Cognizable, Learnable, Expressible, Accessible, and Reasonable Model in Mathematical Thinking, Reasoning and Problem Solving

Hsiu-Ju Chang
changhsiuju@gmail.com
Department of Education,
National Chengchi University
R.O.C.

Abstract: *By means of dynamic visualization, learners have multiple cognitive channels to participate knowledge and information acquisition. However, the acquisition and internalization processes may dependent on the cognitive load of individual learners. The analogy algorithm of dynamic and visual aid learning objects will lead individuals to make concept projections and infer specific analogical and relational conceptions during mathematical problem thinking, reasoning, and solving processes. The Cognizable, Learnable, Expressible, Accessible, and Reasonable Model (CLEAR Model) is to identify the core objectives of mathematic concepts and operations into visible, comprehensible, and recognizable presentations. The transactional analysis, inference, and analogy processes can express the essential conceptions and eventual conceptualizations in thinking, reasoning, and solving mathematical problems. In other words, the analogy algorithm of dynamic and visual aid learning objects usually take ownership and share leadership of instruction processes. In mathematical learning, the expressions of conception may formulate by individual's psychological order reasoning or generate by mathematical logic reasoning. In this paper, we show the parallel learning objectives in 1) two sides of two angles are parallel each other, 2) two sides of two angles are perpendicular each other, and 3) two sides of two angles are one side parallel and another perpendicular each other within using the CLEAR model to evaluate the analogy algorithm of learning objects and try to build an adaptive and reasonable learning paths for individuals to build their mental image.*

Introduction

The complex learning concepts are not easy for novel individuals to obtain and comprehend the characteristics and relationships of the conceptions. The CLEAR model maintains states of forward and backward cognitive processes that learners will experience in cognizable, learnable, expressible, accessible, and reasonable learning environment. An individual knowledge construction implicates he teaching configurations, learning rationalizations and algorithm interpretations which need to take into account for concretely learning cognition, authentication, and conceptualization. In cognitive perspectives, to attain the suitable information about cognitive attributes from learning objects, behaviors, misconceptions, and metacognition in learning processes will be the basic criteria to choose suitable learning objects and to fit the knowledge realization and construction. In mathematical learning, the applications of concepts may formulate by individual's psychological order reasoning or generate by mathematical logic reasoning. However, many of misconceptions are formulated by individuals' psychological order reasoning nor generated by mathematical logic reasoning. In this paper, we show the parallel learning objectives in 1) two sides of two angles are parallel each other, 2) two sides of two angles are perpendicular each other, and 3) two sides of two angles are one side parallel and another perpendicular each other within using the CLEAR model to evaluate the analogy algorithm of learning objects and try to build an adaptive and reasonable learning paths for individuals to build their mental images. The adaptive learning involves suitable interpretation, acceptable participation, sensible comprehension, and reasonable internalization in learning processes. The system-regulation is intended to derive learning processes or learning algorithm form the analogy algorithm of CLEAR model to be the suitable guidance to direct individuals' learning processes. The learning objects will guide learners to look into learning objects and to find, locate, and discover the specific conceptions and operations of specific issues. Consequently, the adaptive educational system will require capabilities for detecting, reasoning, and inferring the cognitive and metacognitive knowledge factors during learning processes. The CLEAR model will give the chances for interaction and communication between learning objects and learning concepts for knowing, sensing, and understanding the specific learning conceptions. Furthermore, the practice-driven learning objects can be refined and identified a well structure for learners to make senses and create their fundamental schemas to complex schemas.

Cognizable, Learnable, Expressible, Accessible, and Reasonable Model

Cognizable, Learnable, Expressible, Accessible, and Reasonable Model (CLEAR Model) is to identify the core objectives of mathematic concepts and operative skills into comprehensible and recognizable learning objects with the concept simplifications, skill concatenations, and structure implementations to help learners to construct their own mental images. The core objectives may comprise of several kinds of conceptions, operations, and applications, whatever to build the fundamental ideas or schemas, to connect the core concepts and related concepts with the meaningful linkages for learners to build their own schemas which are from simple to complex and from imitation to automation gradually. Mayer (2002) mentioned that learner should learn basic problem-solving skills in isolation to build the cognition in what to do and when to do. The CLEAR model will systematically sketch, manage, and monitor the learning processes via the interface of CLEAR system. However, the mathematic learning processes are not only memorize and comprehend the learning concepts, but also to connect and apply the learning concepts to understand, infer and reason the specific problems. Accordingly, the more analyzable, and interpretable knowledge framework with meaningful, connectable, and authenticable concept linkages will lead learning behaviors with enough reciprocal effects between the learners' mental states and concrete applications. Bruner (1966) suggests three modes for processing information allowing human beings to construct their worlds, in the following order: (1) through action → enactive representation (2)through imagery → iconic representation (3)through symbols and languages → symbolic representation. For the multi-feature of phenomena and expressions, each teaching concepts of a cognitive objective which are displayed in more than one expressions will benefit on learners' knowledge comprehension, multi-cognitive channels, and plenty descriptions of each teaching concept. Table 1 illustrates the cognizable, learnable, expressible, accessible, and reasonable to interpret, translate and communicate concepts and operations within CLEAR model in pedagogical perspectives of web-based educational system.

Table 1: Translation and communication of CLEAR model

Knowledge / Role interaction	Cognizable	Learnable	Expressible	Accessible	Reasonable
Acquisition Participation Internalization	Focusing; Realizing; Comprehending;	Knowable; Operable; Generable; Comprehensible;	Framable; Sensible; Analyzable;	Generating; Applying; Learn more and go further easily;	crossover concept boundary: 1) Recombination; 2) Reproduction; 3) re-communication;

Knowledge acquisition in cognizable way means to focus on major concepts to realize and comprehend learning objects in specific presentation. Knowledge acquisition in learnable way means to maintain presentations of learning objects in knowable, operable, and generable comprehensible for learning friendly. Knowledge acquisition via expressible way means to know how to make sense of new knowledge. Accessible means that individual can generate and reason learning concepts easily. Reasonable means to crossover concept boundary:1) recombination; 2) reproduction; 3) re-communication. Individual internalization in cognizable way is to comprehend the concepts with cognitive retrieving and reserving. Individual participation in cognizable manner means to build individual's knowledge via suitable cognitive deconstruction and construction.

Visualization in cognitive retrieving and reserving

The mathematic reasoning, speculating, and solving are constructed by individual's cognition and metacognition to reason, comprehend, connect, and apply the essential concepts and skills. And reading, hearing, thinking, or responding of learners are not always that can be accurate to express what are the cognition or comprehension abilities and situations of individual learners (Chang, 2010). Furthermore, the geometric thinking processes usually involve abstract cognitive reasoning in the mathematical figure, symbol language and operational principle for individual comprehension and manipulation. Accordingly, the inexperience and abstraction of geometric counting and reasoning are not easy enough for grade 8 learners to divide and conquer the complex concepts and problems. Duval (2006) asserted that we needed to determine the cognitive function in diversity of mathematical processes that could understand the mathematical comprehension difficulties of learners. In cognitive load theory (Sweller, 1988), the load of cognition includes learning contents and individual's learning abilities which are individual's processing capacity. The theory has been designed to provide guidelines of instruction design which intended to segment the learning objects and assist in the presentation in order to enhance learning performance. The segmentation, isolation, and representation of the learning materials are the essential and critical key points to mediate and communicate the learning and teaching processes. Accordingly, the segmentation, isolation, and representation of the learning concepts may lead learners to cognize, reason, and comprehend learning concepts on suitable cognition channels. For examples: 1) two sides of two angles are parallel each other (figure 1), 2) two sides of two angles are perpendicular each other, and 3) two sides of two angles are one side parallel and another perpendicular each other. One of the situations of two sides of red angle and blue angle are parallel each other is the degrees of two angles are equal, and another is sum of two angle degrees is equal to 180. And in figure 2 illustrates three situations of two sides of two angles are one parallel and another perpendicular each other. We can realize the results via to manipulate the angle degrees to know the real situations which are $\angle ABC + \angle FDE = 90°$, $\angle ABC - \angle FDE = 90°$, and $\angle ABC + \angle FDE = 270°$.

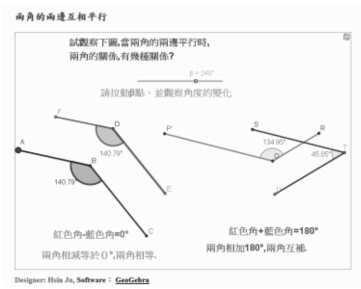

Figure 1: illustrates the two sides of red angle and blue angle are parallel each other

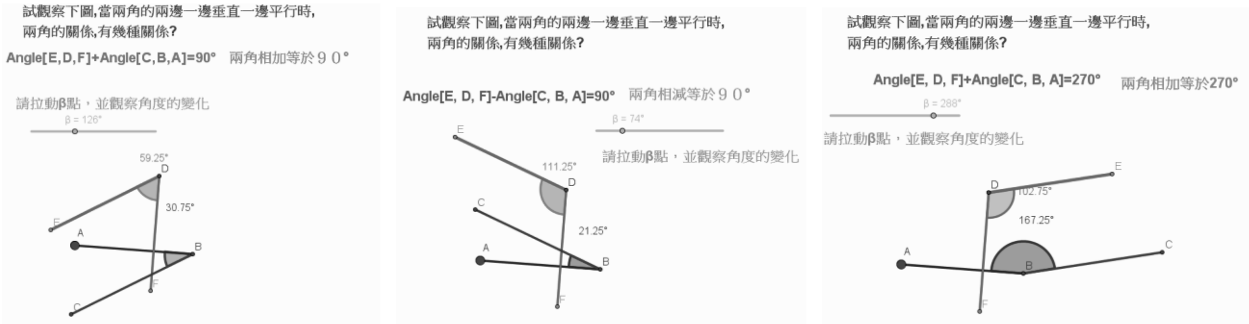

Figure 2: illustrates three situations of two sides of two angles are one side parallel and another perpendicular each other. In figure 2, we can proof it without words to show three situations to individuals that are $\angle ABC + \angle FDE = 90°$, $\angle ABC - \angle FDE = 90°$, and $\angle ABC + \angle FDE = 270°$. In this case, learners can pull the point β to see the changes of the angles and compare the angles' combinations in forms of formulas and figures. Furthermore, the back and forth of point β will support the cognitive retrieve and reserve channels for learners to construct their mental images. Thus the complex mathematic problem and opaque solving processes of 1) algebra counting and reasoning and 2) geometric thinking processes and deictic conjecture need to be directed and represented in suitable visualization ways for individual reasoning, comprehending, and applying. Accordingly, the knowing, conjecturing, connecting, locating, and relating the conceptual knowledge adaptively by serial deductive chains will be the key points during the solving processes. Therefore, the meaningful and suitable cognition channels can support the learners to realize and make sense the semiotic text and abstract concepts.

System-regulation in mathematical reasoning

System-regulation describes that learning system will lead the learning processes of individuals who are metacognitive, motive, strategic, and adaptive in cognitive retrieve and reserve processes. In System-regulation, the derivation of learning processes or learning algorithm are important for learners to know and look into learning objects to find, locate and discover the specific knowledge and operation. Conversely, instructors and system designers need to locate the ambiguity reasons or error types from learners for cognitive retrieve and reserve. However, the mathematical reasoning usually combines mathematical concepts, relational principles, and operational algorithms for learners to interpret and operate the related concepts and skills (Chang, 2008). Moreover, the multiple learning characteristics and reasoning algorithms are distinct in different contents and characteristics. Obviously, the ambiguity algorithm is reasonable for the individual in analogical reasoning and problem solving but not justifiable in mathematical reasoning and problem solving. The expressions of misconception may formulate by individual's psychological reasoning not by mathematical logic reasoning. Learners often use analogies from known domains to fill gaps in mapping and inferring in related domains. In CLEAR model, the practice-driven learning hierarchy need to be refined and identified a well structure for learners to make senses and create their fundamental to complex schemas. The figure 3 illustrates the fundamental elements to build the fundamental schema to complex schema of conceptions and operations in two sides of two angles are one side parallel and another perpendicular each other.

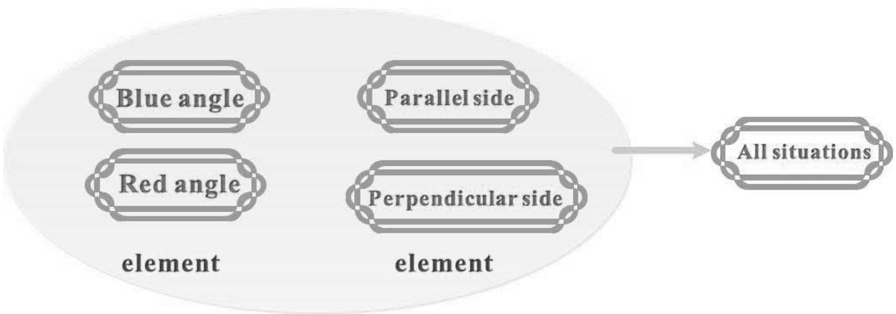

Figure 3:in cognitive load, the fundamental elements of two sides of two angles are parallel side and perpendicular side

The two sides of two angles are one side parallel and another perpendicular each other, after the elements interactions with the degrees of angles then the whole formulas will beyond individuals' imaginations to realize and make sense. Accordingly, the common way to solve this problem will be memorize the results ($\angle ABC + \angle FDE = 90°$, $\angle ABC - \angle FDE = 90°$, and $\angle ABC + \angle FDE = 270°$) under the unclear presentation. Conversely, in CLEAR model, the cognizable, learnable, expressible, accessible, and reasonable learning objects will lead the individual to know the whole situations which the different degrees of the two will become distinct relationships between two angles. The learning objects will be manipulated by individual learner to control the transformation of two angles and to discover the characteristics of the degree computation of two angles. In addition, to give the questions for guiding and assisting with detecting and understanding the specific meanings of learning objects is an another system regulation in CLEAR adaptive system in accessing and reasoning the visual representation. The figure 4 illustrates the presentations and reactions of the two sides of two angles are one side parallel and another perpendicular each other.

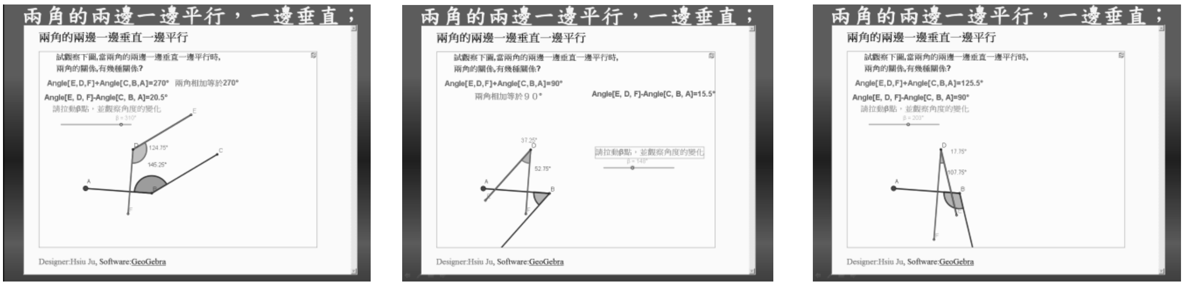

Figure 4: the different presentations and reactions of the two sides of two angles are one side parallel and another perpendicular each other

Consequently, the psychological order reasoning with ambiguity algorithm may rebuild and re-comprehend with the learning objects by system-regulation.

Conclusion

The learning system which was based on CLEAR Model led learning processes in simpler, easier, and clearer ways for individuals to make sense and to build their mental images. There are many ways to enhance individual learning by using psychology aspects**,** pedagogical aspects, technologies, teaching/learning strategies. The adaptive strategies for a web-based educational environment could be more effective when learning is directed by more than one layer of adaptive mechanisms. Otherwise, the more complex, difficult, and nonstructural learning objects will have the chances to lead them to make holes in their learning processes. Cognizable, Learnable, Expressible, Accessible, and Reasonable Model (CLEAR Model) is to identify the core objectives of mathematic concepts and operative skills into comprehensible and recognizable learning objects with the reasonable concept simplifications, skill concatenations, and structure implementations to help learners to construct their own mental images. As we know that teaching processes have more conditions and problems need to be conquered, then the more adaptive teaching effects we need to design in teaching processes. Moreover, less able learners based on their prior experiences to think, generate, and process their learning behaviors individually. Making knowledge communication in manipulative and interactive will be the chances to give a window for instructors and learners into teaching and learning processes. Accordingly, the shareable and interpretable knowledge framework of CLEAR model will benefit individuals' knowledge and schema constructions which are form simplification to complication and form imitation to automation in learning and solving processes. The segmentation, isolation, and representation of the learning materials are the essential and critical key points to mediate and communicate the learning and teaching processes. Consequently, the segmentation, isolation, and representation of the learning concepts may lead learners to cognize, reason, and comprehend learning concepts on suitable cognition channels. In CLEAR model, the practice-driven learning hierarchy refines and identifies a well structure for learners to make senses and create their fundamental to complex schemas gradually.

References

Bruner, J.S. (1966) *Toward a Theory of Instruction*. Cambridge, Mass.: Harvard University Press.
Chang, H. J. (2008). Supporting Fault-Tolerance Learning within Cognitive, Object-Oriented Teaching Model. *ED-MEDIA 2008--World Conference on Educational Multimedia, Hypermedia & Telecommunications, Association for the Advancement of Computing in Education*, Vienna, Austria.
Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 103–131.
Mayer, R. E. (2002). *Metacognition in learning and instruction theory, research, and practice* edit by Hartman, H. J
Sweller, J. (1988). Cognitive load during problem solving: effects on learning. *Cognitive Science*, 12, 257-285.