Some Analogous Forms for Locus – a convenient way for students to deepen their understandings on locus

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Abstract

We investigate HP39GS program APLET, break the limit of static function of HP39GS, fulfil dynamic stimulation of conic curve (Part of locus), and finally design more applicable APLET in line with the actual education situation.

Teachers can send students the APLET stimulator before a class, which was edited earlier, and create a scenario to lead students to observe and test the program of stimulator, so as to ensure students understand "the concept of locus" completely, and inspire the desire of research and explore mathematics questions (see [1]) actively. We notice students prefer this way of teaching style and allowing students to explore mathematics with HP39GS makes a positive impact on teaching. In this paper, we describe three ways of exploring locus: Program, Sequence APLET, and Statistics APLET. The objective is to encourage students learn independently find and yet to stimulate a more natural understanding on locus.

1. Introduction

Understanding the concept of "Locus" plays a very important role in senior high school mathematics curriculum in China. It is a basic concept to study conic curves, lines and circles. Whether students can comprehend conic curve well mostly relies on whether students have a clear understanding on locus (see [2]). When it relates to Locus issues, we find students have difficult time understanding and learning the concept. Traditionally it is described in texts as a static notion. It is so abstract that teachers often need to explain the concept by means of multimedia (Geometry Palette) for example. Although it is intuitive, students remain passive with no opportunity to experiment with a technological tool in a classroom. The authors therefore think of the possibility to take advantage of students being able to access HP graphic calculators, and allow them to edit the programs inside the calculator to suit the teaching and learning. On one hand, students and teachers feel free to use APLET as long as it is useful, and no need to insist on thinking that the APLET is limited to deal with some individual module. On the other hand, for specific problem will need more concrete analysis. For example, for those problems where HP calculators can be used directly in solving, we provide students a customized program, and create scenarios and provide hints to lead students to use the program. We found this makes teaching and learning more dynamic, interactive and effective. We let students operate the calculators individually during a lesson, which allows students to understand the concept of locus independently.

2. Teaching Conic with a Geometric Approach

2.1. The case of the ellipse.

In the chapter regarding Circle Equation in our textbook (PEP version "Mathematic *compulsory 2*), the definition of locus $M(x,y)$ is connection expression of coordinate $M(x,y)$. Let's analyze this definition from two points of view. One is from geometry point of view; what does the graphics look like for different locus? And the other is from algebraic point of view, what is equation of locus? It is easier to solve the latter. According to definition of locus, the locus equation is connection expression of coordinate $M(x,y)$. Now let's answer the former. Take definition of ellipse in chapter 2 regarding ellipse in *"Mathematic elective 2-2"* for example, we define the locus of points $M(x,y)$ to be the sum of distances between $M(x,y)$ and two fixed points F_1, F_2 is equal to a constant, and the sum is bigger then $|F_1F_2|$, as an ellipse.

First of all, from the algebraic point of view-according to the definition- this provides us the sum of distances between moving points $M(x,y)$ in the plane and two fixed points $F_1(-c,0)$, $F_2(c,0)$ is equal to constant $2a$. Then we have

$$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a \Rightarrow \sqrt{(x+c)^2+y^2}=2a-\sqrt{(x-c)^2+y^2}.$$

We square both sides and obtain,

$$(x+c)^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2 \Rightarrow 4a\sqrt{(x-c)^2+y^2}=4a^2-4cx.$$

We square both sides and get

$$a\sqrt{(x-c)^2+y^2}=a^2-cx \Rightarrow a^2x^2-2a^2cx+a^2c^2+a^2y^2=a^4-2a^2cx+c^2x^2$$

$$\Rightarrow (a^2-c^2)x^2+a^2y^2=a^4-a^2c^2 \Rightarrow (a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2),$$ we use $b^2=a^2-c^2$, and the equation can be simplified as follows：

$$b^2x^2+a^2y^2=a^2b^2 \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0),$$ which is algebraic equation of ellipse.

In addition, from geometry point, in the example, we suppose the coordinates of two fixed points, respectively, to be $F_1(\text{-}c,0)$ and $F_2(c,0)$, the fixed value to be $2a$ and the moving point to be $M(x,y)$. With the help ofHP39GS, we can acquire the geometric graphics of locus for M very easily as follows:

- Enter the horizontal coordinate of fixed point $F_2(c,0)$ (Any positive value can be an input). Suppose we enter 3, i.e. $F_2(3,0)$,
- Next, we enter the fixed value 10, i.e. $2a=10 \Rightarrow a=5$ in this way, HP comes up a graphics automatically, which we show in the following screen shots:

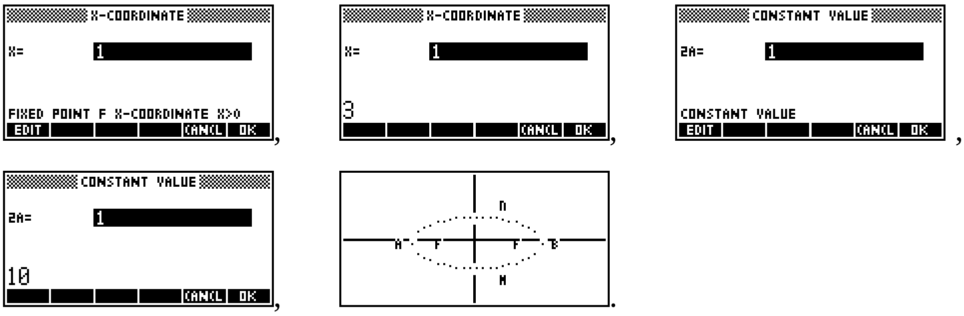


Figure 1. Screen shots of producing an ellipse

Apparently the locus of point M is an ellipse. Let's verify with some particular values. Let's suppose the horizontal coordinate of $M(x,y)$ to be 2, then HP comes up automatically 6.2 and 3.8 respectively as the distance between M and two vertices, $|PF_1|$ and $|PF_1|$.The sum of which is $10=2a$. If we verify with more numerical values, we see the following patterns.

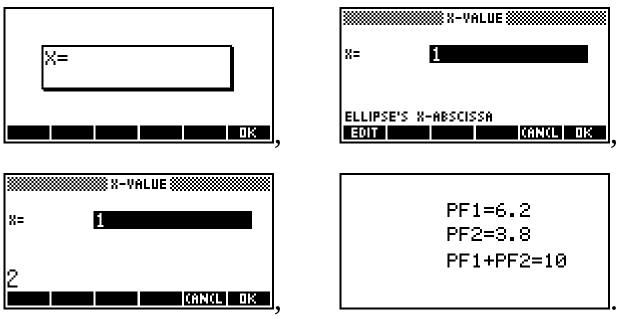


Figure 2. Screen shots of verifying an ellipse

Clearly, geometric graphics of ellipse equation and the corresponding algebraic equation are shown to us intuitively. Analogously, we can continue to studying the graphics of hyperbola using locus from geometric and algebraic points of view.

2.2. The case of a hyperbola

Next, we define the locus of points within plane, the absolute value of difference of distances between a point M and two fixed points F_1, F_2 is equal to a constant (where the difference is less then $|F_1F_2|$) . In such case, the locus is a Hyperbola.

First of all, from algebraic point of view, according to the definition, we suppose the moving points $M(x,y)$ in the plane satisfy that the absolute value of difference of distances between $M(x,y)$ and two fixed points $F_1(-c,0)$ $F_2(c,0)$ is equal to constant $2a$. Therefore, $|\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}|=2a \Rightarrow \sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}=\pm 2a$, simplify it as before to obtain $(c^2-a^2)x^2-a^2y^2=a^2(c^2-a^2)$, we use $b^2=c^2-a^2$ and simplify the equation to: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1(c>a>0)$. This is the algebraic equation of a Hyperbola.

In the same way, we let the moving points be $M(x,y)$, with the help of HP39GS, we can acquire the geometric graph for the locus of M.

- Enter 5 or any positive value as the horizontal coordinate for the fixed point $F_2(c,0)$, i.e. $F_2(5,0)$.
- Next, enter any fixed value 8, i.e. $2a=8 \Rightarrow a=4$, the HP automatically comes up the graph of the hyperbola, which we demonstrate in the following screen shots:

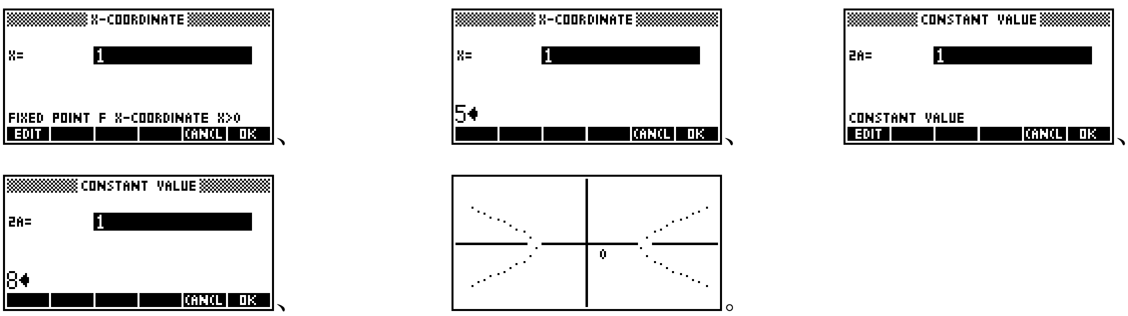
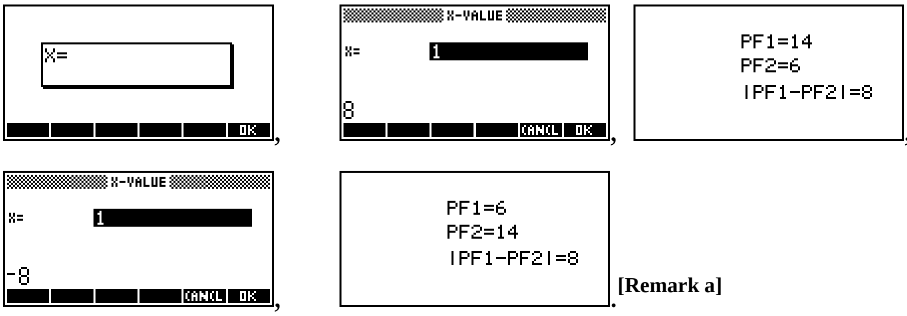


Figure 3. Screen shots of producing a hyperbola

Clearly, the locus of M is a hyperbola. Let's verify with more particular values. Let's suppose horizontal coordinate of $M(x,y)$ to be 8, then HP automatically returns the distance between M and two vertices, $|PF_1||PF_2|$, whose values are 14 and 6 respectively. The difference meets the fixed value $8=2a$. If we verify with more values, we will find the pattern as follows:



[Remark a]

Figure 4. Screen shots of verifying a hyperbola

Through this example, we find a way of exploring locus issue intuitively. Compared with traditional teaching model, by utilizing the program within HP with teachers' proper guidance, we can improve the students' participation and change traditional ways of "Listening, Speaking and Writing" into " ways of Listening, Speaking, Writing and Action, which turns a classroom more lively and make students be active in a classroom. We learn from our experiences that this new model is effective and appreciated by the students. What mathematics knowledge do teachers intend to convey to our students? Are we teaching them how to think and explore mathematics? Or are we advising those ways of seeking necessary knowledge of deducing their findings after experimenting? Authors find the answers are affirmative. The introduction of HP39GS contributes to the supporting role in teaching and learning (see [3]). As far as author's teaching experience is concerned, perhaps the presence of HP39GS in a classroom is not only supporting but sometimes also play a pivotal role in students' learning. Of course, we do not expect any hardware or software to solve problems widely. It can only be used within particular area according to certain lessons. We use the following examples to demonstrate ways of introducing locus to students from different perspectives.

3. Learning Conic with Sequence and Statistics APLETS

3.1 Using Sequence APLET

Example 1 (Sequence APLET) Take Example 2 in Ellipse Chapter of *"Mathematic Elective 2-1"* for example. As the graphics shown in Figure 5, let's pick a point P on the circle $x^2+y^2=4$ and draw the Vertical Line PD through P with D as the foot point. When P is moving on the circle, what is the locus for midpoint M of Line PD, explain why?

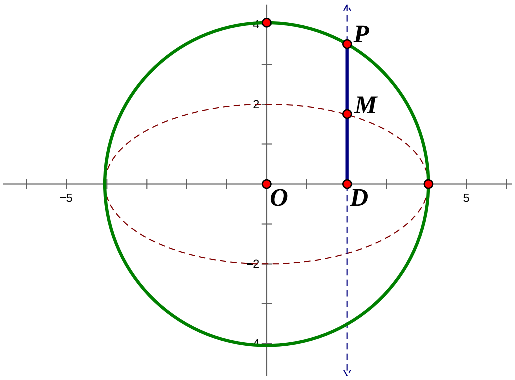


Figure 5. A circle

Let's use sequence APLET with proper calculation, provided $D(N,0)$, then the horizontal coordinate of P is $x = N$ and vertical coordinate satisfy $y^2 + N^2 = 4$, we get: $y = \pm\sqrt{4 - N^2}$, let's get the value of $y = \sqrt{4 - N^2}$ (make same analysis for $y = -\sqrt{4 - N^2}$), so we can get a general graphics with Sequence APLET. In order to get a clearer graphics, let's enlarge from value 4 to 100 and we obtain Figure 6.

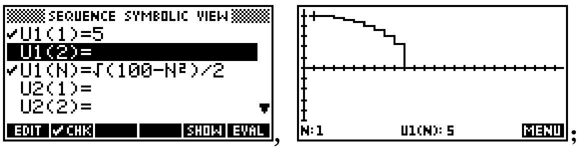


Figure 6. Top half of a sequence plot

In the same way, we get another half graphics in Figure 7.

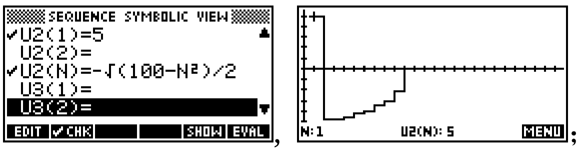
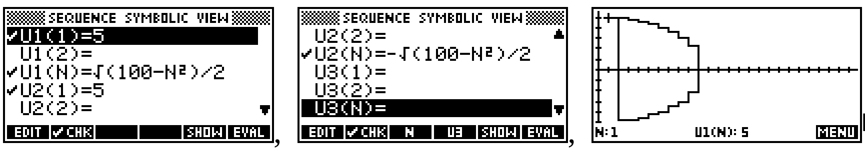


Figure 7. Bottom half of a sequence plot

We can obtain both halves of the graphs as follows, see Figure 8.



[See Remark (b)].

Figure 8. Both halves of a sequence plot

At this point, one may conjecture that the locus of moving points M is an ellipse although we see only right half of an ellipse. We next proceed to a theoretical deduction.

Let's suppose a moving point to be $P(x, y)$, then we use the notations $D(x,0)$ and $M(x_0, y_0)$.

According to our assumptions, we have $\begin{cases} x_0 = x \\ y_0 = \dfrac{y}{2} \end{cases} \Rightarrow \begin{cases} x = x_0 \\ y = 2y_0 \end{cases}$, and substitute them into circle equation, we obtain the followings:

$$x_0^{\ 2} + (2y_0)^2 = 4 \Rightarrow \frac{x_0^{\ 2}}{4} + y_0^{\ 2} = 1.$$

The advantage of using sequence APLET is that we don't have to re-edit the program, but on the other hand the disadvantage exists. Because subscript value of sequence is positive number, we could not show the graphics in the range of $(-\infty, 0)$. This somewhat affects students' intuition on the concept of locus.

Now the author will introduce the third way to study locus of geometry graphics, i.e. to study with scattered diagram and curve fitted of statistics APLET.

3.2 Using Statics APLET

Example 2. (Statistical APLET) Take Example 3 of Curve and Equation Chapter in *"Mathematics Elective 2-1"* for example. The distance between line l and the point F above l is given to be 2, and there is one curve above line l. The differences by distances between every point on this curve and F minus distances between these points and l are always 2 units. Let's build the right coordinate system and figure out the equation of this curve.

Regard the orientation of l as Axis x and the line that goes through F being vertical to l as Axis y, and the point of intersection between x and y is O, then we build a rectangular coordinate system so we can get point $F(0,2)$. Now let's assume $M(x, y)$ and take some particular points and fix value of x then calculate relative value of y and finally fix coordinate of $M(x, y)$.

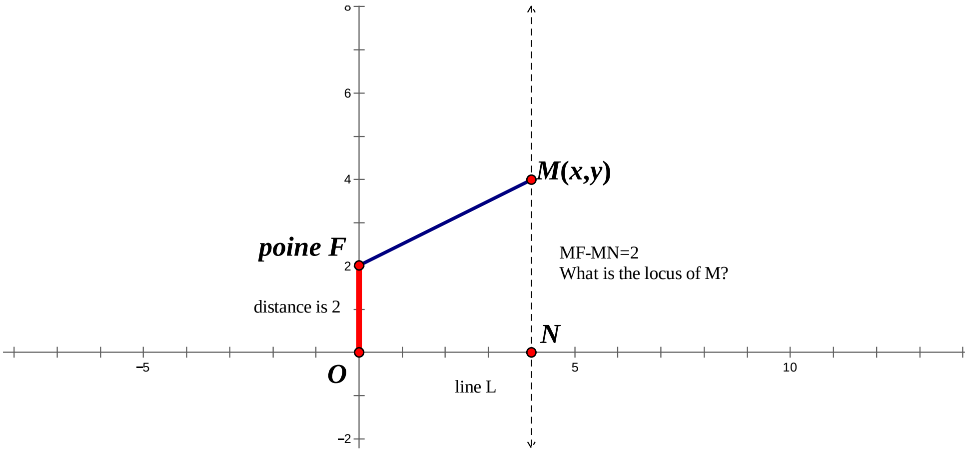
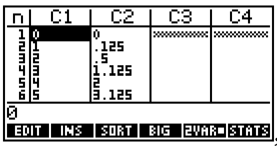
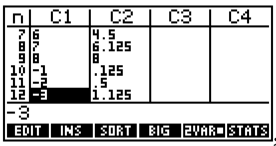


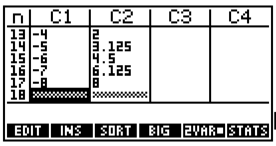
Figure 9.The graphics mentioned in Example2

x	y	$M(x,y)$
0	0	$(0,0)$
1 or -1	0.125	$(1,0.125)$
2 or -2	0.5	$(2,0.5)$
3 or -3	1.125	$(3,1.125)$
4 or -4	2	$(4,2)$
5 or -5	3.125	$(5,3.125)$
6 or -6	4.5	$(6,4.5)$
7 or -7	6.125	$(7,6.125)$
8 or -8	8	$(8,8)$

We use statistic APLET program and input two groups of data,



,


,


[See Remark (c)].

Figure 10.Inputs of Data

We see a locus graphics directly in Figure 10:

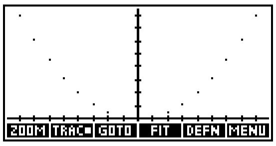


Figure 11.Scattered plot

We may also use embedded program of HP39GS to find the fitted curve which pass through the given data points with equation, which we show in Figure 11 as follows:

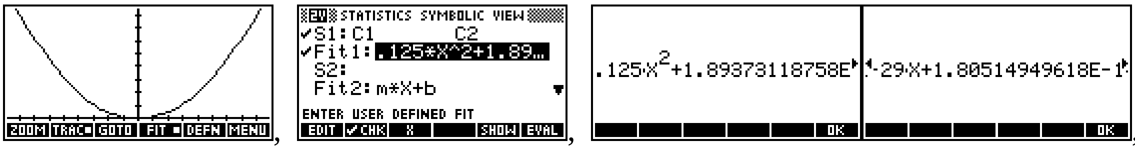


Figure12. Fitted curve and its equation

We need to increase the number of points (data) in our inputs to get a more precise graphics. Nevertheless, the technological tool provides us to conjecture that the fitted curve should be a parabola. Let's confirm this by using the definition of locus. We regard line l as Axis x and the line that go through F being vertical to l as Axis y, and the intersection point of x and y to be origin O. Next, let's build a rectangular coordinate system and get point $F(0,2)$. Let's assume $M(x,y)$ then we see $\sqrt{x^2+(y-2)^2}-y=2$ and we simplify it to be $y=\frac{1}{8}x^2$. This indeed is a parabola and almost follows the conclusion acquired from the calculator.

4. Conclusions

Above 3 ways separately apply program, Sequence APLET and Statistics APLET to figure out locus issues. Although these 3 ways seem to be independent apparently among each other, they are connected to each other in nature as well. This indicates that we should not think independently when we study questions. Sequence APLET is not only be used to solve sequence issue, while Statistics APLET can be used to solve locus issues as well. We shall study the questions from points of connection and development view and search the answers from practice operation, moreover we shall learn and use it flexibly (see [4]). Thanks to powerful function of HP39GS which provides possibility to these practices. In addition, because the program embedded in HP39GS system is enough to solve most courses in senior high school (If not, we can edit program to solve them), this helps students on different levels in class promote themselves either to get answers or to study. Through recent years' education and teaching research, the author has found out that HP39GS has most obvious impact on the students who have weak performance on study. To use HP39GS not only solves some problem which could not be solved previously but also increase students' interest and enthusiasm to mathematics and inspire their confidence to explore knowledge.

Remarks

(a) Relates to recycle command FOR TO STEP END and command CASE IF THEN END;END in "*HP39GS Manual"*

(b) Relates to Sequence APLET in "*HP39GS Manual"*

(c) Relates to Statistic APLET in "*HP39GS Manual"*

5. References

[1] Chinese Ministry of Education Examination Center (2012). Examination outline of College Entrance Mathematics Examination. Higher Education Press.

[2] Fujian Provincial Education Examination Authority (2012).Examination Instruction of Fujian College Entrance Mathematics Examination. Fujian Education Press.

[3] Wang Changpei. Begin from 3A and end to 3Y (2011). HP39GS Proceedings. Beijing Normal University Publishing Group.

[4] Cao Yiming. Look on Function of Graphics Calculator actively (2011). HP39GS Proceedings. Beijing Normal University Polishing Group.

6. Appendix: Source program