} = 3$—as a typical task for recognizing student understanding of the cover-up strategy. Out of the five students who used the cover-up strategy, three students solved the task 3 correctly. Figure 3 presents two examples of written student work on this task. The left and the right screens show a correct and an incorrect solution, respectively.

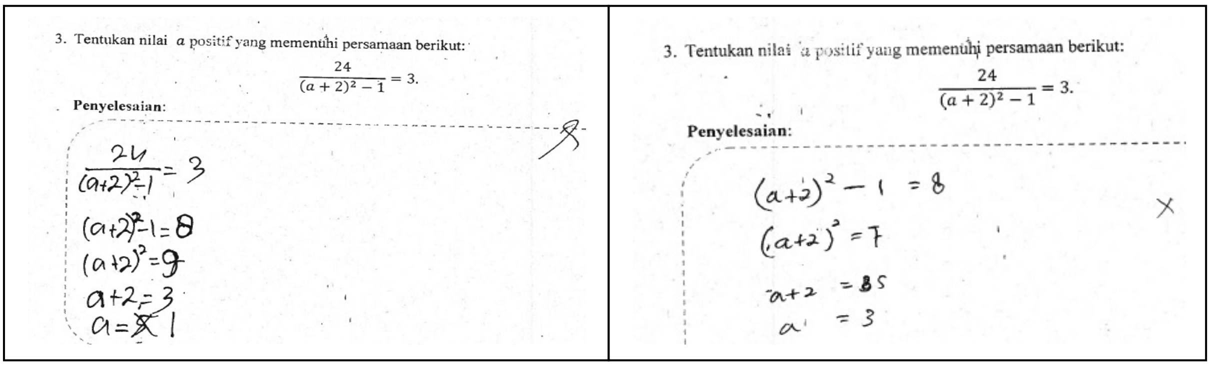


Figure 3: Representative sample of written student work on task 3 of Lesson 2

In Figure 3 (right screen), the student was successful in applying the cover-up strategy for the first step, i.e., determining the part of the equation to cover and filling in a numerical value for it, namely $(a+2)^2 - 1 = 8$. However, in the next two steps, the student made mistakes. In step 2, rather than to fill in 9 for the value of $(a+2)^2$, the student assigned 7, which is an additive inverse mistake. This suggests that rather than using the cover-up strategy, the student used the reverse strategy in an incorrect way. In step 3, the student seems not to understand how to find the inverse of a square: he subtracted 2 from 7 to get 5 rather than to find a square root.

A similar difficulty in applying the cover-up strategy was observed during the digital group work, i.e., students used an improper reverse strategy to solve an equation that can easier be solved with the cover-up strategy. This was probably the origin of student difficulties observed in the daily intermediate assessment, as described in the following excerpt.

The group is working on task 5a in the digital activity, i.e., solve for a: $\frac{18}{5a-2} = 6$. The observer finds that students have difficulties in identifying the expression to cover in the first step.

Observer: Okay, what is the first step you should do?
[The students follow the Step 1 given in the task. But, they are not sure which part of the equation should be covered first.]

Observer: Which part of the equation should you cover at first? [The students are still hesitating to do.]

Saiful:	$6 \times 18 = 108$, and $108 + 2 = 110$, next $\frac{110}{5}$. [He used an incorrect reverse strategy rather than the cover-up strategy to deal with the equation. So, the observer suggests students to follow the Step 1 properly.]
Observer:	Just choose and follow Step 1. [After some guidance, the students are finally able to solve the equation. Their solution is shown in Figure 4.]

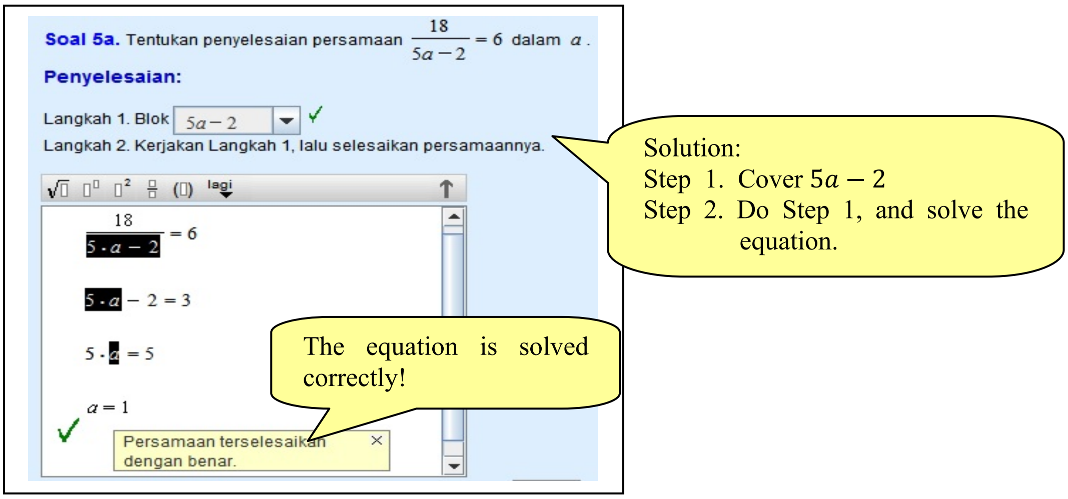


Figure 4: An example of student digital work in the Lesson 2

From the results above, two additional remarks are noteworthy. First, the written work in the intermediate assessment reveals that students have consistently used the cover-up strategy in ways that are quite similar to the cover-up strategy in the applet environment. This suggests a transfer of the applet strategy to paper-and-pencil environment. The transparent and visual character of the Cover-up applet may explain this. Second, mistakes in written student work concern the arithmetical category, including calculation errors and inverses, but they had nothing to do with the algebraic expressions or the variable category. This suggests that the applet invites students to develop on a structural view rather than on an operational view on algebraic expressions.

4.3 Lesson 3: Balance Model activity

Task 3 of the daily intermediate assessment Lesson 3, i.e., solve for x: $3x + 22 = 6x + 1$, is a typical task for recognizing student understanding in the Balance Model activity. The result shows that all students solved this task correctly. Four students presented a solution process similar to the one they had learned—indicating that it had influenced their thinking and actions—and one student provided the final answer only.

A digital activity task similar to this daily intermediate task is task 7, that is, students are required to write an equation from the given model and then to solve it: the equation to solve is $4x + 1 = 2x + 23$. In each step while solving this equation, the students did an action on the model (by moving a bag representing x or a block representing a weight) and represented the action in the form of an equivalent equation. After this group arrived at the equation $3x = 2x + 11$ and one of the students moved a bag (representing x), one of other students concluded that $x = 11$. Their solution to this equation is shown in Figure 5.

A point to note from these results concerns the balance strategy that students used in their written work. Even if the students did not have the balance models at hand during their paper and pencil work, their solution strategies seem to follow directly the Balance Model applet approach. That is, solving an equation boils down to maintaining an equilibrium of the left and the right side of the equation; finding a solution comes down to finding a numerical value of the variable, representing the weight of an object in a balance.

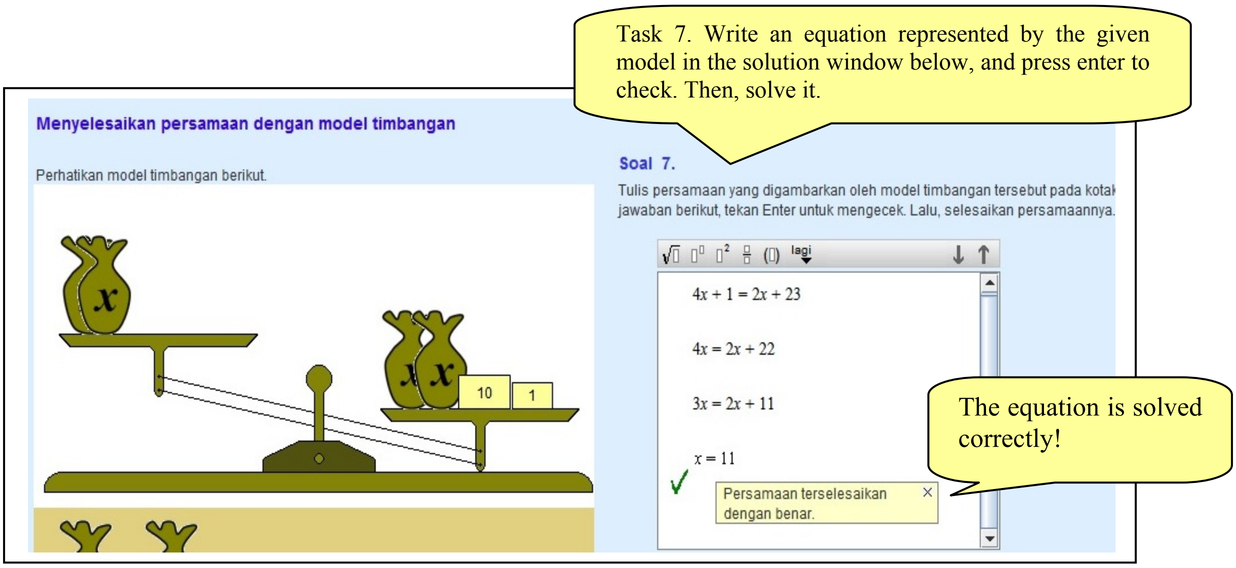


Figure 5: An example of student digital work in the Lesson 3

4.4 Lesson 4: Balance Strategy activity

A typical task to see student understanding in the Balance Strategy activity is task 3 of the daily intermediate assessment, i.e., solve for x: $9(x-1)=2(x-1)+21$. There are at least two different methods to implement the balance strategy for solving this equation. First, we subtract both sides of the equation by $2(x-1)$ to obtain $7(x-1)=21$ in the first step, next divide both sides by 7 and finally add 1 to find $x=4$ as the solution. This first method is actually a combination between the balance strategy and the cover-up strategy. To do so, a structural view on the algebraic expressions in the equation plays an important role. Second, we initially apply the distributive property to remove the brackets in the equation to get $9x-9=2x-2+21$, next we carry out the balance strategy (i.e., for instance, add 9, subtract $2x$, and divide by 7 to both sides, respectively) to get the solution $x=4$. The results show that all five students solved this task correctly using the second method, but that no student used the first method. This suggests that the integration of the balance strategy and other equation solving strategies in this activity is subtle; it was not observed in these students' written work.

Another way to see student conceptual understanding in this activity is by analyzing student work in solving word problems. Our observation showed that it was often difficult for students to transform word problems into appropriate equations. This difficulty is partly caused by, for instance, an inability to translate phrases into correct algebraic expressions as shown in the excerpt below.

Students are working on the following task:

Father is 39 year-old now. If two times Tom's age added to father's age, the result is equal to 5 times Tom's age three years later. How old is Tom now?

After reading the task, students try to represent the word problem into an equation. The observer reads the task phrase-by-phrase to guide students representing the problem into an equation.

Observer: Two times of Tom's age...
Saiful: $2t$
Obsever: Okay, good! Now, it is added to father's age.
Saiful: $2t+39=\cdots$
Observer: Good! Now, it is equal to five times of Tom's age three years later.

Danang & Rafi: [So, it is $2t + 39 =$] $5t + 3$
Observer: Which one should be multiplied by 5?
Danang: t.
Observer: Is it only t or $(t + 3)$? Please you enter what you wrote.
Saiful: [He types $2t + 39 = 5t + 3$, and presses enter.] Incorrect!
Observer: It says that 'five times Tom's age three years later'. So, what should be multiplied by 5?
Saiful: $t + 3$
Observer: Okay, so it means 5 times $(t + 3)$.
[The students represent it correctly as: $2t + 39 = 5(t + 3)$.]
Observer: Good! [Next the students remove the bracket in the equation.]
Saiful: So, now it is $3t + 39 = 5t + 15$
Observer: Good! Now, what is next?
[Next students solve the equation as shown in Figure 6.]

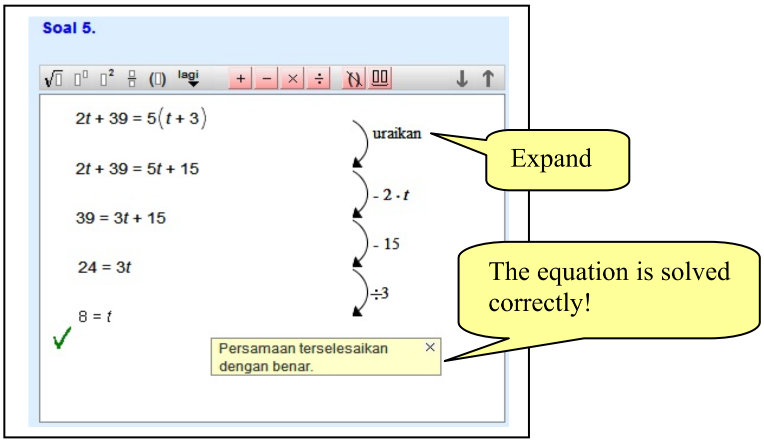


Figure 6: An example of student digital work in the Lesson 4

From these observational findings, we retain two points. First, the scarce use of a combination of equation solving strategies in student work seems to be a consequence of the absence of tasks that require students to do so. Only inserting tasks that can be solved with more than one strategy in the digital activity apparently is not enough to influence student thinking and strategies. Therefore, we conjecture that the integration of different equation solving strategies requires more attention. Second, concerning student difficulties in transforming word problems into appropriate equations, intensive attention from the teacher during the learning process seems to be a good way for future research.

5. Conclusions

From the results described earlier, we draw the following conclusions. The impact of the use of digital technology—four applets embedded in Digital Mathematics Environment in particular—includes problem solving strategies that used by students and observable difficulties that they made in both digital group work and written work. We argue that written work strategies similar to digital work strategies used by students reflects a direct effect of the applets use. The difficulties emerged in both observations of digital activity and written work to certain extent show student conceptual understanding and procedural skills.

We acknowledge that the results are still limited and focused only on the outcomes of student written and digital work, in terms of difficulties in algebra and the solution strategies that students use. We do not yet, however, consider to a larger extent about the role of applets on student conceptual understanding and procedural skills in particular. This will be a main theme in a future study addressing the intertwinement of techniques for using the digital tool and students' conceptual understanding and procedural skills.

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