# Mono-unary algebras and functional graphs in upper secondary school mathematics

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**Abstract:** *In this paper, alternative descriptions of functions are demonstrated with the use of a computer. If we understand them as mono-unary algebraic functions or functional graphs, it is possible, even at the school level, to suitably present many of their characteristics. First, we describe cyclic graphs of constant and linear functions, which are a part of the upper-secondary level educational curriculum. Students don't expect to see the surprising characteristics of such simple functions which can not be revealed using traditional Cartesian graphing. The next part of the paper deals with characteristics of functional graphs of quadratic functions, which play an important role in school mathematics and in applications, for instance in the description of non-linear processes. We show that their description is much more complicated. In contrast to the case with functional graphs of linear functions, it is necessary to use computers. Students can find space for their own individual exploration to reveal lines of interesting characteristics of quadratic functions, which give students a new view on this part of school mathematics.*

## 1. Introduction

In this paper, we present representations of functions as algebras with one unary operation and as special orientated. The theory of unary algebras is today a profoundly elaborated algebra theory enabling an original view on the theory of homomorphism (see for example [6] and [8]) or the theory of automata (see [7]) etc. Most of the concepts, which will be later introduced, can be used in school mathematics and it is possible to work with them on an intuitive level and to demonstrate to students, in a natural way, the advantages of the mathematisation of various situations.

## 2. Unars and functional graphs

First of all, we summarise the basic terms of the field of the theory of mono-unary algebras and functional graphs. Let $f$ be a mapping of set $A$ to itself. Then, we may consider $f$ to be an operation on set $A$. Briefly, we call the given mono-unary algebra $(A, f)$ the unar. We can assign unambiguously to this unar an orientated (directed) graph so that $A$ is a set of vertices and from vertex $u$ there is an oriented edge to vertex $v$ only if $v = f(u)$. We call this graph the *functional graph* respective to the unar $(A, f)$. It is obvious then that the oriented graph is a functional graph only if there is just one edge from every vertex. The transition from the given mapping, the described unar and the respective functional graph will obviously be used commonly in the following text.

A unar $(A, f)$ is called *connected*, if for each two elements $a,b \in A$ there is a couple of non-negative integers $m$, $n$, so that $f^n(a) = f^m(b)$. Otherwise, such a unar is called *disconnected*.
A unar $(B, g)$ is called a sub-unar of unar $(A, f)$, if $\emptyset \neq B \subseteq A$ and mapping $g$ is a restriction of mapping $f$ on set $B$. The maximum connected sub-unar within the meaning of the given relation is called a *component* of the unar $(A, f)$. The connected unar consists only of one component. Components of the given functional graph are called *orbits*. The sub-unar $(B, g)$ of the connected graph $(A, f)$ is called a *cycle of lenght* $k > 0$, briefly a $k$-cycle, if set B consists of $k$ elements $\{x_0, x_1, \ldots, x_{k-1}\}$ and it is true that $f(x_m) = x_{m+1}$ for $0 \le m < k-1$ and $f(x_{k-1}) = x_0$. The component of the functional graph is called *cyclic*, or *acyclic* respectively depending if it contains a cycle or not.

Let's say that vertex $a$ of a functional graph of the respective unar $(A, f)$ is of degree $\infty$ if there is a progression $(x_n)_{n=1}^{\infty}$ such that $a = x_0$ a $f(x_{n+1}) = x_n$ for every $n \in N$. If vertex $a \in A$ is not of degree $\infty$, we assign to it a definitive degree in the following way: vertex $a$ is of degree 0 if there is no vertex $b$ such as $f(b) = a$, i.e. there is no edge leading to vertex $a$. If vertex $a$ has no degree and all vertices $b$ satisfying $f(b) = a$ have a degree, we determine the degree of vertex $a$ as the lowest natural number which is greater than all degrees of the given vertices $b$. Every component of a unar apparently contains one cycle in maximum. Every definitive orbit is cyclic.
Let $(A, f)$, $(B, g)$ are unars. Mapping $h: A \to B$ is a *homomorphism* of a unar $(A, f)$ to a unar $(B, g)$, if $h \circ f = g \circ h$. If $h$ is a bijection, then unars $(A, f)$, $(B, g)$ are isomorphic. In this case we say that $f$, $g$ are *conjugate*. It is obvious that functional graphs of conjugate functions are isomorphic.

**Examples of linear functions and their orbits**

First, let's consider a real constant function, for instance $y = 1$. The description of a corresponding functional graph is "simple", as from each point (real number) there is only one edge and all edges lead to number 1. Number 1 maps to itself, so in this case there is a cycle of length 1 called a *loop*. The functional graph is connected. In Figure 1, there is a Cartesian graph and an orbit representing this function. Let's explore the linear function $y = x + 1$. It is obvious that the number of orbits of the functional graph is, in this case, infinite. All orbits are infinite, acyclic and all of them "go through" a point from interval $\langle 0,1 \rangle$. In Figure 2, there are two such infinite orbits.

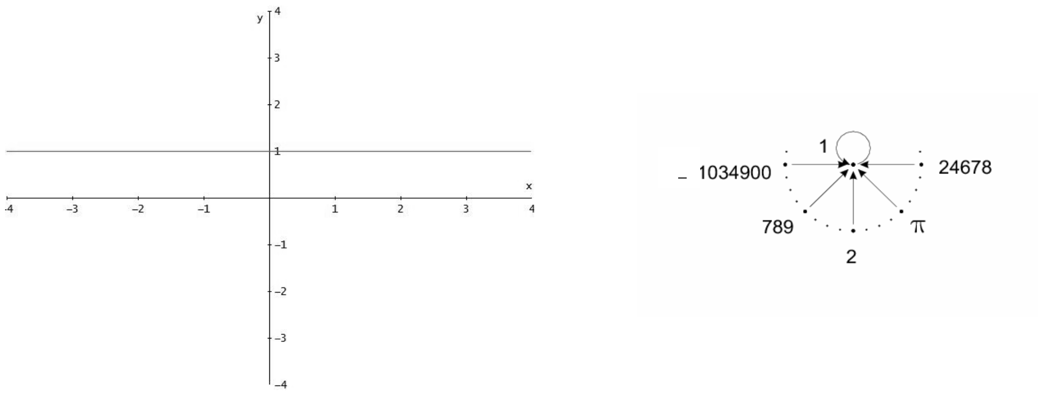


**Figure 1** Constant function

The functional graph of the linear function $y = x$ is totally different. In every real number, there is a loop, as every point maps to itself.
We can see that the orbit of function $y = c$ is only one and it is cyclic, while linear function $y = x$ has an infinite number of orbits and all of them are cyclic. If we move this function by a certain non-zero parameter the situation changes. Its orbits are infinite, acyclic and there are infinite numbers of them. However, the graphs of these functions are lines.

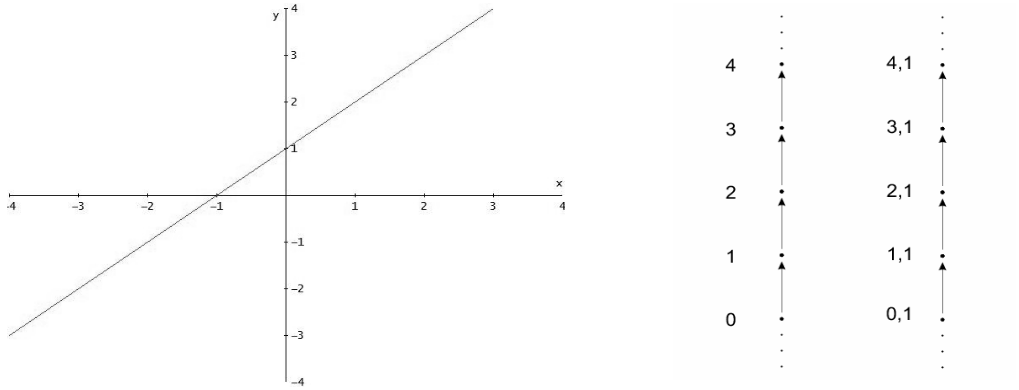


**Figure 2** Function $y = x + 1$

## 3. Functional graphs of quadratic functions

Now, we can demonstrate that the description of the functional graphs of quadratic functions is much more complicated. Let's deal with system $\tau$ of quadratic functions $f(x,t,s) = (x+t)^2 + s$. Our aim is to decompose this system into blocks of mutually conjugate functions. The description of this decomposition is significantly influenced by intersections of the given quadratic function $f(x,t,s)$ with identity $g(x) = x$, as the functional graphs will have loops in these points. Let's solve the equation $x = x^2 + 2tx + t^2 + s$. In the case that $s + t > \frac{1}{4}$, the given equation has no solution; for $s + t = \frac{1}{4}$, the equation has one solution; and finally for $s + t < \frac{1}{4}$, it has two solutions. The blocks of the desired decomposition, i.e. blocks of mutually conjugate functions, cannot be determined only using the found intersections. However, it enables us to discover new findings.
Let's consider the function $y = x^2 + 1$. In Figure 3 are its graph, and on the right, two of countless orbits into which the functional graph of the given unar is decomposed. All elements $x \in (-\infty, 1)$ are of degree 0. Element 1 has, as the only element, only one predecessor – element 0. All other elements of the given orbit have two predecessors. All other orbits are mutually isomorphic and in each of them lies an element of interval (0, 1). The following statement is important for the description of blocks of conjugate quadratic functions of the given system.

**Theorem 1.** The given functions $f(x,t_1,s_1)$, $f(x,t_2,s_2)$ such that $t_1 + s_1 = t_2 + s_2$ are conjugate.

The proof is simple. As the Cartesian graphs are mutually moved in the direction of line y = x, the given functional graphs are evidently isomorphic. The given functions are, therefore, conjugate.

**Consequence 1.** According to Theorem 1, it is possible in other considerations to represent all parabolas $f(x,t,s)$ for which $t+s$ equals the given constant $c \in R$ with parabola $y = x^2 + c$.

**Theorem 2.** Let $(R,q)$ be a unar on set *R* of real numbers. We define a binary relation $\leq$ on $R$ as follows: for $x, y \in R$ we make $x \leq y$, if there is a $n \in N_0$ with characteristic $q^n(x) = y$. Then $\leq$ is quasi-ordered (i.e. reflexive and transitive relation) on set *R*. This relation is antisymmetric, i.e. it is an order only if orbits of the given unar contain one-element „cycles“, i.e. loops.
Proof: The reflexivity and transitivity of the defined relation is obvious. If there is in a unar $(R,q)$ a cycle of length $k > 0$, in *R* there are such elements $x_0, x_1, ..., x_{k-1}$ that $x_0 \leq x_1 \leq ... \leq x_{k-1} \leq x_0$, so that relation $\leq$ is not antisymmetric. Antisymmetry is evidently not broken by the loops. This makes the statement proven. Now, we can describe one decomposition block of two mutually conjugate functions of the studied system $\tau$.

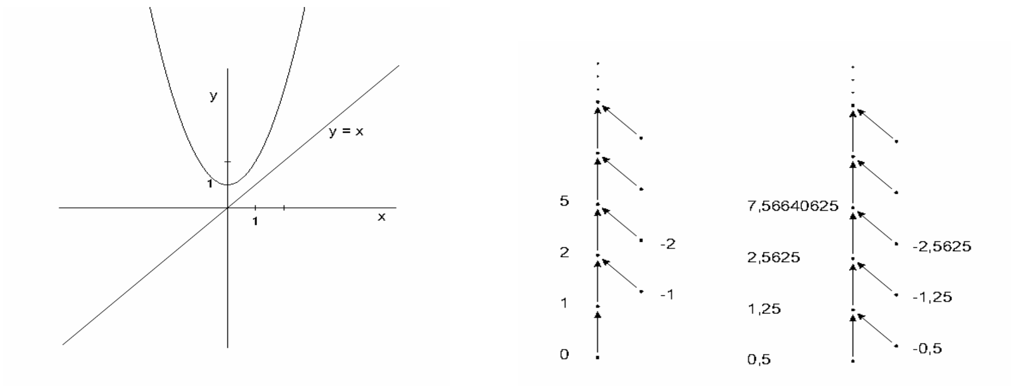


**Figure 3** Cartesian graph and orbits of function $y = x^2 + 1$

**Theorem 3.** All functions $f(x,t,s)$, where $s+t > \frac{1}{4}$, are mutually conjugate and form one decomposition block.

Proof: Orbits of the unars of system $\tau$ generally contain cycles. In the case of function $f(x,t,s)$, $s+t = \frac{1}{4}$ the corresponding unar contains no cycle, so the orbits are ordered by the given relation. The node graph of any function $y = x^2 + c$, $c > \frac{1}{4}$ is isomorphic with the graph in Figure 3. At the same time, it is true that in the node graph of every function $f(x,t,s)$, where $s+t \leq \frac{1}{4}$ there is a least one point (intersection of a quadratic function with identity), i.e. point, in which there is a loop. The node graph of such function may not be isomorphic with the graphs of functions

$f(x,t,s)$, $s+t>\frac{1}{4}$, so according to Theorem 1, none of the given functions can be conjugate with a function not intersecting the identity. This proves the given theorem.

Other blocks of the studied decomposition are not easy to describe. It is not true that every two functions of system $\tau$ intersecting the identity, for example, in two points are conjugate. The decomposition blocks are, in this case, much more complicated. We should consider how powerful functional graphs are when exploring conjugate functions. If we wanted to determine, according to the given definition, whether two functions $f_1 = f(x,t_1,s_1)$ and $f_2 = f(x,t_2,s_2)$ of system $\tau$ are conjugate, we would have to prove the existence of bijection $h: R \to R$ such that it is true that $h \circ f_1 = f_2 \circ h$. The exploration of this problem leads to the exploration of non-trivial functional equations. An even simpler problem than the solution of the last equation, i.e. the question of the interchange of bijection $h: R \to R$ with the quadratic equation $\varphi = (x+t)^2 + s$, i.e. the validity of relation $f(\varphi(x)) = \varphi(f(x))$ leads to the functional equation $f((x+t)^2 + s) = (f(x)+t)^2 + s$, $[f(x)]^2 + 2tf(x) + t^2 - f((x+t)^2 + s) + s = 0$. A non-trivial question of the solution of such a functional equation is, however, for quadratic equations of system $\tau$, for which $s+t>\frac{1}{4}$, is positively answered according to Theorem 3 (see for instance [1], [4] and [5]). To demonstrate this, we illustrate two different types of orbits of the quadratic function $y = x^2 + c$. We analyse particular situations regarding the classic school point of view (Cartesian graphs) and regarding mono-unary algebras' point of view. We show that without the help of sophisticated computer software we cannot analyse completely node graphs of a purely quadratic function with students. Doing this, we can emphasise the usefulness of various views on the given problem, which may appear simple, when approached from only one point o view. According to the previous conclusions, an important role is played in this description by constant *c*.

*Sample 1*: constant $c = \frac{1}{4}$

Consider function $y = x^2 + \frac{1}{4}$. The functional graph of the corresponding unar is created by one cyclic orbit in the touch point of the parabola with the identity (Figure 4), i.e. in point $\frac{1}{2}$ (the first obit from the left). As the function is even, also $-\frac{1}{2}$ maps to this value.

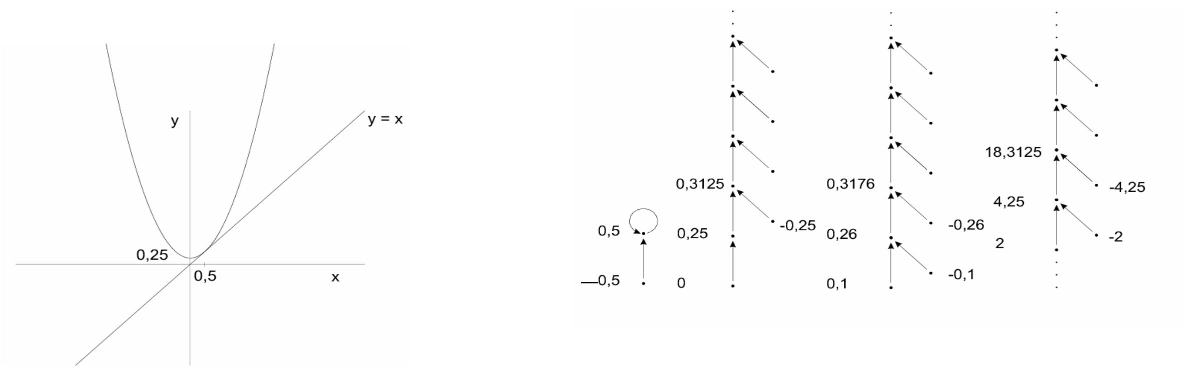


**Figure 4** Cartesian graph and orbits of function for $c = \frac{1}{4}$

Then, there is the only orbit "growing" from 0 (in Figure 4 the second from the left), which is acyclic and infinite. Nondenumerably many orbits grow from values of interval $\left(0,\frac{1}{4}\right)$ (the third orbit from the left). The last type of orbits (in Figure 4 on the right) is infinite, acyclic orbits. There are nondenumerably many of them and they grow from values of interval $\left(\frac{1}{2},0\right)$.

*Sample 2*: constant $0 < c < \frac{1}{4}$

A graph of such a quadratic function has two intersections with the identity, so there are two loops in the functional graph. The vertex of the parabola is not in 0, so for sure one orbit will be of the same type as in Sample 1.

If constant $c = 0.24$, we can easily calculate the intersections of function $y = x^2 + 0.24$ with function $y = x$ using the equation $x^2 - x + 0.24 = 0$, and we get roots $x_1 = 0.4$ a $x_2 = 0.6$. In these values, 0.4 and 0.6, there are loops and edges from nodes $-$ 0.4 and $-$ 0.6 direct to these nodes (Figure 5).

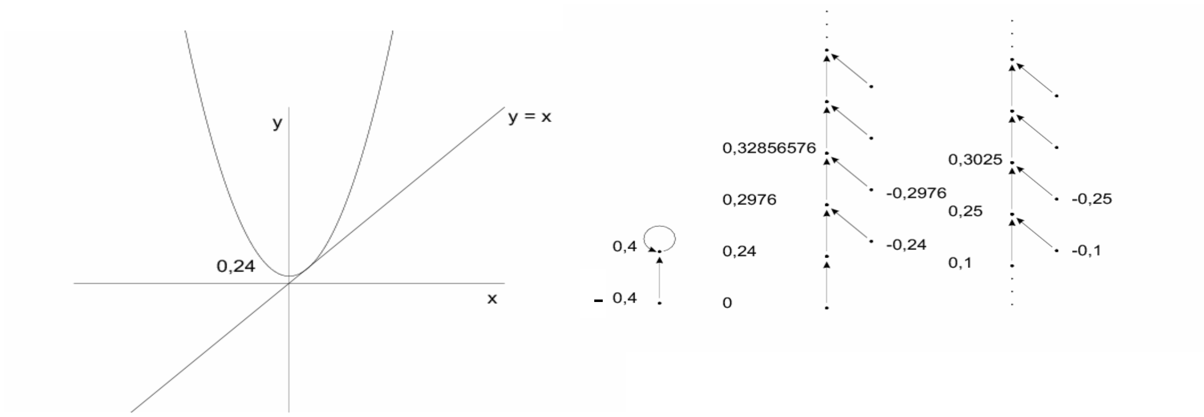


**Figure 5** Cartesian graph and orbits of function for $c$ = 0.24

The quadratic function is represented by two-cyclic orbits of this type (see Figure 5 on the left). The problem is, as we have already mentioned before, point 0. Another orbit type is the only orbit which grows just from 0 (in Figure 5 in the middle). The last orbit type of the node graph of this unar is infinite, acyclic orbit (in Figure 5 on the right). There are an infinite number of such orbits and they grow from interval $(0,c)$.

*Sample 3*: constant $c = 0$

In an analogy with the previous considerations, it is obvious that intersections with axis *x*, i.e. the values $x = 0$ and $x = 1$ are displayed as loops. Zero (in Figure 6 on the left) represents only one orbit – a loop. The second orbit type of the node graph of this unar (in Figure 6 in the middle) grows from value $-1$, which is, from the essence of the even function, the symmetric value to the other intersection with the identity $x = 1$. Nondenumerably many acyclic orbits (in Figure 6 on the right) grow from intervals $(0,1)$ and $(1,\infty)$.

*Sample 4*: constant $c < 0$
If constant $c$ is negative, the situation changes significantly. We can see that points in which may be cycles or which behave “suspiciously” are much numerous than in the previous cases: The vertex of the parabola; two intersections with axis *x;* two intersections with the identity. Up to this time, only loops could appear as cycles. Let’s pose a question if there are cycles of the length of two and more. To be able to answer it, we have to solve the following system of equations:

$$a^2 - t = b$$
$$b^2 - t = a$$

where $t$ presents a quadratic equation parameter.

These equations can be simply solved by subtracting them to obtain $a^2 - b^2 = b - a$, assuming that $b - a \neq 0$ and we obtain $a + b = -1$. Simple substitutions provide the results immediately. The other solutions are when $a = b$, and again, substitutions can be used. Secondary school students should be able to solve this system of equations. However, we can use suitable software as a preparation for more complicated calculations in the next more difficult problem where the calculation without software is not possible.

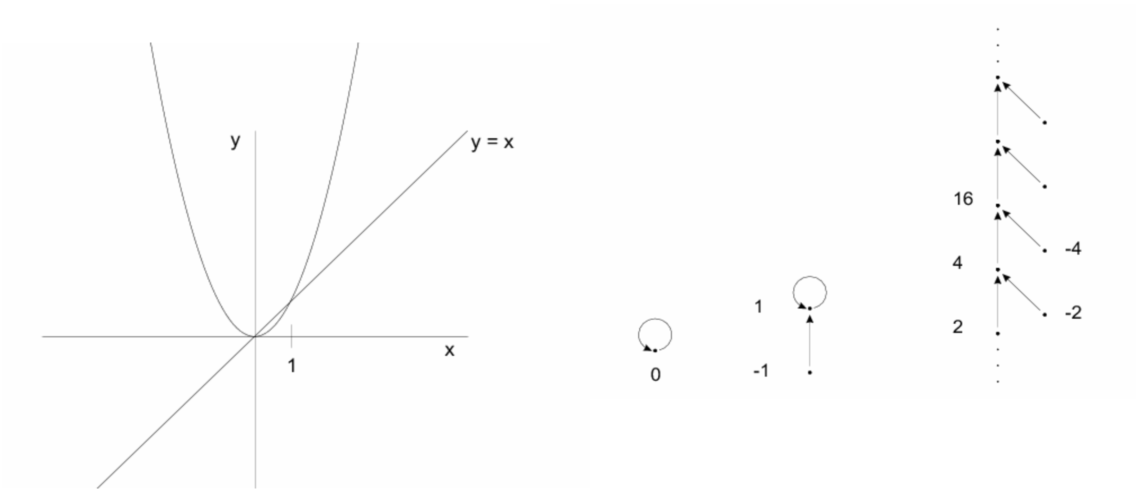


**Figure 6** Cartesian graph and orbits of function for $c = 0$

We solve it using mathematical software CoCoA 4.3, which was chosen because it can eliminate variables in systems of equations. In this case, it can transform the system of equations into a product of polynomials. In Figure 7, we can see the result. Using command Elim, we get a polynomial denoted, in Figure 7, as Ideal(-a^4 + 2a^2t - t^2 + a + t). If we use command Factor, program makes a decomposition into the product of two polynomials $a^2 + a - t + 1 = 0$ and $a^2 - a - t = 0$.

The roots of the first polynomial are $a_{1,2} = \dfrac{-1 \pm \sqrt{4t - 3}}{2}$ and they determine the values of two-cycles. Next, the solution of this equation is the second polynomial $a^2 - a - t = 0$, whose roots have the form $a_{1,2} = \dfrac{-1 \pm \sqrt{1 + 4t}}{2}$ and determine values for one-cyclic orbits. When we ask for which values of constant $c$ we can expect cycles of the length two, we can see, from the equation for the first roots of the first polynomial, that if $4\alpha - 3 > 0$, i.e. for $\alpha > \frac{3}{4}$, in our notation for the quadratic equation parameter $c < \frac{3}{4}$. This is the value discovered by Mitchell Feigenbaum in his research on the behaviour of quadratic function iterations.

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--      Version       : 4.3                          --
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-- The current ring is R ::= Q[x,y,z];
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Use R::=Q[abt];
I:=Ideal(a^2-t-b,b^2-t-a);
Elim(b,I);
Factor(-a^4 + 2a^2t - t^2 + a + t);
Ideal(-a^4 + 2a^2t - t^2 + a + t)
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[[a^2 + a - t + 1, 1], [a^2 - a - t, 1], [-1, 1]]
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**Figure 7** Calculation of two-cycles using computer

The presented considerations may make students think about the question of when there are also cycles of the length of four and more. We will try to answer this question.

Let's try to generalize our consideration and search for a solution of the following system of equations

$$
\begin{aligned}
a^2 - \alpha &= b \\
b^2 - \alpha &= c \\
c^2 - \alpha &= d \\
d^2 - \alpha &= a
\end{aligned}
$$

If we obtain a "reasonable" solution, we will have a relationship for cycles of length 4.

To solve this problem, we use a mathematical program. We tried to make calculations in program Maple9, Derive6 and CoCoA 4.3. The third one seemed to be better for factorisation of polynomials. For other adjustments, we used the program Derive6. After elimination of the variables, we get a product of three polynomials. The first two are the same as in the case of the previous set of equations, $a^2 + a - t + 1 = 0$ and $a^2 - a - t = 0$, and determine the values for one-cycles and two-cycles. The third polynomial is

$$a^{12} - 6a^{10}t + 15a^8t^2 + a^9 - 3a^8t - 20a^6t^3 - 4a^7t + 12a^6t^2 + 15a^4t^4 + 6a^5t^2 - 18a^4t^3 - 6a^2t^5 + a^6 - 2a^5t + 3a^4 4t^2 - 4a^3t^3 + 12a^2t^4 + t^6 - 4a^4t + 4a^3t^2 - 6a^2t^3 + at^4 - 3t^5 + 5a^2t^2 - 2at^3 + 3t^4 + a^3 - a^2t + at^2 - 3t^3 - 2at + 2t^2 + 1 = 0$$

We tried to find roots of this polynomial with students using Derive6 and then also CoCoA 4.3. Taking into account the obtained results (Table 1), it is apparent that we cannot derive easily the relationship for cycles of length 4. Here, we have an example of a reasonable utilisation of the computer for creation concepts in mathematics and the demonstration of its beauty. We react to the current teaching situation on the secondary school level in the Czech Republic when mathematics is, very often, presented by teachers as a boring theory full of formulae which should be memorized and when students are not led to their own discoveries. The ongoing curricular reform has ambitions to change it and there are tendencies to change teachers' view on the teaching and learning processes in mathematics lessons. Our paper is considered to be an inspiration for such a change

We are fully aware that a massive 12 degree equation is not "beautiful" at the first sight, but we present it as we think, based on our experience, that secondary schools students have never met this result before. The results are for them astonishing (considering their reaction in mathematics lessons) and it is important to make them interpret the obtained result in the table. When working with students on demonstrated problems, we found out, that with a teacher as a moderator and with the use of a computer, the students wanted to go beyond their previous possibilities. The computer helped them, to a certain extent, open the way to further exploration.

Going back to the description of decomposition of the node graphs of the corresponding unars and taking into account all mentioned considerations, we can see that the situation is not so clear and is more complicated and that the description of particular node graphs is not unambiguous. To illustrate this, we will choose various values for constant *c* and describe the corresponding graphs. Let's choose the parameter of the quadratic equation $y = x^2 + c$, value $c = -1$. The first root equation tells us that a two-cycle appears for values $\dfrac{-1 \pm \sqrt{4 \cdot 1 - 3}}{2}$, i.e. for 0 and −1. The node graph of this function is one cycle of length 2 (Figure 8) growing from the symmetric value −1 towards value 1 because of the even nature of the function and nondenumerably many acyclic orbits. Other two orbits typical of the function in this form are one-cycle infinite orbits. The values of their nodes can be determined form the equation for the second root.

Similarly, if we change parameter *c*, we obtain, apart from infinite acyclic orbits, one-cycles and two-cycles. So for instance, for parameter $c = -3$, there is a cycle of the length 2 (see Figure 8). The nodes of this two-cycle are the values 1 and −2, which is a two-cycle orbit similar to the case when $c = -1$, but there is an edge leading from 2 to 1 from −1 to −2. The nodes of one-cycle orbit are calculated from the equation for the second root, these values are $\dfrac{1+\sqrt{13}}{2}$ and $\dfrac{1-\sqrt{13}}{2}$. However, also the symmetric points $\dfrac{-1-\sqrt{13}}{2}$ and $\dfrac{-1+\sqrt{13}}{2}$ map to these values.

These orbits are two. If we choose $c = -2$, there appears, apart from infinite acyclic orbits, a one-cycle orbit growing from 1 (in Figure 8 in the middle) of the same type as in the case of the function $y = x^2 - 1$ with the difference that the values of its nodes are whole numbers and can be more easily estimated. Moreover, we will find a "new" one-cycle orbit growing from nondenumerably many iterations ..., $-\sqrt{2}$, $\sqrt{2}$, which all map gradually on 0, on $-2$, this on 2 and there is a cycle. The function is described by an two-cycle orbit and we can determine its nodes calculating the equation for the first root, which is $\dfrac{-1\pm\sqrt{5}}{2}$.

**Table 1** Determination of polynomials using program CoCo A 4.3

```
Use R::=Q[abcdefght];
I:=Ideal(a^2-t-b,b^2-t-c,c^2-t-d,d^2-t-e,e^2-t-f,f^2-t-g,g^2-t-h,h^2-t-a);
Elim(b..h,I);
Factor(a^256 - 128a^254t + 8128a^252t^2 - 64a^252t - 341376a^250t^3 +
8064a^250t^2 + 10668000a^248t^4 - 504000a^248t^3 - 264566400a^246t^5 +
2016a^248t^2 + 20832000a^246t^4+.......................171 pages of calculations.............................-
14680t^11 - 60a^7t^3 + 86a^6t^4 - 170a^5t^5 + 752a^4t^6 - 1008a^3t^7 +
2722a^2t^8 - 2698at^9 + 5368t^10 + 6a^7t^2 - 8a^6t^3 + 90a^5t^4 - 196a^4t^5 +
274a^3t^6 - 856a^2t^7 + 962at^8 - 1944t^9 - 2a^7t + 2a^6t^2 - 26a^5t^3 +
34a^4t^4 - 102a^3t^5 + 262a^2t^6 - 334at^7 + 698t^8 + 8a^5t^2 - 8a^4t^3 +
44a^3t^4 -68a^2t^5 + 120at^6 - 248t^7 - 16a^3t^3 + 20a^2t^4 - 48at^5 + 84t^6 +
4a^3t^2 - 4a^2t^3 + 20at^4 - 28t^5 - 8at^3 + 8t^4 + 1, 1], [a^12 - 6a^10t +
15a^8t^2 + a^9 - 3a^8t - 20a^6t^3 - 4a^7t + 12a^6t^2 + 15a^4t^4 + 6a^5t^2 -
18a^4t^3 - 6a^2t^5 + a^6 - 2a^5t + 3a^4t^2 - 4a^3t^3 + 12a^2t^4 + t^6 - 4a^4t +
4a^3t^2 - 6a^2t^3 + at^4 - 3t^5 + 5a^2t^2 - 2at^3 + 3t^4 + a^3 - a^2t + at^2 -
3t^3 - 2at + 2t^2 + 1, 1], [a^2 + a - t + 1, 1]]
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Making a complete and exhausting appraisal of the whole process raises the question of whether there can also appear four-cycles or more-cycles? It is sure that such orbits represent the given unars, but we are not able to determine their shape.
There is a possibility to solve this equation numerically for different values of *t* too and plot the results with different kinds of software. We do not present that due to the limited extent of the paper.

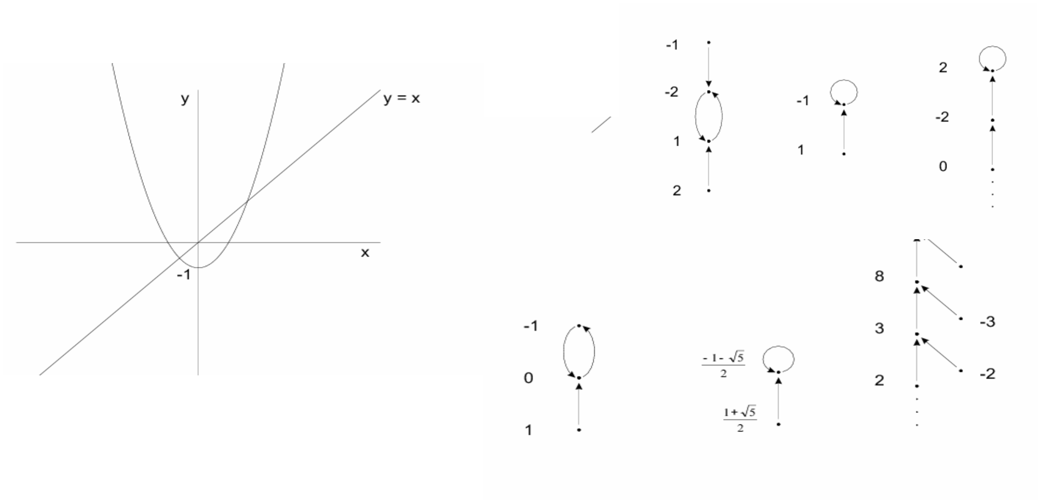


**Figure 8** Cartesian graph and orbits of function for $c=-1$

There is the question of the use of directed graphs of mono-unary algebras in the context of real value functions. We have evidence of there phases in the development of functional thinking – the creation of quantitative links and causal phenomena, the intuitive use of obtained experience, and

the phase of systematic work with functions. In our opinion, supported with our experience, all there phases are not seized adequately. Results of international comparisons (TIMSS and PISA) show that Czech students struggle with understanding of the concept of function. The reason may be the mentioned formal way of teaching. The presented view on functions opens possibilities how to enable students to understand it more deeply, it hides a new view on Cartesian and nod graphs of functions and their mutual comparisons Moreover; such algebra is said [2] to correspond to the topology.
All the previously mentioned previous considerations show that the presented problem is not trivial. For this reason, we have demanded its exhausting description, but only tried to show to readers the complexity of the whole problem. We tried to demonstrate in the example of quadratic functions the beauty of mathematics which offers various views on the same problem. From one point of view, the problem is trivial within the scope of the school mathematics. From another point of view, the problem is difficult demanding a deeper insight into the problem. It enables students to see another view on mathematics and gives space for individual mathematical experimenting using a computer. The presented problems are suitable for special facultative seminars and for talented students who participate in the mathematics Olympics.

## 4. Conclusion

Recently, a powerful tool has appeared in education and this tool is the computer which enables mathematics teachers to show and present their students previously unforeseen possibilities of mathematical experimenting, linking various educational subjects, mathematical "vision" of the real world etc. Iterations can help to model many phenomena occurring in the real world and they have a close connection with the theory of chaos and fractals and can be applied in technical sphere, economics or IT technologies.
A didactical utilization of the above mentioned approaches consists, among other, in the following directions: a discrete description of various functions in sets of real and complex numbers, a relationships between discreet and continuous mathematical structures, stimulation of special-interest activities of students and their motivation by simple applications from the real life, the use of computer, creation of tasks for gifted students, etc.

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