ICT (I see it) in Math education

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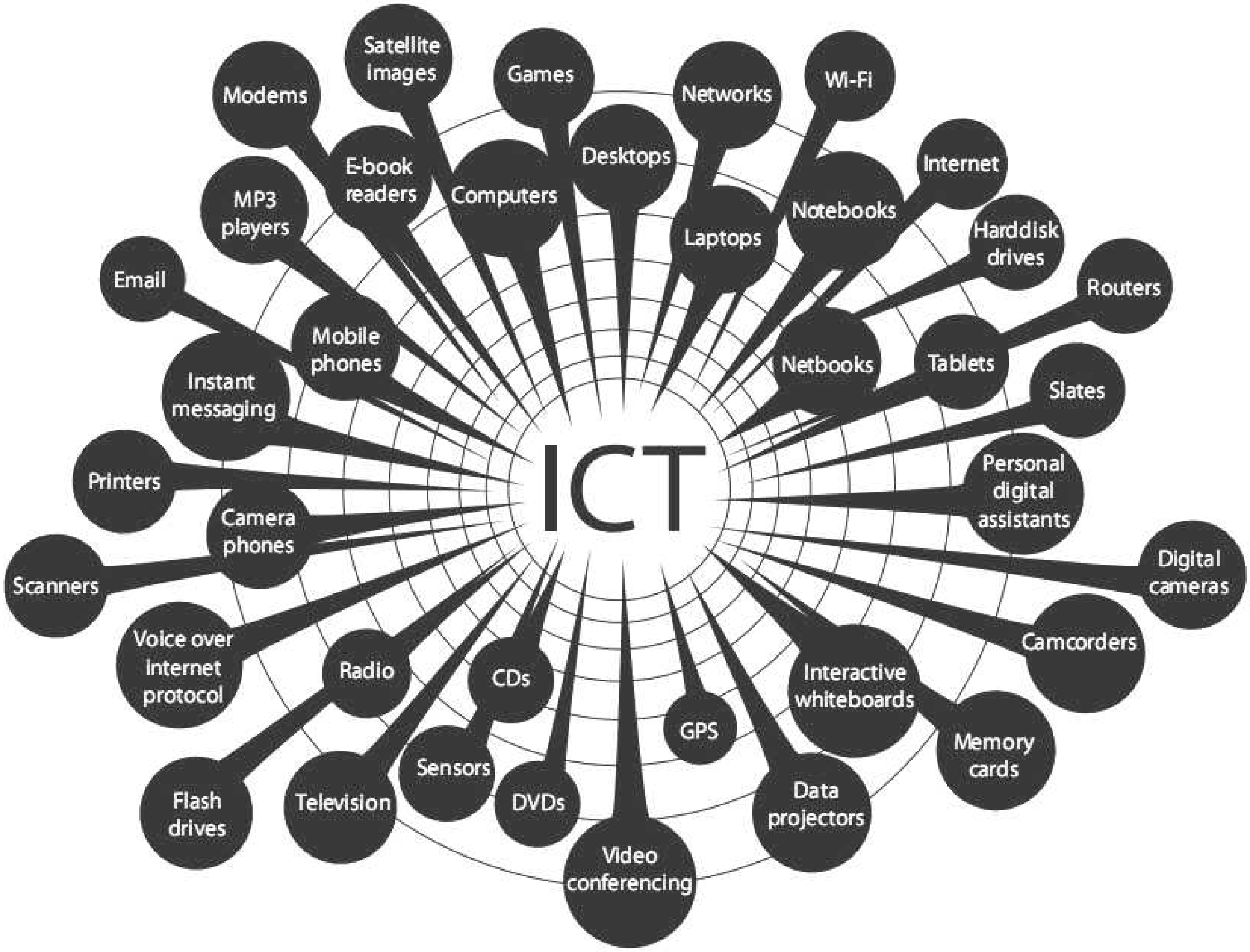
Abstract

ICT in education has been a buzz word for about 15 years now; still the debate is on the various issues of integrating ICT in education: -what is ICT? -What role it can play? - The level at which it is suitable (primary/middle/secondary/higher education)?-how effective it is and it can be? - Issues like teachers' orientation?-outcomes versus the cost? - and some on. Each specific country has its own answers to above questions (see [1]). For some of the Indian initiatives and outcomes, one may look at [2]. The aim of this paper is to look at some suggestions for answering first two questions in the context of math education. We will illustrate with examples how ICT tools can help effectively support various pedagogical components in math education at various levels of math education.

1 What is ICT and how it can help

1.1 What is ICT

There appears to be a misconception that ICTs generally refers to computers and computing related activities. It was about a quarter of a century ago that the first micro computer was marketed; about twenty years back the first graphing calculator was launched; and about fifteen years ago the World Wide Web started. Pelgrum and Law (2003) state that near the end of the 1980s, the term computers was replaced by IT (information technology) signifying a shift of focus from computing technology to the capacity to store and retrieve information. Here are some of the technologies that ICT encompases:



ICT comprise many technologies
for capturing, interpreting, storing and transmitting information [5]

ICT is the most astounding breakthrough in educational technology since the invention of the blackboard.

1.2 Advantages and Challenges in implementing ICTs

Like all innovations that we have come to accept, ICTs also have strengths and weaknesses. We should list these because it is important to know what they are especially if we are to plan and use them effectively.

1.2.1 Strengths:

Some of the strengths of the ICTs are as follows:

1. **Individualization of learning:** This means that people learn as individuals and not as a homogenous group. ICTs allow each individual to relate to the medium and its content at his/her own time and own pace. Learner can access the content, go forward

and backward in the content, start at any point depending upon prior knowledge instead of always in a sequential way.

2. **Pedagogical gains:** ICTs can help bringing interactivity, multiple representations, collaborative learning, independent thinking, bringing topic to real life context, and so on.

3. **High speed delivery, wide reach at low per unit cost:**

   There is instant delivery of information. Further per person, ICTs reduce the cost of education from very high to very low.

4. **Distance and climate insensitive:** It does not matter where you are, or how the weather is, you can still access and learn from ICTs.

5. **Can serve multiple teaching functions and diverse audiences:**

   ICTs, especially the computer and Internet based can be useful in drill and practice; to help diagnose and solve problems, for accessing information and knowledge about various related themes.

6. **Uniform quality:** If content is well produced and is of good quality, the same quality can be delivered to the rich and the poor, the urban and the rural equally and at the same low cost.

1.2.2 Challenges:

Implementation of ICTs also poses challenges:

1. **High infrastructure and start up costs:** It costs money to build ICT systems and to maintain them. Tend toward centralized uniform content in economies of scale: The larger the numbers, the lower the cost. This means that sometimes we try to reach large numbers so we make content common, not taking into account individual differences.

2. **Are not ideally location and problem sensitive:** Address problems in a general way, but cannot, without special effort, solve local and culturally sensitive problems.

3. **Problems of reach, access, remain:** Not everyone has equal access; so not everyone benefits equally from the use of ICTs.

4. **Tend to create new class of knowledge rich/knowledge poor:** Those who have access and knowledge through the media become richer and those who do not become poorer, widening the "knowledge or digital gap" between rich and poor.

5. **Essentially delivery systems:** A medium is different from the content; and often we forget that we can deliver any content, because ICTs are essentially meant only to deliver content, not to change attitudes or bring about behavior change.

6. **Hard to assess impact:** Learning from ICT delivered content is difficult to assess since such learning is of a multidimensional and long term kind, rather than from immediate learning assessment as in a classroom test.

7. **Officers, trainers need reorientation and retraining:** Just as people learn to use ICTs, trainers and officers also need training - something they sometimes resent.

8. **Call for attitudinal change to understanding of teaching and learning:** These are different media and have a different way of teaching from what we are accustomed to. Therefore, teachers need different ways of understanding what teaching and learning is all about.

2 Some data and initiatives (India)

All education systems are complex and varied, and India's is as complex and varied as any education system in the world. Only China rivals India in the vast scale of its education sector.

In India, policy framework, financial support and guidelines to ensure a national standard of education is provided by the Government of India through the Ministry of Human Resource Development (MHRD). The implementation of the policies and guidelines is primarily done at the state level through the various state level departments in the country. The MHRD functions through two departments, the Department of School Education and Literacy and the Department of Higher Education. The National Council of Educational Research and Training (NCERT) is an autonomous organization under the MHRD to assist the central as well as the state governments in implementing policies and programs pertaining to education, particularly school education. The Central Board of Secondary Education (CBSE) under the MHRD prepares the syllabus for schools and conducts board examinations for classes X and XII. The National University of Educational Planning and Administration (NUEPA) assists the MHRD with capacity building and research in planning and management of education.

The Department of Information Technology (DIT) in the Ministry of Communications and Information Technology (MCIT) is responsible for formulating, implementing and reviewing national policies pertaining to information technology. In terms of IT education and IT enabled education, DIT is responsible for imparting ICT skills as well as encouraging the implementation of ICT in the teaching learning process. There are various autonomous organizations under DIT to assist it with its functioning such as the National Informatics Center (NIC) which provides network backbone and e-governance support and Center for Development of Advanced Computing (CDAC) which encompasses multilingual computing, free and open Software, education and training et cetera. Department of Telecommunications (DOT) in the MCIT is responsible for policy formulation, licensing, coordinating, standardization and research and development of telecommunications in India. The Center for Development of Telematics (C - DOT) is the telecom technology development center for DOT.

As per [9],adult literacy rate in India is relatively high (70% for males and 48% for females) when compared to other countries in the South Asia region, but a strong gender disparity exists although not as severe as Afghanistan and Nepal. Primary Gross Enrolment Rate (GER) stands at 114% for males and 109% for females. This high percentage can be attributed to the governments commitment to ensure universalization of elementary education, for which it launched the flagship scheme, Sarva Shiksha Abhiyan (SSA). Secondary GER however is significantly lower at 59% for males and 49% for females indicating a near 50% drop at the secondary level. To address gender equality and to ensure universalization of secondary education, the government has implemented the Rashtriya Madhyamik Shiksha Abhiyan (RMSA).

Though the IT and Telecom sector in India has made significant progress the challenge lies in strengthening the ICT infrastructure in rural areas, particularly in terms of internet penetration and electrification.

There is need for greater focus on developing relevant content and applications and using them to enhance learning across subjects, to ensure improvement in quality of education. There are no standards or guidelines available at a national level to develop or choose relevant content. Most content used in schools are developed by teachers themselves as a result there is no uniform content used. While content creation by the teachers and students themselves is a positive trend enabling ownership; one needs to weigh the pros and cons of not having a professional content development team who can involve teachers and faculty in the process. Off the shelf products which are available need have some scope for flexibility and customization to give a sense of ownership to users

Still the fact that the impact of ICT use to date on learning outcomes is negligible in most places, at least partially attributable to the fact that, in most places, computers are only used to teach 'ICT literacy'. It is hoped that initiatives like "Rashtriya Madhyamik Shiksha Abhiyan (RMSA)" and "National Mission for Education through ICT"[8] will make a difference in coming areas. It is however clear that we no longer have a choice.

It is no longer an "if" but "how" to deploy the technologies optimally.

3 Illustrations

Before we see how ICT can help, let us briefly look at education and pedagogy.

3.1 Aim of education

According to George Polya $(1887-1985)$ a Hungarian mathematician, also called "The Father of Problem Solving in Mathematics Education", there are two kinds of aims of education [3]:

1. Good and narrow aims: The schools should turn out employable adults - adults who can fill a job.

2. Higher aim: To develop all the inner resources of the growing child.

Mathematics plays an important role in achieving both of the above aims.

3.2 Pedagogy

"The science of teaching", (Oxford English Dictionary) is called **pedagogy**. However, pedagogy encompasses:

1. What is taught/learned - the content?

2. How it is taught or learned - approaches to teaching and learning?

3. Why it is taught or learned - the underpinning values, philosophy or rationale?

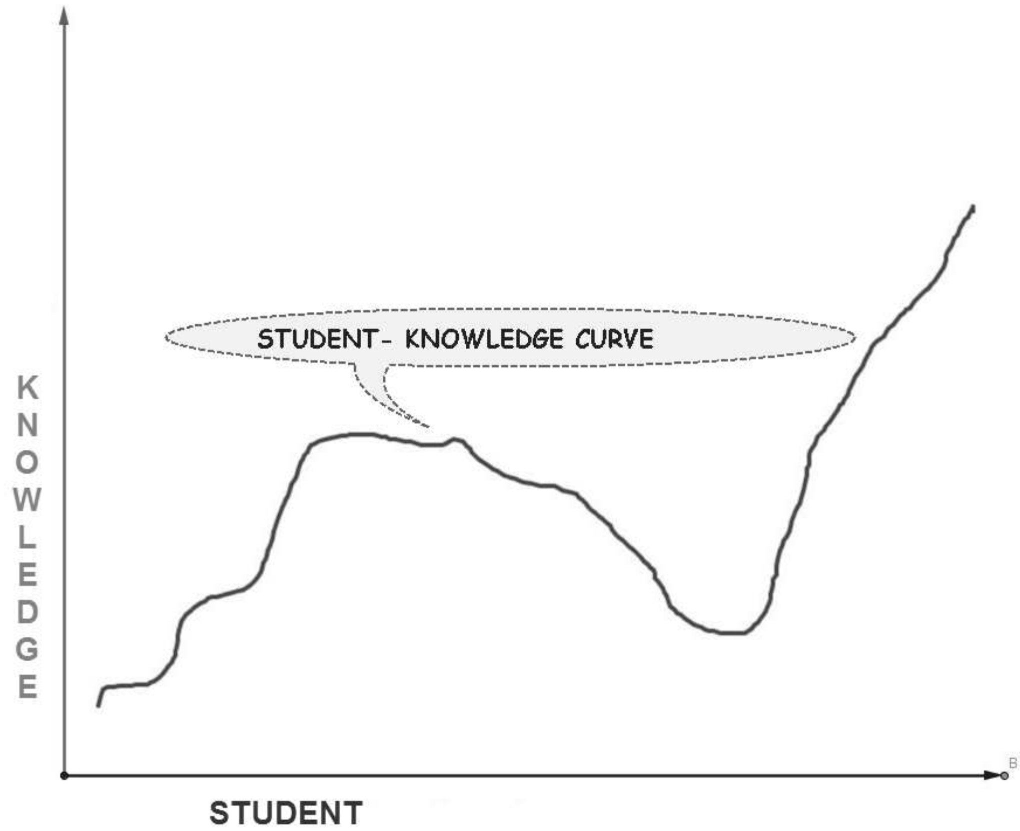
The three are intertwined and the 'why' will have a strong influence on what is taught or learned and how. Some of the pedagogy approaches, supported by research findings, that are desirable for effective teaching and learning are as follows (see [4]):

1. Caring classroom communities that are focused on mathematical goals help develop students' mathematical identities and proficiencies.
2. Effective teachers provide students with opportunities to work both independently and collaboratively to make sense of ideas.
3. Effective teachers plan mathematics learning experiences that enable students to build on their existing proficiencies, interests, and experiences.
4. Effective teachers understand that the tasks and examples they select influence how students come to view, develop, use, and make sense of mathematics.
5. Effective teachers support students in creating connections between different ways of solving problems, between mathematical representations and topics, and between mathematics and everyday experiences.
6. Effective teachers use a range of assessment practices to make students' thinking visible and to support students' learning.
7. Effective teachers are able to facilitate classroom dialogue that is focused on mathematical argumentation.
8. Effective teachers shape mathematical language by modelling appropriate terms and communicating their meaning in ways that students understand.
9. Effective teachers carefully select tools and representations to provide support for students' thinking.
10. Effective teachers develop and use sound knowledge as a basis for initiating learning and responding to the mathematical needs of all their students.

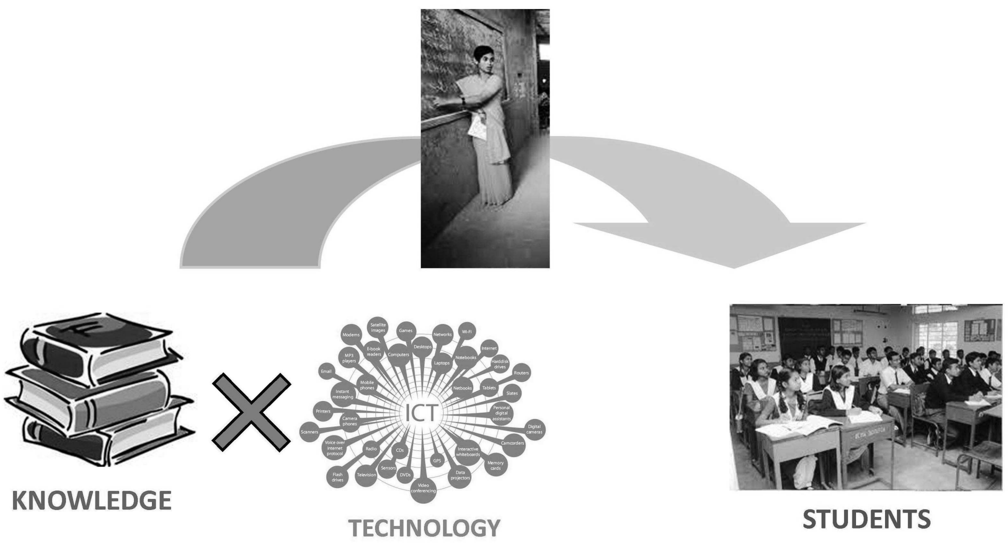
3.3 Role of a teacher and teacher preperation

Teachers remain central to the learning process.

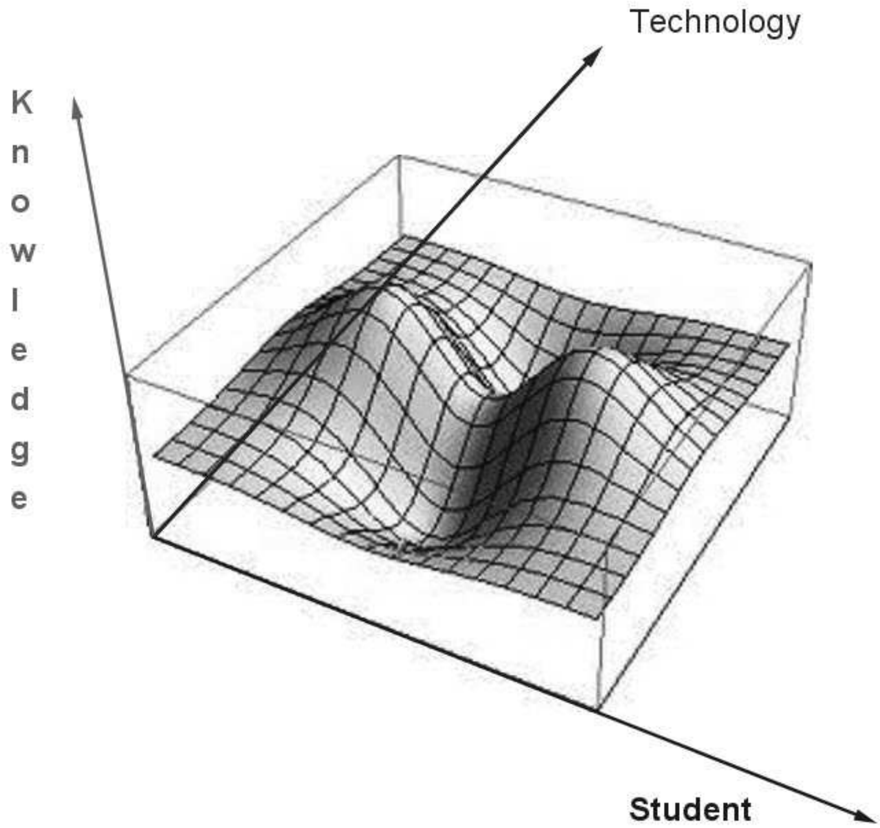
This gives a geometric representation of learning curve:



Technology adds a new dimension to teaching and learning:



This converts knowledge plane to knowledge space.



A shift in the role of a teacher utilizing ICTs to that of a facilitator does not obviate the need for teachers to serve as leaders in the classroom; traditional teacher leadership skills and practices are still important (especially those related to lesson planning, preparation and follow-up). Integration of ICT can help in achieving many of the above objectives of pedagogy. Lesson planning is crucial when using ICTs. Research shows that where little planning has occurred, student work is often unfocused and can result in lower attainment.

At present technology training appears to focus mainly on technology knowledge and skills while overlooking the relationships between technology, pedagogy, and content[7]. As a result, teachers learn about "cool" stuff, but they still have difficulty applying it for their students' learning. ICT is often used simply as a supplement for existing pedagogical practices. However in order to fulfil the potential of ICT as a tool for enhancing teaching and learning, ICT must be fully integrated into pedagogical processes, which requires a cognitive shift on the part of educators, curriculum developers, administrators and policy-makers.

Further teacher candidates need opportunities to practice effective technology integration strategies in supportive contexts during technology courses, technology-integrated methods courses, and field experiences. Experienced teachers also need opportunities to learn about new technologies and ways to integrate them effectively in their classroom. At the same time here is a strong need for teaching technology-tools to pre-service teachers. For a rich variety of ICT resources on integrating them in teaching and learning, see [10] and [11].

3.4 Motivation and algorithm for illustrations

Some of the illustrations that I want to present are motivated by the inputs from cognitive psychology and neuro-science:

1. Concepts are learned best when they arise in a variety of contexts, are represented in a variety of ways, and when students have a chance to use the concepts on authentic tasks.

2. Learning to do well involves: think critically, analyze information, communicate scientific ideas, make logical arguments, work as part of a team, and acquire desirable skills.

3. Effective learning requires feedback. Students must be able to feel free to express ideas and to receive analysis and comment from their peers.

Here are some ingredients for developing ICTs enhanced instructional material:

- **Hands-on:** Students are actually allowed to perform science to construct meaning and acquire understanding.It takes these subjects out of the realm of the magical or extraordinary.

- **Minds-on:** Activities focus on core concepts, allowing students to develop higher-order thinking processes and skills,and encouraging them to question and seek answers that enhance their knowledge, and thereby acquire an understanding of the physical universe in which they live.

- **Reality-on:** Students are presented with problem-solving activities that incorporate authentic, real-life questions and issues in a form that encourages drawing on multi-disciplinary knowledge, collaborative effort, dialogue with informed expert sources, and generalization to broader ideas and application.

The objective is to promote students' insight into the real scientific, technological, business and every-day world, and the skills needed to live and work effectively.

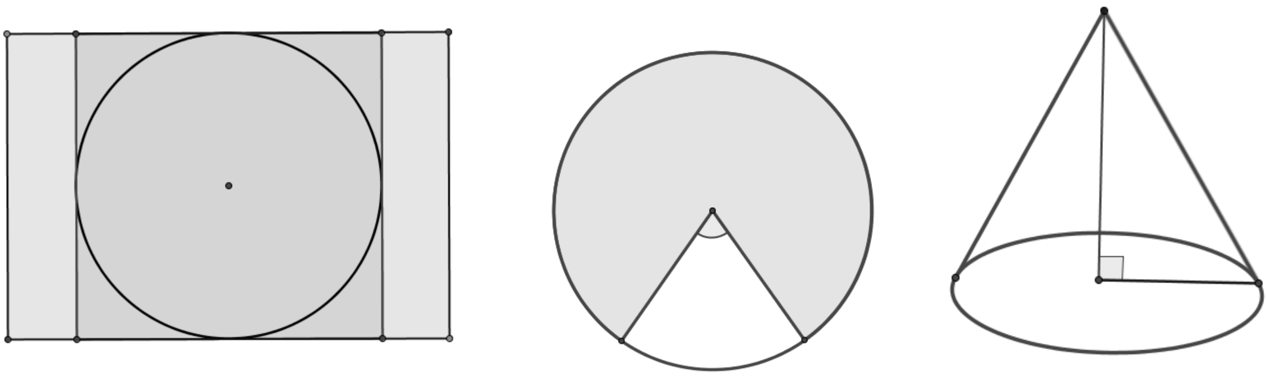
3.5 Illustration :

The aim of this illustration is to show how some of the text book problems can be turned into a series of explorations that included much of the desired qualities of math pedagogy, as mentioned in section 3.4.

Problem: *Given a rectangular sheet of size* 30cm $\times$ 24cm., *construct a right circular cone of maximum volume.*

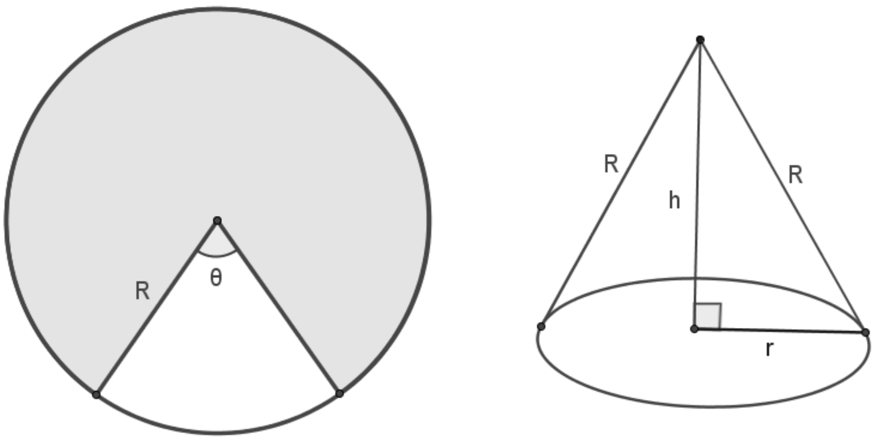
To start solving the problem, recollect that to make a right circular cone one needs circular disc from which a sector has been removed. The cone is obtained by joining the edges where the sector is removed. Problem is to find a cone with maximum volume. Similar problems occur in textbooks of secondary level (Exercises 8.1, No. 12 page 192 of [12]).

To solve the problem, one may proceed with cutting largest possible circular disc from the given sheet and then use it to construct cone of maximum volume.



This motivates one to look at the following problem:

Problem: *A circular disc of radius R is used to make a right circular cone by removing a sector with angle θ radians and then joining the edges where the sector was removed. Find the maximum volume of such a cone.*



Let h be the height of the cone with base radius r. The cone will have slant height R. If $\phi = 2\pi - \theta$, then $2\pi r = R\,\phi$. Since

$$h^2 = R^2 - r^2 = R^2 - \left(\frac{R\,x}{2\pi}\right)^2,$$

the volume V of the cone is given by

$$V(R,\phi) = \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{R\,\phi}{2\pi}\right)^2 \left(\sqrt{R^2 - \left(\frac{R\,\phi}{2\pi}\right)^2}\right) = \frac{1}{12\pi} R^3\,\phi^2 \sqrt{1 - \frac{\phi^2}{4\pi^2}}.$$

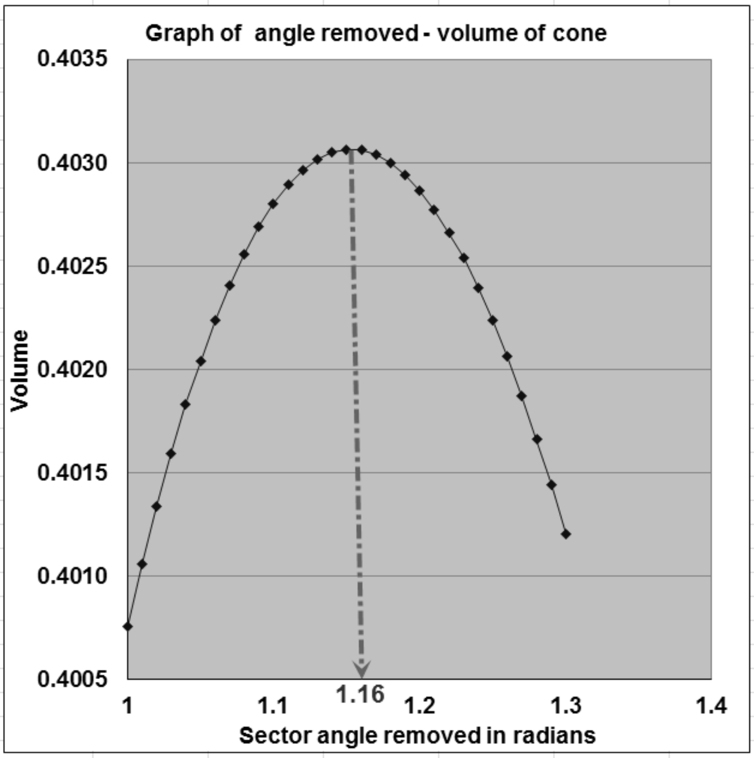
As a function of ϕ the the maximum of V can be found with a variety of tools:

1. **Using spreadsheets and graphs:**
   For $R = 1$,

$$V(1,\phi) = \frac{\pi}{3}\left(\frac{\phi}{2\pi}\right)^2 \sqrt{1 - \left(\frac{\phi}{2\pi}\right)^2}.$$

Graphing $V(1,\theta)$ as function of θ using excel and locating maximum, gives $\theta = 1.16$ radians:

θ	Volume	1-(θ/2pi)
1	0.4008	0.8408451
1.01	0.4011	0.8392535
1.02	0.4013	0.837662
1.03	0.4016	0.8360704
1.04	0.4018	0.8344789
1.05	0.4020	0.8328873
1.06	0.4022	0.8312958
1.07	0.4024	0.8297042
1.08	0.4026	0.8281127
1.09	0.4027	0.8265211
1.1	0.4028	0.8249296
1.11	0.4029	0.823338
1.12	0.4030	0.8217465
1.13	0.4030	0.8201549
1.14	0.4031	0.8185634
1.15	0.4031	0.8169718
1.16	0.4031	0.8153803
1.17	0.4030	0.8137887
1.18	0.4030	0.8121972
1.19	0.4029	0.8106056
1.2	0.4029	0.8090141
1.21	0.4028	0.8074225
1.22	0.4027	0.805831
1.23	0.4025	0.8042394
1.24	0.4024	0.8026479
1.25	0.4022	0.8010563
1.26	0.4021	0.7994648
1.27	0.4019	0.7978732
1.28	0.4017	0.7962817
1.29	0.4014	0.7946901
1.3	0.4012	0.7930986



2. **Using Dynamic geometry software:**
We can also use dynamic geometry software like Geogebra to plot

$$V(1,\phi) = \frac{\pi}{3}\left(\frac{\phi}{2\pi}\right)^2\sqrt{1-\left(\frac{\phi}{2\pi}\right)^2},$$

See figure on next page. This also gives maximum $V(1,\theta)$, $\theta = 66.837^o$.

3. **Using Calculus**
Let us analyze this using calculus: The volume function, for R fixed, is

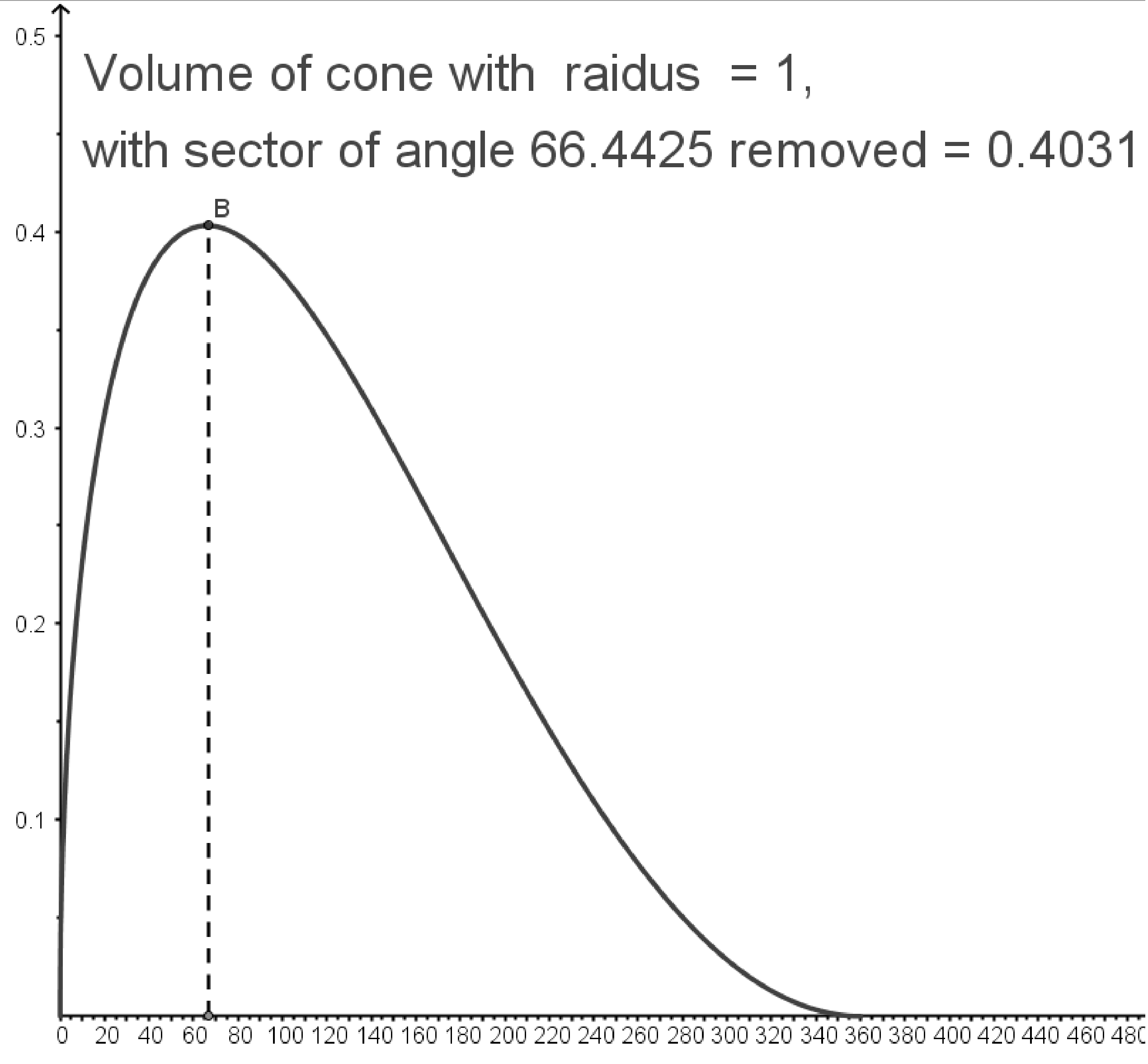
$$V(\phi) = \frac{1}{12\pi}R^3\phi^2\sqrt{1-\frac{\phi^2}{4\pi^2}}. \qquad (1)$$

Computing $\frac{dv}{d\phi} = 0$:

$$\left(\frac{R^3}{12\pi}\right)\left(\sqrt{1-\frac{\phi^2}{4\pi^2}}\right)\left(2\phi-\frac{3\phi^3}{4\pi^2}\right) = 0,$$

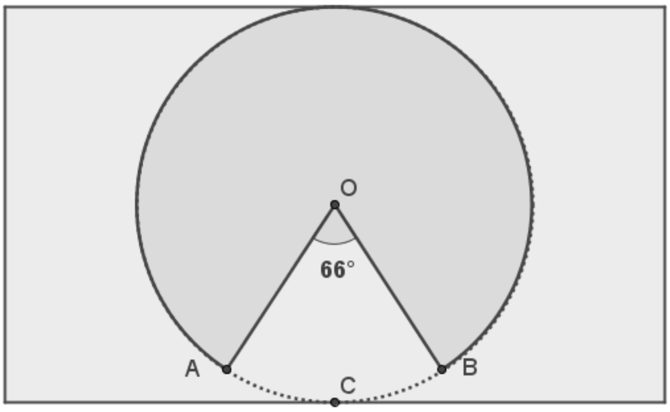
gives $\phi^2 = \frac{8\pi^2}{3}$. Thus, for a given R, the volume is maximum for $\phi = \sqrt{\frac{8}{3}}\,\pi = 5.131$ radians, or 293.94^o. The angle of the sector removed is $\gamma = 360 - 293.94 \simeq 66^o$ The maximum volume is for $\phi = \sqrt{\frac{8}{3}}\ \pi = 5.131$. Thus using equation(1), the maximum volume is

$$V(5.131) = \frac{1}{12\pi} R^3 (5.131)^3 \sqrt{\frac{1}{3}}. \tag{2}$$



Plotting volume function using Geogebra

Let us go back to the problem of constructing a right circular cone of maximum volume, from a rectangular sheet of paper? We can first inscribe a circle in the rectangular sheet and delete a sector of angle 66^o and fold to get the required cone! If our paper size is 30cm. $\times$ 24cm.,



to construct a right circular cone as suggested above we will cut largest possible circular disk, which will have diameter 24cm. Thus the cone of maximum volume, using equation (2) is

$$V = \frac{1}{12\pi}(12)^3(5.131)^3\sqrt{\frac{1}{3}} \simeq 3576. \tag{3}$$

However this does not give cone of maximum volume made from the rectangular sheet. Note that to construct cone of maximum volume, we should have the sector of maximum area with cut of 66^o. The circular sector of radius R inside $ABCD$ will have maximum area when CD is tangent to the circle and the "corners" of the sector are on AB, as shown in the figure 1 below:

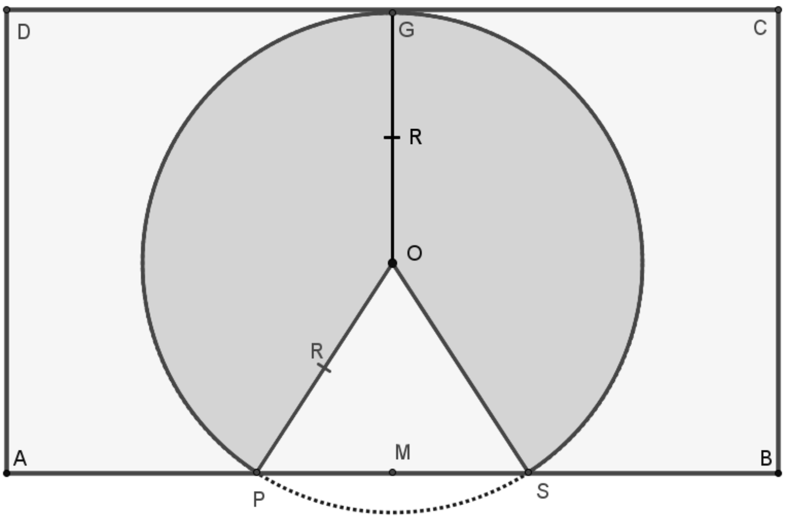


Figure 1

Using a simple geometric-trigonometric computation (using Figure 2,) we can find the relation between R and θ :

$$\begin{aligned} MG = b &= PG\ \cos\theta/4, \\ PG &= 2\,OP\ \cos\theta/4 = R\cos\theta/4, \\ \text{Thus, } b &= 2\,R\,\cos^2\theta/4. \end{aligned}$$

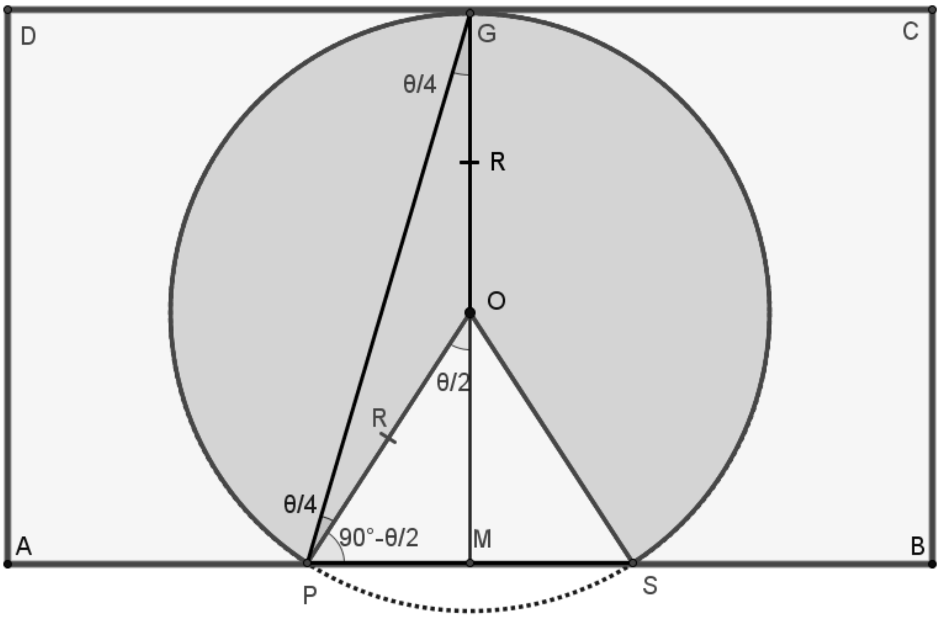


Figure 2

Hence,

$$R = \frac{b}{2\cos^2(\theta/4)} \tag{4}$$

Thus to construct cone of maximum volume, we should have

$$R = \frac{24}{2\cos^2(5.131/4)} = \frac{12}{.919} = 13.05. \tag{5}$$

Thus the maximum volume of right circular cone that can be made from a rectangular paper of size 30cm. $\times$ 24cm. is

$$V = \frac{1}{12\pi}(13.05)^3(5.131)^3\sqrt{\frac{1}{3}} \simeq 4600. \tag{6}$$

4 Conclusions

ICTs are used in education in two general ways: to support existing traditional pedagogical practices (teacher-centric, lecture-based, rote learning) as well as to enable more learner-centric, constructivist learning models. Research suggests that both are useful, but that ICTs are most effective when they help to enable learner-centric pedagogies. However, despite the fact that ICTs can enable new types of teaching and learning styles, at present mostly part they are being used to support traditional learning practices. To bring a change effort should be to empower teacher to develop

- With an inquiry-based culture.
- With a community of explorers.

- Where curiosity, creativity, and questioning are valued.
- Where resources and opportunities are made readily available.
- Where students can "work" like scientists engaged in the process of collective sense-making.

A variety of changes must be implemented to optimize teacher use of ICTs. Shifting pedagogies, redesigning the curriculum and assessment, and providing more autonomy to the schools help to optimize the use of ICT. With sufficient enabling factors in place, teachers can utilize ICTs in as constructivist a manner as their pedagogical philosophies would permit.

Functioning technical infrastructure is (obviously) crucial Teachers must have adequate access to functioning computers, and be provided with sufficient technical support, if they are to use ICTs effectively.

Introducing ICTs takes time. Adequate time must be allowed for teachers to develop new skills, explore their integration into their existing teaching practices and curriculum, and undertake necessary additional lesson planning, if ICTs are to be used effectively. Support from school administration and the community is important.

Lessons learned from introducing ICTs in education need to be shared As the introduction of ICTs to aid education is often part of a larger change or reform process, it is vital that successful uses of ICTs are promoted and disseminated.

References

[1] ICT in Education `http://www.unescobkk.org/education/ict/`

[2] UNESCO `http://tinyurl.com/unesco-india`

[3] Polya `http://mathematicallysane.com/goals-of-mathematical-education/`

[4] Pelgrum,W. J.,Law, N.(2003) *ICT in Education around the World: Trends, Problems and Prospects"UNESCO-International Institute for Educational Planning.* `www.worldcatlibraries.org/wcpa/ow/02d077080fcf3210a19afeb4da09e526.html`

[5] Glenda Anthony and Margaret Walshaw(2009). Borwein, Jonathan; Bailey, David *Effective pedagogy in Mathematics.* International Academy of Education.

[6] Jonathan Anderson (2010)*ICT Transforming Education* Regional Guide, UNESO publication.

[7] Geer, R. & Sweeney, T. (2012). *Students Voices about Learning with Technology.* Journal of Social Sciences **8 (2)**, 294.

[8] National Mission on Education through ICT `http://http://mhrd.gov.in/nme_icthindi`

[9] Survey of ICTs for Education in India and South Asia, Extended Summary 2010. `http://www.infodev.org/infodev-files/resource/InfodevDocuments_890.pdf`

[10] Adrian Oldknow(2008)*ICT bringing mathematics to life and life to mathematics* http://atcm.mathandtech.org/EP2008/papers_invited/2412008_14594.pdf

[11] Adrian Oldknow and Ron Taylor(2000) *Teaching Mathematics with ICT*, Continuum Press, 2000.

[12] Mathematics textbook standard IX, Department of school education, Government of Tamilnadu (2012).http://www.pioneermathematics.com/content/Std09-Maths-EM.pdf