Graphic Input System in Elementary Geometry for Non-visual Communication

Ryoji Fukuda
rfukuda@oita-u.ac.jp
Faculty of Engineering,
Oita University
Japan

Masato Kojo
v13E6012@oita-u.ac.jp
Faculty of Engineering,
Oita University
Japan

Abstract: *In this study, we developed a graphical input system for expressing elementary geometry in non-visual communication. In geometry, figures contain various pieces of information , that are expressed by positional relations between pares of elements. Understanding this communication is done subconsciously among sighted persons using, for example, handwritte figures. We analyze these tasks and recognized their intentions based on the hand-written curves.*

1. Introduction

In a figure of elementary geometry, several properties are expressed by its lines and points. Even a simple diagram contains numerous pieces of information. It may take a long time to gather all the possible information that could be relevant to all situations. Usually, only a few properties are relevant in a given situation, and other elements may often be ignored. For example, consider straight lines that cut through two quadrangles, (see Figure A in the Appendix). If a person was only interested in the area ratio of the two quadrangles, he would construct some triangles to calculate their areas. There are also corresponding angles in this figure; however, it is probable that he would ignore them. If text or spoken information exists on corresponding angles, this could distract from a quick understanding of the problem, when this angle equivalence has no connection with the area calculation, thereby leading to wastage of the precious time and efforts.

In visual communication, recipients obtain adequate elements of information from a simple graph. The sender does not have to consider details about the figure since the human eye and brain will select a relevant set of informational elements by inspection. Only the intelligence of the recipient is required to solve the target problem or understand a concept. On the other hand, non-visual communication does not have a common method to express geometric information. Tactile representations are often adopted; however, their ability to communicate the intended content is quite different from visual representations. In some cases, verbal descriptions are more helpful. In this study, we considered input methods for graphic content using elementary geometry for non-visual communication where the inputs are shapes, positions, names of the targets, and their mutual relations.

In our previous study, we developed a function graph input tool for non-visual communication [1] that we assumed it would be used in real-time communication. A user inputs a curve by hand; the system recognized the curve type, and an approximated curve was displayed. This graph was sent to a tactile display, and the curve type and corresponding parameters were listed in a text area. We aimed to use our input system for real-time communication and have adopted a handwriting input method. This input method recognizes the sort of the input element and relations for verbal descriptions. Elementary geometry does not have many element types: our targets a a point, angle, line segment, triangle, quadrangle, and circle. Essential contents consist of the element type(properties) and the relations among them. There are many types and relations, and we cannot grasp all of them. In this study we selected and created some functions for decision-making and input. The user's input intentions appear in the written input curves. We analyzed and evaluated these intentions to create decision and input functions.

2. Expression of Graphical Information

We focus on elementary geometric figures, where informational elements are represented by visual shapes that are connected to each other. In non-visual communication, we often use expressions, tactile output systems; a tactile display, a Braille printer, and so on. However, such methods may not reproduce the figures effectively, and hence, they cannot convey the relevant information. Therefore, we consider verbal expression methods for such content.

2.1 Nodes of Information Graphs

In this study we consider the node categories listed in Table 2.1. We can construct most figures using these concepts; however, they are not sufficient for expressing the full extent of mathematical properties. For example, a triangle consists of three line segments; however, the concepts of a triangle and a set of three line segments are quite different. The concept of area calculation is associated with the triangle, whereas the ratio of edge lengths is associated with a family of edges.

Table 2.1 Categories

Node category	**Sufficient condition for drawing**
Point	*x, y* coordinates
Angle	1 point, start and end radian.
Line Segment	2 points.
Circle	center point and radius
Arc	center point, radius, and angle
Triangle	3 points
Quadrangle	4 points

To describe various properties, we define different types of elements for each node category, as listed in the table below.

Table 2.2 Types of Nodes

Node category	**Type of Element**
Point	Origin(base point)
Angle	Right angle, Straight angle
Line Segment	Horizontal line, Vertical line
Triangle	Regular, Isosceles, Right Angled, Obtuse, and Acute
Quadrangle	Regular, Rectangle, Lozenge, Parallelogram, and Trapezium

2.2 Edges of Information Graphs

Some concepts expressed in a figure are interlinked because of the relations between two elements. We consider the following relation types:

1. Typical part of others:
   center of a circle, vertices of a triangle or quadrangle, vertex point of an angle, start and/or end point of a line segment, and so on.
2. Included in other, a picked up part:
   point on the circumference of a circle, midpoint of interval, an angle of a triangle or quadrangle, base of a perpendicular line, and so on.
3. Simple positional relation:
   intersection of two line segments, inscription and circumscription of a circle and a triangle, line segment that is a chord of a circle, a line segment that is tangential to a circle, and so on.
4. Specific positional relation:
   two orthogonal or parallel line segments, two congruent triangles, midpoint of a line segment, and so on.

Although there are many concepts, we have listed only a few commonly used above.

3. Recognition of a Closed Curve

Our system has three categories of closed curve: a circle, triangle, and quadrangle. We judge that whether an input curve is closed, using the distance between the start and end points, while the type of closed curve is determined by counting the number of angle points. This recognition therefore essentially depends on the judgment "bend or break." For example, an obtuse angled triangle is sometimes incorrectly recognized as a half circle. In this section, we present our method for evaluating this situation, to determine the correct curve.

3.1 Curvature of Handwritten Curve

In our system a user draws a curve using a mouse, and the system recognize the category, properties, or relation to other element. Let $\{p_j\}_j \subset \mathbb{R}^2$ be a handwritten curve and

$$D_j = \frac{1}{2d+1}(p_{j+d} - p_{j-d})$$

where d is constant set to $d=5$. We approximate the curvature of this point by

$$K_j = \frac{1}{|p_{j+d} - p_j| + |p_j - p_{j-d}|} \left| \frac{D_{j+d}}{|D_{j+d}|} - \frac{D_{j-d}}{|D_{j-d}|} \right| ,$$

where $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^2$. Figure 3.1 is a graph of curvature values K_j of for obtuse angles and small circles. The maximum values of the curvatures in both groups do not show a significant difference. We cannot distinguish these two curve types by a simple thresholding of the curvature. However, drawing an angle requires a clear peak with very small curvatures outside the peak point.

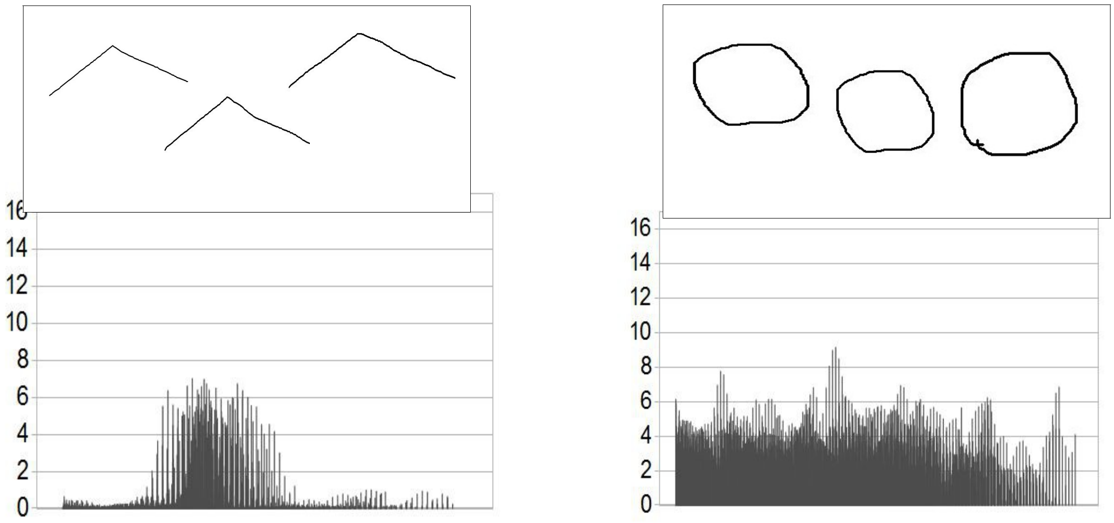


Figure 3.1 Curvature values of obtuse angles and small circles

3.2 Distinction between an Arc and Angle

If there is an angled point in a drawn curve, this point is the peak point of curvature. However, the curvature values are not always larger than those of small circles. It is not practical to distinguish arcs and angle points using curvature values alone. In the case where one draws a broken line with an obtuse angle, the lines are drawn to be very flat except in the neighborhood of the angle point with the result that the average value of the curvature may be small. We define the following evaluation value for the angle point.

$$A_j = \frac{K_j}{\overline{K_j}} \quad , \quad \overline{K_j} = \text{Average of } K_j \text{ in neighborhood of } j \text{ ,} \qquad (3.1)$$

for the neighborhood average we use 30% $\{K_i\}$ near j . Figure 3.2 is a graph of K_j and A_j . An average value of K_j is rather small for in obtuse angle point data, and ranges of the A_j are completely separated for angles and arcs.

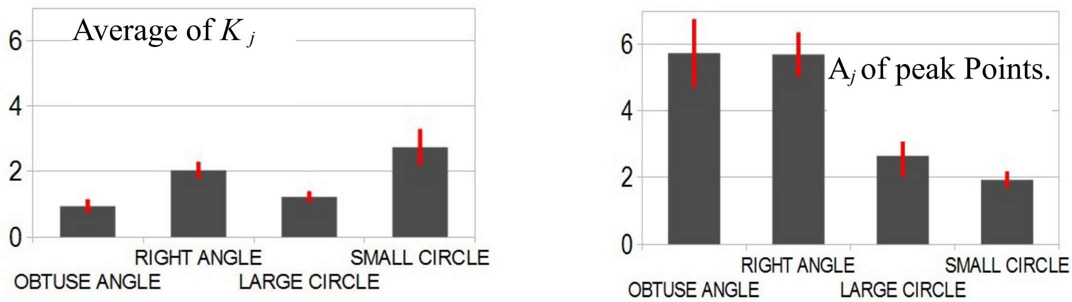


Figure 3.2 Average Curvature of Angles and Evaluation Values for Angle Points.

4. Angle Recognition

A right angle and straight angle have special definitions. These are often input without any explanation, and recipients understand the intent without confusion. Consider the case where a user draws a right-angled triangle; he will draw a typical figure that everyone recognizes as a right angle triangle using the symbol shown in Figure 4.1. Conversely, a user will never draw a triangle with an angle close to a right angle when he does not intend to draw a right-angled triangle. This input system must recognize a right angle, when the user draws one. We analyze the angle data for several situations.

4.1 Right Angles

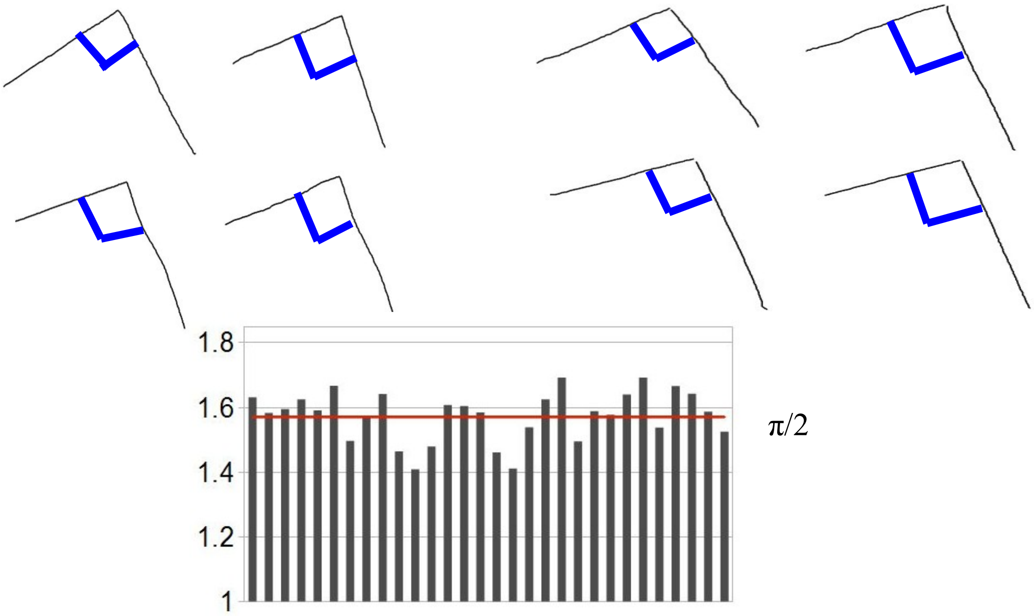


Figure 4.1 Right Angle Values

Triangles and quadrangles are sometimes drawn in one-stroke. These angles tend to be rounded especially when they are drawn quickly. First, we extract the angle points by using the method described in the previous section. Let $\{p_j\}_j \subset \mathbb{R}^2$ be a line segment, i.e., a family of points between two angle or edge points, and A be the 2×2 covariance matrix of $\{p_j\}_j$. Next, we define the direction of this line segment by the normalized principal vector (i.e. normalized eigenvector corresponding to the highest eigenvalue). Figure 4.1 shows example angle. The left subfigure shows a set of angles from one-stroke data and the right shows angle from two-stroke data. Each data set contains 30 points of angle data. Using the two direction vectors $v_{1,} v_2$, the angle value θ is given by $\theta = \cos^{-1}\left(\frac{v_1 \cdot v_2}{|v_1||v_2|}\right)$ (in radians). In both cases, the distributions of angle values are similar. Almost all values are in $[\frac{\pi}{2} - 0.2, \frac{\pi}{2} + 0.2]$.

4.2 Recognition of Parallel Angles

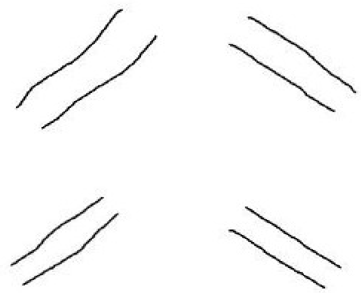


Figure 4.2 Parallel Angles

The word "parallel" is one of the most important concept in elementary geometry. Two line segments are sometimes input by hand, with the intention of being parallel to each other. It might be confusing when a line segment is drawn almost parallel to another line in a situation where these lines are in fact not parallel to each other. Thus, it must be recognized when a user has drawn a line segment, intending them to be parallel.

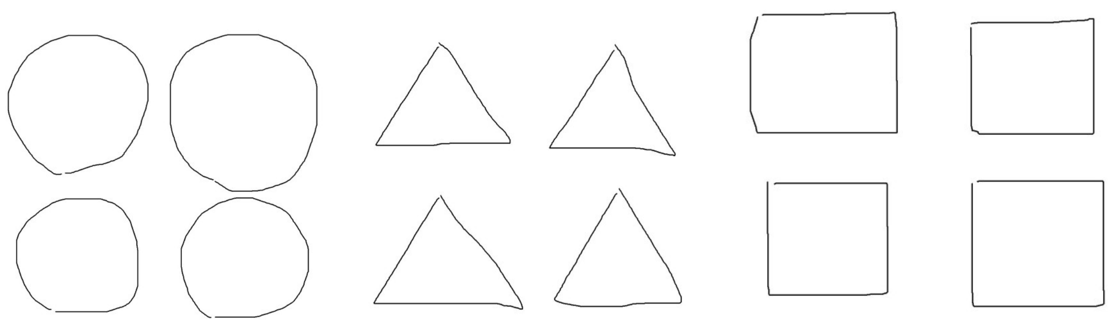
Thirty pairs of line segments were drawn by hand, with intention of drawing parallel line segments. The direction vectors were given as the principle vectors of the covariance matrices. The angle between the two direction vectors is defined as a value in $[0,\pi]$. All angles in the data are within $[\pi-0.2,\pi]$. Thus we consider that 0.2 (radians) is a negligible difference between two handwritten input line segments.

5. Positional Relation Recognition

The concept of an input curve influenced by the positional relations among the elements of a figure. Consider the case when one draws a line segment in a circle. For example, its meaning changes when the end points fall on the circle's edge, or when it passes through the center. In a narrow sense, a handwritten curve never starts on a circle's edge and never passes through the center point, however, one can usually express these intentions by rough handwritten figures.

5.1 Closed Curve

A one-stroke element is a closed curve if the start point is same as the end point. This condition is not satisfied in actual curves. Graphs of the distances are shown in Figure 5.1. These distances are between the ends of a drawn curve, that was input as a closed curve. There are 30 circles, triangles, and quadrangles. For all of them, the distance between the end points is less then 0.1.



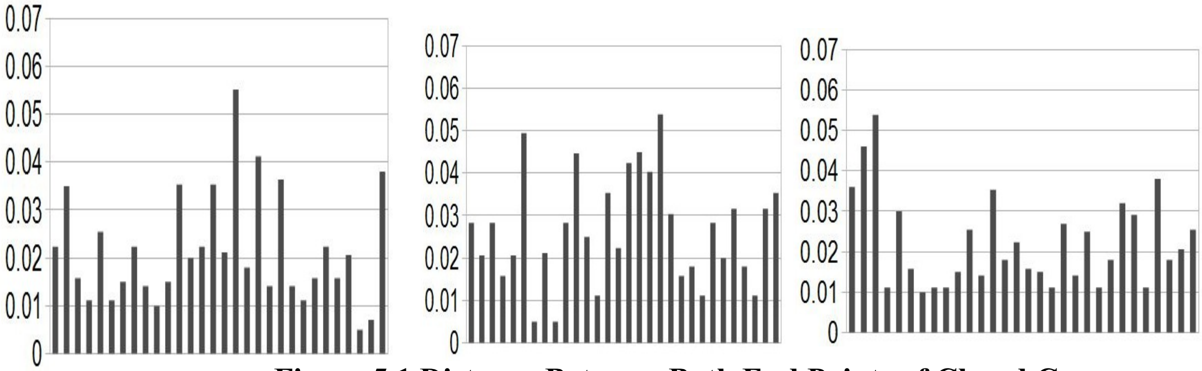


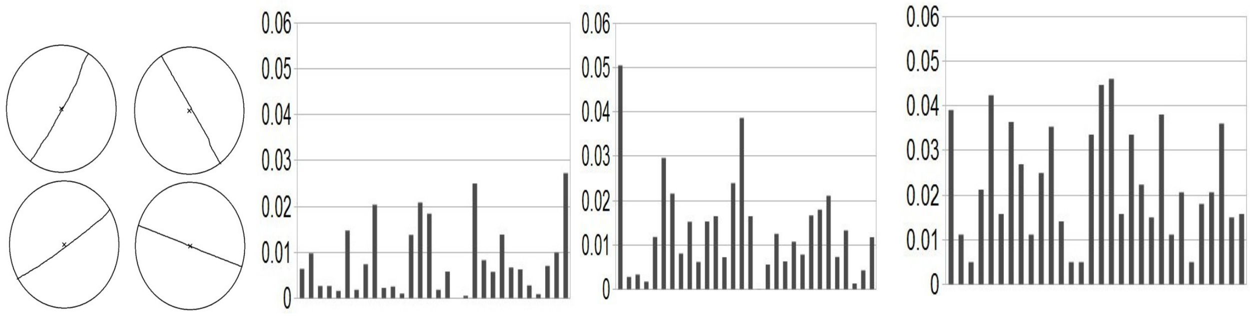
Figure 5.1 Distance Between Both End Points of Closed Curves

5.2 Circle Chord, Diameter

Circle chords, diameters, and radii are often used in elementary geometry. A line segment correspond to a circle if:

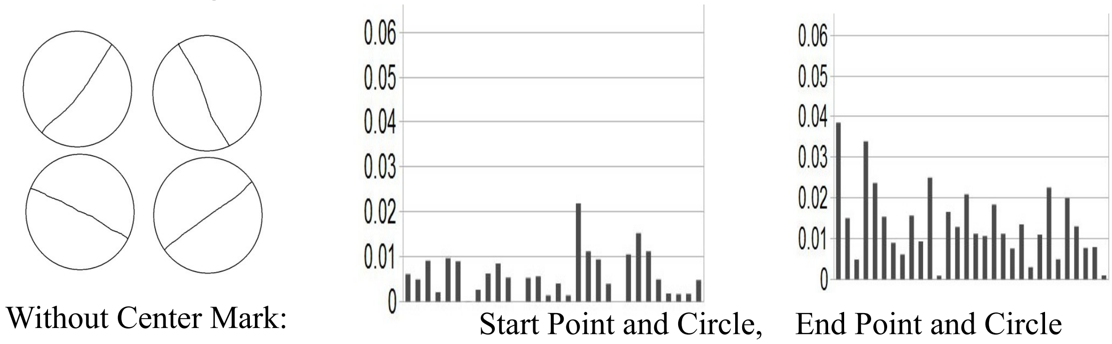
1. The start and end points fall on the circle's edge line.
2. The circle center is on the line segment.
3. The segments ends fall on the circle's center and edge.

We prepared machine-drawn circle figures of two types: with and without center marks. For each figure, a circle chord was input by hand and the input curve deleted after saving the data. We iterate this task 30 times. For the figure with a center mark, chords were to be input to pass through the center mark, i.e., to indicate a diameter. Figures 5.2 and 5.3 show graphs of the distances distributions between each end point and a circle and between the circle center and an input line, respectively. The line data used in Figure 5.2 were drawn with center marks, and lines in Figure 5.3 are drawn without center marks.



With Center Mark: Start Point and Circle, End Point and Circle, Center Point and Line

Figure 5.2 Distance From Points

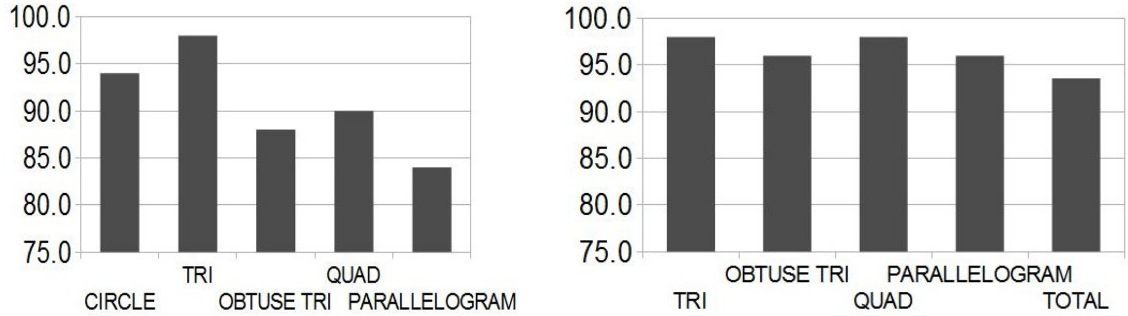


Without Center Mark: Start Point and Circle, End Point and Circle

Figure 5.3 Distance From Points (no center mark)

6. Recognition Results

In this section we evaluate the recognition method for closed curves. Target closed curves are circles, triangles, and quadrangles. Essentially, this recognition consists of the extraction of angle points and closed curve recognition. For the extraction, we used the Equation (3.1) defined in Section 3. We considered the following closed curve types: circles, acute or right angle triangles, obtuse angle triangles, squares (nearly equal to right angle), and quadrangles with obtuse angle. Fifty curves were drawn for each curve type. Triangles and Quadrangles were written with one-stroke and in multiple-srokes. Figure 6.1 shows graphs of their recognition ratios.



One Stroke Lines Several Strokes Lines

Figure 6.1 Recognition Ratio

The main reasons for miss-recognition were the large distances between start and end points and obtuse angle recognition errors. Figure 6.2 shows examples of misrecognized curves.

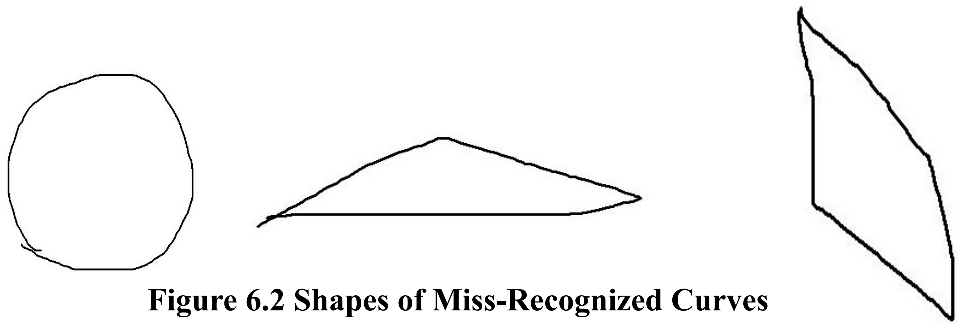


Figure 6.2 Shapes of Miss-Recognized Curves

7. Conclusions

We have created a graphical input system for non-visually communicating elementary geometry . The system recognizes several curve categories and positional relations, however, these may not be sufficient considering various situations. More curve types and relations need to be automatically recognized. The graph structures of a created document could then become very complicated, and we will need to identify relevant ones according to the situation. These are problems to be addressed future work.

References

[1] Fukuda, R. Miura, A. Ubiquitous Mathematical Graphic Viewer for Visually Impaired Students, Proceedings of the 16th Asian Technology Conference in Mathematics,2011

Appendix: Example Figure for Introduction

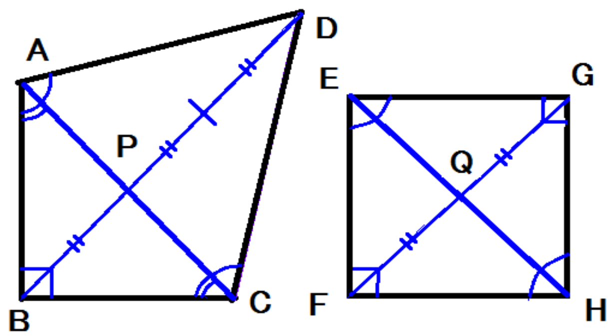


Figure A Two Quadrangles

Purpose of the figure:

Calculate the area ratio of two quadrangles.

Necessary Information:

In Figure A:

2 BP = PD, FQ=QG.

Additional Information (Not In Figure A):

AC=EH, BP=FQ.

Unnecessary Information:

∠DAP=∠DCP, ∠GEQ=∠GHQ.

∠B=∠F=∠G=∠R(right angle).

Unnecessary informational elements do not distract from problem solving if they are presented as visual information. However they distract when expressed as speech or text.