Expanding Plane Geometry Using The Geometer's Sketchpad

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Abstract: *This paper describes a very broad meaning of "expandable plane geometry". It includes any plane geometry that transforms by using the tessellation transformation concept, a two-space expansion. The symbols used to describe the tessellation forms (regular and semi-regular) use naming conventions by choosing a vertex, then look at one of the polygons that touch that vertex. How many sides does it have? The notation of a regular tessellation of triangles has six polygons surrounding a vertex, and each of them has three sides: the symbol used to describe is "3·3·3·3·3·3" or "3^6" means there are six triangles at each vertex. A semi-regular tessellation is a set of regular polygons of two or more kinds so arranged that every vertex is congruent to every other vertex.*

Keywords: *Construct mathematics knowledge, tessellation, expanding plane geometry, the Geometer's Sketchpad, and constructivism*

1. Introduction

Since teaching geometry classes in the past, has changed from the classical "ruler and compass" constructions. These constructions have an accuracy far surpassing any person with a physical compass. Dynamic Geometry Programs such as the Geometer's Sketchpad has appeared over the last decade as a critical new tool in geometry. The features are expanding to encourage visual investigation and exploring relationships dynamically so that the students can see change in geometric figures as they manipulate them [1]. Its dynamic behavior comes from dragging or animating the initial objects generates a range of examples and possibilities. This changes the entire quality and impact of students' experiences.

An important area of this paper includes any structure that geometrically tessellation transforms. There are two classifications of given plane expandable structures:

(1) Regular tessellation;
(2) Semi-regular tessellation.

2. The Geometer's Sketchpad and Plane Geometric Figures

Plane geometric figures are collections of points that all lie in the same plane (coplanar). Formally, a plane figure is any set of points on a plane. The words "point", "line", and "plane" are undefined concepts. A point, which is 0-dimentional, can lie on a line, in a plane, or in space. A line, which is 1-dimensional, can line in a plane or in space. A plane, which is 2-dimensional, can lie in space, and space is 3-dimensional.

Computer software, the Geometer's Sketchpad, allows students to construct plane geometric figures on a screen and then flip, turn, stretch, shrink, or slide them to view from a new perspective. Exploring and observing to represent plane geometric figures in various positions by drawing and constructing with computer software also help students develop spatial sense, see Figure 2.1.

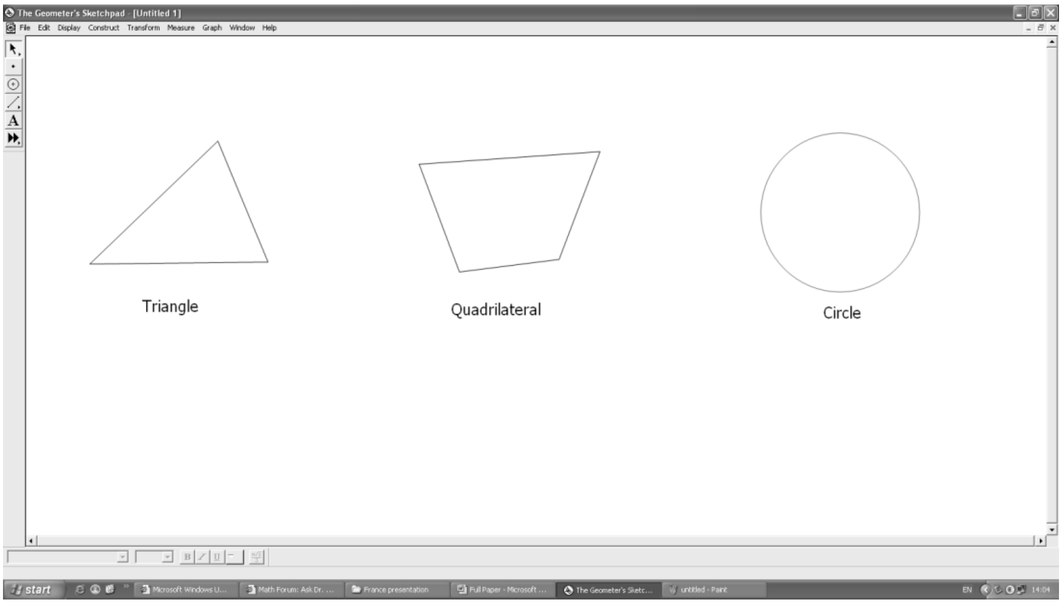


Figure 2.1 An example of plane geometric figures

Students first learn to recognize whole shapes and then to give the definition and analyze properties of a shape by their own words. Students are able to identify and classify plane objects and understand the relationship among the sides of plane geometric figures.

3. Regular Polygons

A regular polygon (also equiangular polygon) is a 2-dimensional figure that has all sides equal and all interior angles equal. Computer software, the Geometer's Sketchpad, allows students to construct regular polygons on a screen. Students can adjust the polygon by dragging any vertex point and notice that the length of all sides and the interior angle remains the same. Individual regular polygons are named (and sometimes classified) according to the number of sides, for example, a regular polygon that has five sides is named a pentagon and that has *n* sides is named *n*-gon. A regular polygon that has three sides is called an equilateral triangle and a regular polygon that has four sides is called a square. A diagonal is defined to be a line connecting two nonconsecutive vertices of a polygon. All polygons (regular and non-regular polygons) with the same number of sides have the same number of diagonals, so the number of diagonals of regular *n*-gon will be the number of diagonal of *any* polygon with *n* sides.

Based on the Geometer's Sketchpad activities, students can draw inferences and make logical deductions from geometric shapes. Students will count the total number of line segments that connect two vertices, including the sides. The result is the following expression:

$$(n-1)+(n-2)+(n-3)+\cdots+1+0=\sum_{i=1}^{n-1} i=\frac{n\,(n-1)}{2}. \tag{1}$$

and then subtract *n* from the result to derive an expression for the number *D(n)* of diagonals in an *n*-gon.

$$D(n) = \frac{n\,(n-1)}{2} - n = \frac{n\,(n-3)}{2} \tag{2}$$

In general, there is the common interior angle of an *n*-sided regular polygon. The sum of angles of any *n*-gon is 180(n – 2). Since all the angles of *n*-gon must have the same measure, the common interior angle is $\frac{180(n-2)}{n}$. A relationship between number of sides and interior angle of regular polygons is shown in Table 3.1.

Name of a polygon	Number of sides	Sum of interior angles	Interior angle
equilateral triangle	3	180	60
square	4	360	90
pentagon	5	540	108
hexagon	6	720	120
heptagon	7	900	$128\frac{4}{7}$
octagon	8	1080	135
nonagon	9	1260	140
decagon	10	1440	144
undecagon	11	1800	150
dodecagon	12	2340	156
n-gon	*n*	180 (*n*-2)	$\frac{180(n-2)}{n}$

Table 3.1 Relationship between number of sides and interior angle measure in regular polygons

The measures of the interior angles are extremely important in the exploration of expanding the regular polygons by using tessellation transformations.

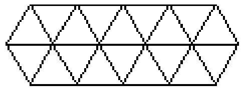
4. Tessellation Transformation Concept

This study was limited to the regular and semi-regular tessellation forms. A tessellation is created when a shape is repeated over and over again covering a plane without any gaps or overlaps.

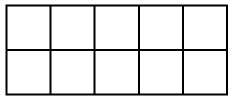
A regular polygon has 3 or 4 or 5 or more sides and angles, all equal. A regular tessellation means a tessellation made up of congruent regular polygons. A regular tessellation must contain only one kind of a plane polygon, that is equilateral, equiangular, and rectilinear. Every side of each polygon must belong also to one other polygon with every vertex of each polygon belonging also to one other polygon vertex.

Here are some examples of regular tessellation forms :

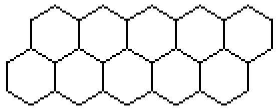
a tessellation of equilateral triangles



a tessellation of squares



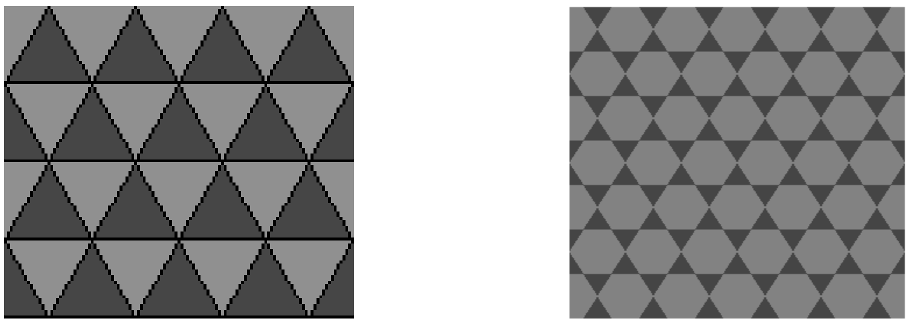
a tessellation of hexagons



Notice these three samples the squares are lined up with each other while the triangles and hexagons are not. If you look at 6 triangles at a time, they form a hexagon, so the tiling of triangles and the tiling of hexagons are similar and they cannot be formed by directly lining shapes up under each other - a slide is involved.

The symbols used to describe the tessellation forms (regular and semi-regular) use naming conventions by choosing a vertex, then look at one of the polygons that touch that vertex. How many sides does it have?

The notation of a regular tessellation of triangles has six polygons surrounding a vertex, and each of them has three sides: the symbol used to describe is “3·3·3·3·3·3” or “3^6” means there are six triangles at each vertex shown as Figure 4.1a.



a. b.

Figure 4.1 (a) a regular tessellation of triangles (b) A semi-regular tessellation

A semi-regular tessellation is a set of regular polygons of two or more kinds so arranged that every vertex is congruent to every other vertex. The rule for the semi-regular tessellation in Figure 4.1b is “3·3·6·6” or “$3^2 \cdot 6^2$”

In mathematics classroom, students are interested in constructing the expansion of plane geometry by using dynamic geometry programs such as the Geometer’s Sketchpad to design tessellation patterns and form the rules for various patterns. Based on the Geometer’s Sketchpad activities students enable to develop higher-order-thinking skills to construct body of knowledge by themselves.

References

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[3] B. Eckel (2006). *Thinking in Java, 4th Edition*. CA: Prentice Hall.