The MCY-Activities: Constructing and Sharing Three Types of Pattern

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Abstract: *In recent years, an increasing number of viewpoints hold that students should be engaged in a learning environment where understanding and knowledge transfer take place. This study introduces Mathematics Created by You (MCY)-mentoring program, which allows students to construct pattern artefacts to explore and share. This program is Web 2.0 online-based and so can be shared by several people and mathematics leaning takes place through interactions within this carefully designed environment. In addition, this studies the activities about three types of patterns and three types of mathematical patterns (building block pattern, motion graph pattern and recursive and probabilistic pattern) included in the activities, which are currently taking place for a project that builds an amusement park called 'Mathland' as a part of MCY-mentoring program. It is observed that the symbol expression and pattern that students designed to create their pattern artefacts in the context of play were progressed from a personal expression to a mathematical expression.*

1. Introduction

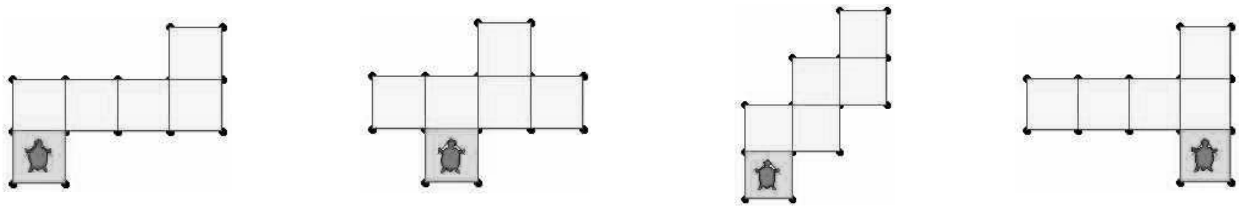
It is widely accepted by many researchers that active learning approaches such as asking questions, searching for constructions and creating abstract concepts should be adopted in order for students to understand and transfer the knowledge when they learn mathematics and science. There has been recently a growing interest toward the technology-based learning which requires students to actively participate in their learning by developing simulation activities. At the same time, numerous studies have demonstrated the efficiency of technology-based learning (see [5], [6] and [9]).

As STEM(Science, Technology, Engineering and Mathematics) education coalition has been emphasized in U.S., STEAM(Science, Technology, Engineering, Arts and Mathematics) education coalition has been emphasized in Korea. The Science Created by You (SCY) project is also currently underway in Europe (see [5]). This project reflects the spirit of constructionism which believes knowledge construction takes place when students are engaged in building objects. In fact, SCY learners are supposed to work on missions by producing 'emerging learning objects (ELOs)' that are created by them. In order to accomplish these missions, students are required to equip themselves with integrated knowledge of different domains (e.g., physics and mathematics, or biology and engineering). In these activities, students perform like scientists who are exploring and creating science. Their productive learning activities include designing a plan and stating a hypothesis, doing experiments, analyzing data and discussing the results. In this study, we propose MCY(Mathematics Created by You)-mentoring which applies mathematics to SCY principles and discuss the pedagogical implications of MCY by investigating the examples that were posted on a mentoring program website 'http://mentoring.snu.ac.kr/siheung/index.html'. MCY-mentoring is a kind of informal mathematics program which reflects the spirit of STEAM education coalition. Currently, a collaborative project is in progress that builds Mathland in MCY-mentoring program. We will study the three types of activities that are required to build Mathland and the mathematical patterns hidden behind the activities, and then find its educational implications.

2. The MCY-Mentoring Pedagogical Approach

Constructionism proposed by Papert is based upon Piaget's constructivism, but it is more pragmatic because it contains specific methods and strategies. As pointed out in [1], both theories hold the same view, while Papert's constructionism focuses more on the role of media. Papert's constructionism, in other words, holds that learning occurs when 'physical construction' like building artefacts transforms into 'mental construction' through media. This notion has been well explained in [7]. MCY-mentoring program is basically inspired by constructionism theory. It helps learners build specific artefacts in media context that aims to boost the idea of 'learning by making' or 'learning by designing'.

In MCY-mentoring program, constructed artefacts are represented by symbols. Thus, artefacts made by learners are identified as semiotic symbols and visual objects which correspond to each other. It means semiotic symbols remind learners of visual objects, and vice versa. Learners will notice the semiotic symbols when they see visual objects. A Logo-based symbol using turtle metaphor of 'forward' and 'rotate' is what a learner projects himself to the turtle on the screen and it is embodied to the learner physically and mentally. In [8] defined it as 'embodied symbols'. Figure 1 shows the process of constructing a regular hexahedron using embodied symbols. As can be seen in Figure 1, the turtle is constructing a rectangle faces with three embodied symbols – m (moving forward), L (rotating to left), and R (rotating to right), and the green face is the initial position where the turtle starts to construct an object.



(a)	(b)	(c)	(d)
mmRmmmLm	mmLmRRmmLmRRmLm	mmRmLmRmLm	mmmRRmRmmm

Figure 1 Constructing folding nets of regular hexahedron using embodied symbols

The symbols represent how to construct an object, while visual objects show the result. There are four different types of symbols that can construct the same 3*3 quadrangle as in Figure 2. M1 constructs in zigzag pattern. M2 rotates from outside to inside in a spiral pattern. In M3, the turtle constructs a facet starting from left bottom to upward using symbol '[ ]' which means the turtle remembers its position and then branches out to the right to make two more

visual product	symbolic executable expression
	(M1) do mmmRmRmmLmLmm (M2) do mmmRmmRmmRmRm (M3) do m[Rmm]m[Rmm]m[Rmm] (M4) A='m[Rmm]′; do AAA

Figure 2 Diverse symbol expressions that construct the same visual object

facets. Also, we can see the basic module in M3 'm[Rmm]' has been replaced by a simple one- A in M4. In this way, learners' thinking process can be investigated by analyzing the symbols since those symbols reveal the whole process.

Moreover, the embodied symbol-based artefacts can lead to the connection between 'aretefact-learner' and 'artefact- third party'. For instance, the main subjects-'learners and artefacts'- can interact with each other because learners can reflect their thoughts by observing the visual objects and correct mistakes by operating the symbols. 'Artefact-third party' interaction is noticeable along with 'artefact-learner' interaction. The 'Third party', which refers to peer students or mentors, can perceive how the learner developed thinking when constructing an artefact by considering the symbols. This indicates that social communication is taking place through comments or writing on the bulletin board since other people can evaluate the outcomes or give feedback. This social interaction is exactly reflected in our Web 2.0 online MCY-mentoring environment. In this sense, social interaction can provide scaffolding that leaner's inner communication lacks of. In addition, social interaction is more specific and accurate than usual interaction since it occurs among people mediated by symbol-based artefacts, and also it may derive more fruitful communication related to learning.

MCY-mentoring is a Web 2.0-based learning environment where learners can construct their own artefact with embodied symbols through JavaMAL microworld and share and explore it with others. In this microworld environment, the typed expression appears on the editor screen, and executing the expression makes a polycube appear on the execution screen. Because the typed expression can be saved and shared with others, students can ask questions or exchange their ideas through by writing comments. We were actually able to observe that students asked questions on their peers' artefacts and solved the raised issues cooperatively.

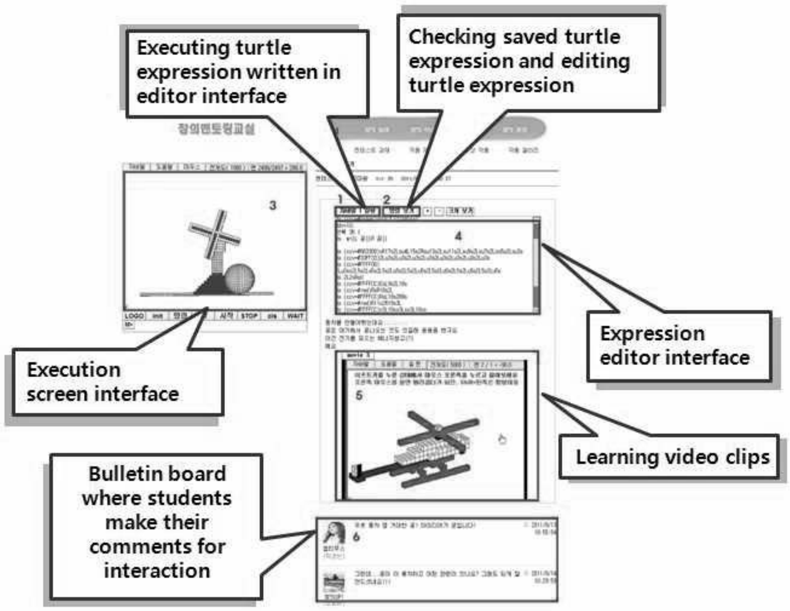


Figure 3 MCY-mentoring environment

With the development of media, activities to manipulate 3D object have been expanded and mathematics education also stresses its importance. With the stream, in [4] developed 3D representation system where 3D objects can be constructed in a virtual space, based on the idea of traditional 2D-Logo and physical LEGO bricks. While Logo makes a figure in 2D plane by using basic symbols of 'forward' and 'rotate', JavaMAL microworld designed in [3] can make a 3D shape with polycubes. A polycube is a solid figure formed by joining one or more unit cubes face to face. Figure 4 shows 5 basic embodied symbols which would construct polycubes. 's' is a symbol for a turtle to make a cube as moving one step forward from its position, and 'R' and 'L' are symbols for a turtle to change its direction to right and left respectively. In addition, 'u' and 'd' are symbols for a turtle to make a cube by lifting on floor up and down as if a turtle is in an elevator. The left-side image in Figure 4 is a polycube constructed by the symbols of 'ssRsusLssd'.

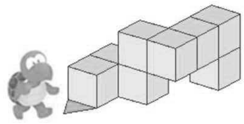
 ssRsusLssd	**s** : one **s**tep forward **R** : turn **R**ight **L** : turn **L**eft	horizontal movement
	u : move **u**p **d** : move **d**own	vertical movement

Figure 4 3D representation system

It is also described that the meaning of '[ ]' and 'substitution command' which were usefully used in MCY-mentoring program in Figure 5.

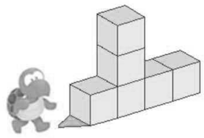
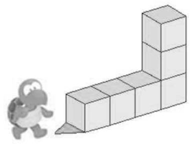
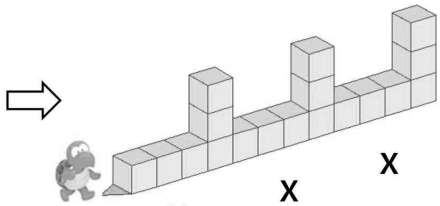
 ss[uu]ss	 X='ssss[uu]'  X X X
'A[B]C' orders a turtle to make A, memorize its position P, makes B, then come back to P, and make C.	Recognize a repeated module (ssss[uu]) and substitute it with one symbol (e.g. X). Then, students can simply create an applicable object by using X.

Figure 5 '[ ]' symbol and substitution command

3. MCY-Activities: Constructing and Sharing Three Types of Pattern

We conducted three activities about three types of patterns and three types of mathematical patterns (building block pattern, motion graph pattern and recursive and probabilistic pattern) included in the activities, which are currently taking place for a project that builds an amusement park called 'Mathland' as a part of MCY-mentoring program.

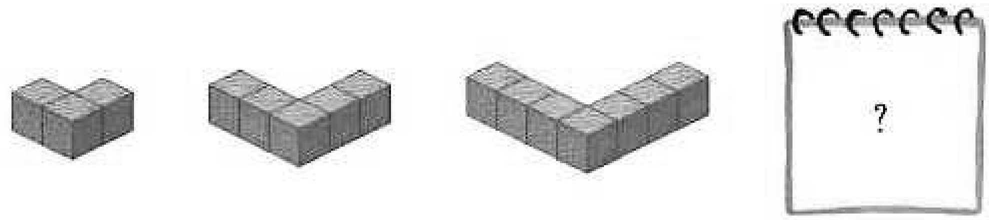
3.1. Virtual LEGO Contest: Building Polycube Pattern

A virtual LEGO contest where students make creative artefacts, using 5 embodied symbols, '[ ]', and substitution command were carried out. Elementary and middle school students as well as mathematics teachers participated in the contest in different periods respectively. It took about 6 weeks in total to learn the basic symbols and submit the final artefacts. They were also to ask questions by writing on the Q&A bulletin board or putting comments. After the learning period, participants created their own artifacts for the last two weeks by using the symbols they learned and finally submitted their final artefacts. Figure 6 shows the artefacts submitted by 6th grade students.

This JavaMAL microworld could be used for students to construct and explore mathematical patterns by using symbols that they feel already familiar with while creating their own artefacts. Students can discover a pattern through a pattern construction activity and represent it by generalizing with a microworld language system. When doing this, symbols become a tool for a pattern construction and manipulation.

Figure 6 Participants' contest artefacts using building polycubes

Students are able to understand the structure as they construct a figural pattern with symbols and also become able to represent it in a more sophisticated and compact form from a complicatedly constructed symbol in the beginning as they recognize repeated and changing factors. If a personal language learned in the context of play can be progressed to a mathematical expression, learners will find such activities very meaningful (see [11]).



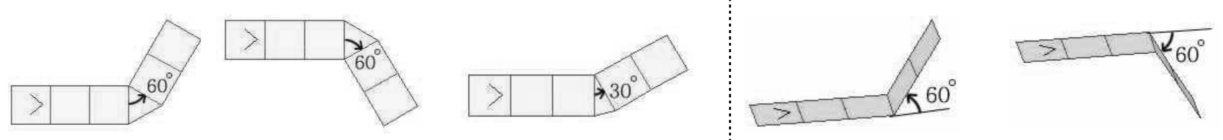
do ssLs		do 2sL1s		do (n+1)sL(n)s
do sssLss	→	do 3sL2s	→	
do ssssLsss		do 4sL3s		
do sssssLssss		do 5sL4s		general form

Figure 7 Generalization of symbols, representing a pattern activity in elementary school mathematics textbook

3.2. Constructing Dynamic Animation Construction: Motion Graph Pattern

The 5 basic embodied symbols (s, R, L, u, d), '[ ]', and 'substitution' symbols presented in 3.1 are the basic tools for students to construct static objects. During the contest, students wanted to make their artefacts move. As physical construction kits, like Lego WeDo, equipping active motor or motion sensor with traditional LEGO bricks, have been recently introduced, in JavaMAL microworld that can be viewed as virtual construction kits, symbols to animate artefacts were designed as students put their artefacts on the rail and make them move. This is based on [3]'s 'turtle strap' idea, constructing a strap by connecting the faces that the turtle makes as moving forward. The activity to construct dynamic animation along the rail can arouse students' interest as well as serve as a tool to explore motion graph pattern.

2D Rotations			3D Rotations	



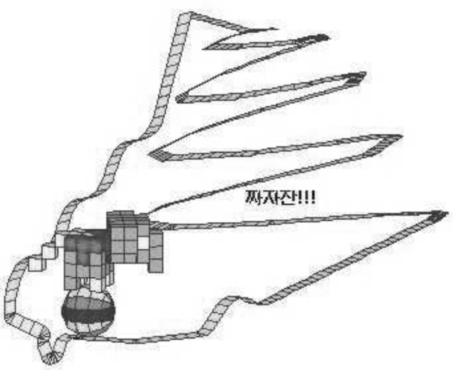
mmmomm	mmmOmm	mmm(rrv=30)omm	mmm(60#)mm	mmm(-60#)mm

Figure 8 2D and 3D Rotations

As a turtle makes a 3D cube as moving forward in a virtual LEGO contest, turtle strap applies this idea to the 2D plane. We introduced symbol ‘m’ that makes a square and symbols ‘o, O’ that make a regular triangle. Here, small letter ‘o’ and capital letter ‘O’ are used for 60-degree counter-clockwise rotation and 60-degree clockwise rotation respectively. That is, the symbol ‘m’ is used to make a straight-line route and ‘o and O’ are used to make changes in direction. The symbols ‘o and O’ are to rotate 60 degree to make a regular triangle, but the symbol ‘rrv’ including a variable can be used to adjust a rotation angle. By putting (rrv=‘a’) before o or O symbol, students can define a rotation angle in ‘a’ (See Figure 8).

In addition to the rail on a 2D plane, students also wanted to make the rail that move in a 3D space like a rollercoaster in an amusement park. To make a 3D rail, a symbol to change the direction not only to left and right, but also upward and downward is required. In our environment, we use ‘(a #)’ symbol to change the direction upward and downward. If this symbol is used before the symbols ‘m’, ‘o’ or ‘O’, the newly created face is rotated towards the previous face, making a dihedral angle. Both positive and negative number can be put in ‘a’ and the direction changes upward at an angle of ‘a’ if a positive number is used and changes downward if a negative number is used.

We developed animation symbol to make the 3D object move on the rail, engine symbol to make it rotate, and spring symbol to move it straight-line. By adding dynamic effects into their artefacts created in the virtual LEGO contest, students started making rides which they will put in Mathland like a roller-coaster or a merry-go-round. For this, students calculated a degree of angle to make a rail that they designed and also used engine symbol and spring symbol to add dynamic effects. For example, a student made the elephant on the ball turn round with engine symbol as it moves along the rail shown in Figure 9. Students also used symbols and mathematical calculation

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약속 A{dd=10}
약속 a{dd=-10}
약속 B{do mmmmmmmm}
ㄱ
window 200
ddv=10
rrv=20
do mmmmmoommmmmooAmAmAmamamamamamamAmAmAmmmmmmmmmmm
do mmmmmmmmmmmmmmmmmmm>oB>ooB>ooB>ooB>ooB>ooB>ooB>ooB
do >oooB>ommmmmm>oommmmm>oommmmmm>oommmmmmmm>oommmmm>oo
do mmmmmmm>oommmmm>oommmmmmm>ooommmmmmm>ommmm>oommm>oommmmm
do >oommmmmm>oommm>oommmmm>oommm>oommmmm>ooommmm>ommm>ooooommm
do >oommmm>ooooomm>oom>oomm>ooooomm>ooooooommmmmmmmm>>>>>>
do mmmmmmmmmmAmAmAmAmAmamamamamamamAmAmAmAmAmamamamamamamAmAmAmAmAm
do amamamamamamamamooommm0000oomoooooooom>>>>>m
do RmHuuuuuuuu
do Cd(ccv=#red)s[do (ccv=#ADADAD)sssLssLssLsLsusRCsRssRssR(ccv=#ADADAD)sCsR
(ccv=#ADADAD)susRss[LsdLsuLs]RssRs[LsdLsuLs][Rs]s
do RsdLsdd]
do (ccv=#red)Ls(ccv=#red)LssLs[sLss(ccv=#ADADAD)dLssL(ccv=#red)s(ccv=##ADADAD)
sL(ccv=#red)s(ccv=##ADADAD)sL(ccv=#red)s(ccv=##ADADAD)
dsLssLssLssdLHsCsLHsCsLHsCs
do uuuLsRsHsCdHsCdHdLLsssE
ddv=30
do (ccv=#green)m[>m>m>m>m>m>m>m>m>m>m]
do (ccv=#E0F4FF)R>mL[>m>m>m>m>m>m>m>m>m>m]
do (ccv=#FF3FF7)R>mL[>m>m>m>m>m>m>m>m>m>m]
do (ccv=#68269A)R>mL[>m>m>m>m>m>m>m>m>m>m]
do (ccv=#FFE033)R>mL[>m>m>m>m>m>m>m>m>m>m]
do (ccv=#FF7E00)R>mL[>m>m>m>m>m>m>m>m>m>m]
```

Figure 9 Students’ Artefacts and Its Symbols in the Contest to move their artefacts dynamically. That is, these activities that build such patterns can be used later when constructing function graphs in mathematics.

3.3. Green Project: Recursive and Probabilistic Pattern

Recursion plays an important role in both inductive and deductive thinking, and it is covered in secondary education or university curriculum due to a difficult form of recursive subroutine. However, Math in Context (see [2]), the textbook written by Freudenthal Research Institute, introduces the mathematical concept of recursion in the context where a string of symbols composed of R(Red) and B(Black), representing rattlesnake's colour pattern, is sequentially grown shown in Figure 10. Unlike a ready-made mathematical formula, a string of symbols that consists of R and B is semi-formal. Hence, manipulating a string of symbols helps learners conjecture, observe, reflect and explore mathematical ideas (see [8]).

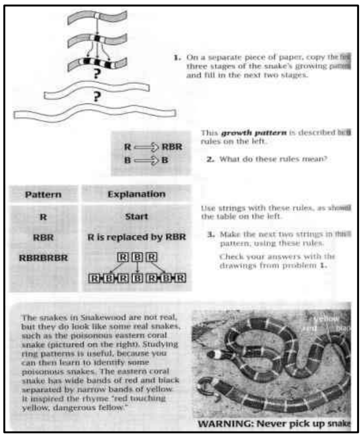
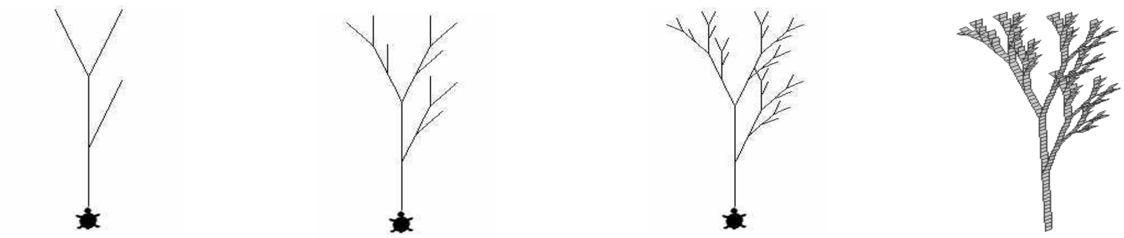


Figure 10 String of Pattern Symbols (see [2])

Recursion reflects natural phenomenon like fractals. Lyndenmayer, a biologist, proposed L-system, a grammatical system to describe the processes of plant development which consists of an initial value and a collection of production rules. In [12] introduced turtle symbols and led to turtle actions in L-system. Similarly, in [3] also designed the turtle metaphor-based L-system environment by using simple symbols of 'f, <, >' in JavaMAL microworld environment. 'f' is a symbol to move a turtle forward at a certain length '<,>' is a symbol to rotate a turtle at a certain angle, and '[ ]' is a symbol to save a turtle's position and recall it. Figure 11 shows the tree growing rules and tree images in each growing step.



do_2 X do_3 X do_4 X do_5 X

Production rule : X= 'f [> X] f [< X] > X' ; f = 'ff'

Figure 11 Growing Trees and Using Recursion

In JavaMAL microworld environment, recursion can be constructed by a similar rule through 3D turtle blocks in addition to 1D turtle line and 2D turtle strap. Figure 12 shows the recursion context in Math in Context textbook (see Figure 10) moved to JavaMAL microworld. This shows the growing process from step 0 to step 3 according to the production rule of R->RBR, B->B. In fact, the production rule that makes recursion can be viewed along with substitution symbol students used above. For

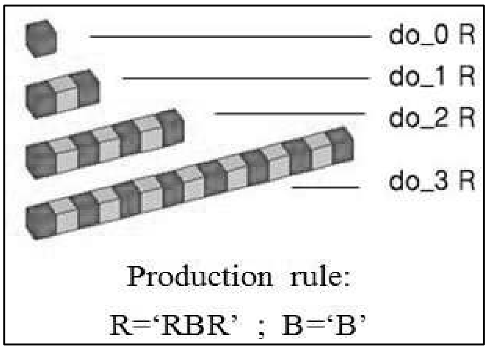


Figure 12 Symbol-Based Microworld

example, the production rule, R=‘RBR’, is similar to the substitution shown in Figure 5 (e.g. X=‘ssss[uu]’), but previous stage’s self-replication occurs in the next stage by including existing ‘R’ in ‘R’ newly substituted.

Probability can also be included in the production rule that constructs recursion in JavaMAL microworld. For example, Figure 8 is the context where a turtle slides down on skis from the top of the hill to the bottom. When making recursion by using dummy variable B, probability of 50% of ‘RB’, coming down to the right, and probability of 50% of ‘LB’, coming down to the left, are ordered by B=‘RB, LB’ symbol. Recursion occurs three times by ‘do_3 B’ symbol, and the turtle ends up with different positions via different routes every time. Such a recursion experimental environment including probability can be extended to combinational thinking or mathematical exploration on normal distribution.

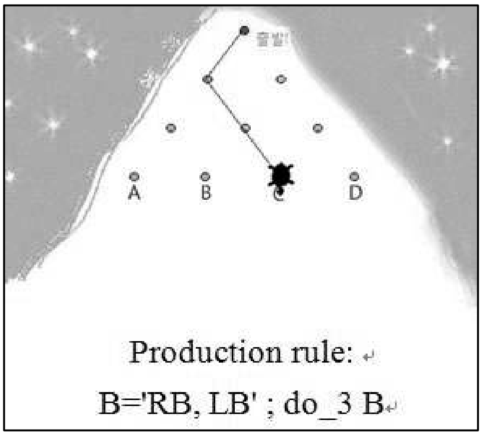


Figure 13 Recursion Using Probability

We conducted Green Project to the students who mastered the symbols in the contest. Green Project is an activity to create various kinds of flowers or trees by using ‘growing rules and probability’ explained above, in commemoration of Tree-Planting Day. Figure 9 shows the artefacts of undergrad school students who participated in Green Project. A variety of artefacts were submitted including cosmos, orange tree, sun flower and mathematics etc.

Recursion used to construct students’ artefacts is along with symbols and substitution symbol presented in the previous activity; however this leads us to focus more on the extrinsic perspective which recognizes the relation between the whole and parts, rather than turtle’s intrinsic perspective. In addition, students can reflect their thought process by receiving feedback through visual images represented by symbol-based recursion rules. Such activities in JavaMAL microworld practice ‘learning by design’ in that students accept mathematics concepts as a concrete idea in their own context. Furthermore, such activities would enable students to observe, explore and conjecture various mathematic situations from their intuition and generate new mathematics concepts through ‘what if questions’.

Figure 14 Students’ Green Project Artefacts

4. Conclusion & Discussion

Based on educational philosophy of 'Constructionism' and 'Web 2.0 Interaction', MCY-mentoring is designed as a learning environment where learners can construct their own artefact and explore it in the internet space shared with others. This is related to mathematics, but not limited to mathematics only. This, in fact, aims for students to experience 'Creative Mathematics', starting from a fun play and moving to learning and exploration.

In preparation of building Mathland in MCY-mentoring program, we studied these three types of pattern activities in this paper. The symbols used respectively in the activities like creating LEGO-like objects with 3D building blocks, making them move by using rotation, straight-line movement and animation symbols, and creating a tree with recursion and probability were introduced as a personal language that helps learners construct meaningful objects in the context of play, not as an abstract mathematical expression. Such symbols can be connected to mathematical exploration like number patterns, motion graph patterns and recursive patterns. In addition, artefacts constructed in each activity will be used in the collaborative project to build Mathland.

We can find educational implications of MCY-mentoring environment in two perspectives. First of all, from a constructionism perspective, we observe that students can use the basic symbols that they learned to create their own artefacts as tools for mathematical exploration. That is, the tools students feel familiar with can be 'cognitive tools' in the next stage of exploration activity and may help Low floor & high ceiling/wide wall (see [10]) practice. Secondly, MCY-mentoring is a Web2.0-based interactive learning environment; therefore students' artefacts are shared in the internet space and can be interacted with others through comments. Students actually raised questions on their peers' artefacts and solved them through interaction.

On the other hand, MCY-mentoring environment also needs improvement in technology and management. From a technology standpoint, MCY-mentoring environment enables learners to represent and deliver what they construct, but does not play a guide role intelligently. For this, automation functionality can be considered to provide learners with feedback or guides on appropriate learning contents according to learner's level without peers or mentors involved. Additionally, it may systemize and automate database by students or by activities with e-portfolio functionality enhancement. From a management standpoint, it is very important to manage learners effectively and attract their voluntary participation given that MCY-mentoring occurs in the online environment. Thus, it needs to be discussed regarding how to encourage interactions with mentors and peers, how to develop tasks that stimulate learner's intellectual curiosity, how to give an educational compensation to excellent learners and how to attract their participation.

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