Mathematical Modelling as a Learning Experience in the Classroom

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Abstract: *Mathematical modelling has been gaining attention and becoming a part of classroom practice in many countries. In Singapore, despite recognizing its importance and relevance, curriculum planners and teachers face various challenges in including and incorporating mathematical modelling in their teaching curriculum. Nonetheless, the recommended practice is to expose students to learning experiences in mathematical modelling whenever and wherever possible. In this paper, a framework which serves a practical guide for teachers in planning instruction in mathematical modelling will be introduced. Examples illustrating the application of this framework by teachers in crafting classroom learning experience in mathematical modelling will be presented. In addition, a learning experience implemented for a group of teachers in an in-service course will also be discussed. It is no coincidence that technology had featured quite prominently in these examples as mathematical modelling in practice would often involve the use of some specific technological tools.*
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1. Introduction

In recent years, mathematical modelling has become increasingly important in the school mathematics curricula of many countries. Interest in this area has grown significantly in the last two decades, stimulated by the numerous and varied studies and research work done on the learning and teaching of mathematical modelling in the classroom. Many of these studies have been presented at international meetings such as ICTMA Conference, ICMI Study Conference, and so on and received much attention from both researchers and classroom practitioners alike (Blum, 2002).

In Singapore, mathematical modelling has a rather short history and has only begun to feature in the school mathematics curriculum framework around 2007. In fact, it was probably Ang (2001) who first proposed the idea of introducing mathematical modelling in Singapore schools. Since mathematical problem solving is a central theme in Singapore's school mathematics curriculum, it would not be unreasonable to expect mathematical modelling and applications to play a key role in the delivery of the local curriculum in schools.

However, many teachers in Singapore find it difficult to teach mathematical modelling or include modelling tasks in their mathematics lessons. A typical mathematics teacher faces a number challenges, such as a lack of ready and relevant resources, a set of good exemplars of modelling problems, and the resistance from students to engage in activities not directly related to examinations and assessment (Ang, 2010a).

Among many other issues, one of the challenges that teachers in Singapore face when trying to implement mathematical modelling tasks or carrying out mathematical modelling lessons in the classroom is the lack of experience in mathematical modelling itself, and in teaching mathematical modelling.

2. A Framework for Instruction in Mathematical Modelling

Perhaps one good point to start is to provide a strong and relevant framework that can assist the teacher in planning and preparing lessons, activities or tasks in mathematical modelling. Such frameworks do exist elsewhere, such as the one proposed by Stillman et al (2007) for Australian schools. Although Stillman's framework has been reported to be successful, it applies to situations where students may have a different mathematical background from those in Singapore. In addition, a contextualized framework can help provide a shared understanding of key pedagogical ideas in the teaching mathematical modelling in the local classroom.

Three Levels of Learning Experiences

Classroom learning experiences in mathematical modelling may be classified into three levels, each of which may roughly correspond to a certain level of cognitive demand or expectation.

Level 1
At **Level 1**, the most basic level, the lesson will focus on learning of skills directly or indirectly related to mathematical modelling. Sometimes, these could be simply mathematical skills or even ICT skills that can be used or applied in a modelling context. For instance, a real life problem or situation may be examined or represented by some mathematical function or graph. Alternatively, the problem may involve some curve fitting using an IT tool, such as a graphing calculator or some other computing applications. Then, the activity may be considered a Level 1 modelling learning experience.

Level 2
At **Level 2**, the learning activity should be aimed at developing modelling competencies. Here, we differentiate modelling competencies from modelling skills. By competency, we mean the ability to apply specific modelling knowledge such as making assumptions to simplify a problem, identifying variables in a problem, validating a model and so on.

A Level 2 modelling activity may also be one that examines and applies some existing or standard models to a real life problem or situation. The learner demonstrates the ability to recognise the behaviour of a model, and apply it to the problem.

Level 3
At **Level 3**, the highest level, the activity should require students to tackle a fairly substantial real life problem. The task usually would be quite complex and students will be expected to apply various modelling skills and competencies. At this level, very often, group work may be necessary, and students may even be required to make a presentation or write a report.

Level 3 modelling experiences are what most mathematics educators would have commonly called "modelling tasks". Sometimes, to complete a Level 3 activity may require a few hours of instructional time, or even require students to work on it over a few days.

It is imperative that before the teacher plans a modelling learning experience, he has a clear idea of the level at which to pitch the lesson because this would help the teacher set a clear goal and develop a sound structure for the lesson.

With the three levels of learning experiences in mathematical modelling described above, a simple framework that teachers can use to plan or design a lesson on mathematical modelling is now proposed (Ang, to appear). The structure of this framework is in the form of a set of five questions listed in Table 1.

Table 1: Framework for Planning/Designing Mathematical Modelling Learning Experience

Framework Component	Explanation
1. WHICH Level of Learning Experience?	Decide which level (Level 1, 2 or 3) of mathematical modelling learning experience that we wish to focus on.
2. WHAT is the Skill/Competency?	List all the specific skills and competencies (mathematical or modelling) that we target in this learning experience; State the problem to be solved, if applicable.
3. WHERE is the Mathematics?	Write down the mathematical concepts or formulae or equations that will be needed in this learning experience.
4. HOW to Solve the problem/model?	Prepare and provide plausible solutions to the problem identified in this learning experience.
5. WHY is this experience a Success?	List factors or outcomes that can explain why this experience is considered successful and look out for them during the activity.

3. Examples of applying the framework

The framework was applied by practicing teachers who attended a professional development course on the teaching of mathematical modelling conducted in 2011 by the author. In general, the response to the framework has been positive although not all lessons designed based on the framework have been reported to be successful.

In this section, three examples are presented to illustrate how the framework may be applied. We also note that in all the lessons planned, invariably, some form of technology has been used. This is an observation that suggests that very often, mathematical modelling lessons and activities would usually involve (and sometimes require) the use of technology of some form.

Example 1: Mountain Climbing

In this example, a set of data obtained from the public domain on atmospheric pressure and altitude is presented to students (see Table 2). The data is related to a physical phenomenon often experienced by mountain climbers who may feel the air getting thinner as they climb higher.

Table 2: Atmospheric Pressures at different altitudes above sea level.

Altitude (km)	0	1	2	3	4	5	6	7	8	9	10	11
Pressure (mb)	1013	899	795	701	616	540	472	411	356	307	264	226

The question that one may is whether there is any relationship between altitude and air pressure. Of course, we know there is, but this serves as a good exercise for students to find or *model* this relationship by examining the data given. By doing so, students may apply what they know from functions, and relationships between variables, and also practise using some form of technology to estimate values of parameters that may appear in the model. In this case, the technique found in Beare (1996) may be useful.

This learning experience could be a Level 1 activity, and in applying the framework, the following table is obtained.

Framework Component	**"Mountain Climbing"**
1. WHICH Level of Learning Experience?	Level 1 – about 40 minutes lesson
2. WHAT is the Skill/Competency?	Knowledge and understanding of the exponential function, and ability to use the Solver Tool in Excel to find parameter value.
3. WHERE is the Mathematics?	Given a set of data, find a function of the form $f(x) = Ae^{-kx}$ that fits the data.
4. HOW to Solve the problem/model?	Use of *Excel*'s Solver Tool to find the best value of *k* that minimizes error between data and model.
5. WHY is this experience a Success?	Students learn a specific skill, and are able to apply it to a problem with real data.

Example 2: Mouldy Bread

In this example, we attempt to construct a model for the growth of bread mould. The approach is essential one of empirical modelling, and we estimate parameters in the model from collected data. The whole process begins with growing mould, collecting data on the growth, examining and studying the data to construct a reasonable model, and refining the model if necessary. Mould is a form of fungi and can start to grow on bread after a few days of exposing the bread to a humid environment. Data may be collected by taking pictures of a piece of bread each day as mould grows on it.

Using a square grid placed on the bread, the total area of the bread may be estimated. Then, from the pictures taken, the area of the mould-covered part of the bread may then be estimated (see Figure 1). As an example, the circled portion in Figure 1 is enlarged in Figure 2. For this portion, the area covered by mould is estimated to be around 2 cm^2. Using the collected data, a model may be constructed.

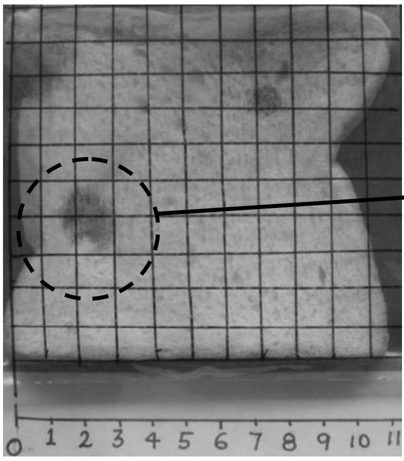
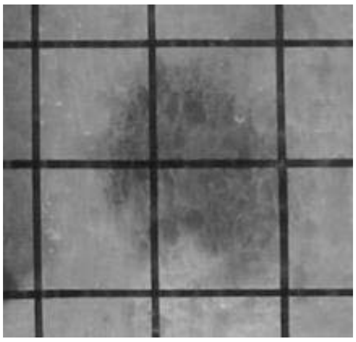


Figure 1. Estimating the amount of mould



Area of mould ≈ 2cm^2

Figure 2. Example of an estimate

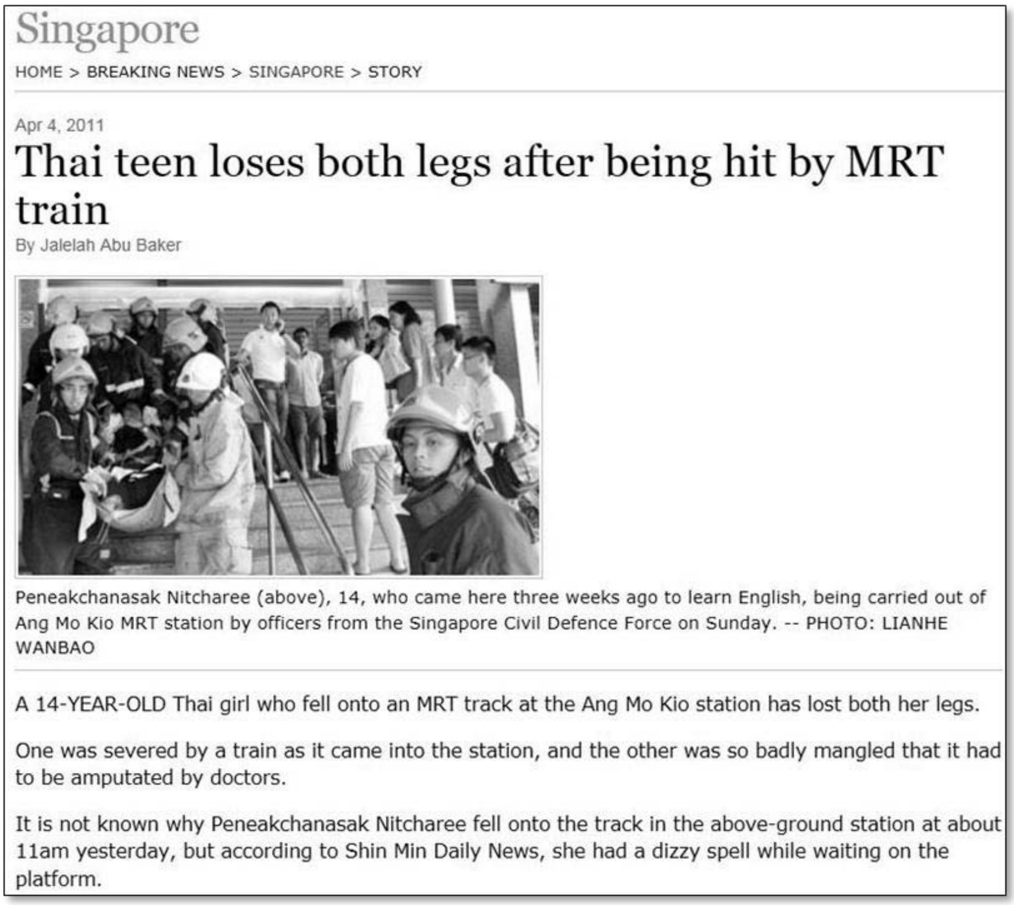
This task may be a Level 2 learning experience based on the framework. It involves collecting data and making assumptions about a model. Students may or may not have knowledge of the logistic equation, but with suitable and timely scaffolding, they can always learn and apply. If students are not ready for differential equations, they may use the solution as a model and proceed from there. If first order differential equations have been taught, then this task presents itself as a good learning experience to see how ODEs may be used as models in real life. The table below shows how this task may be framed as a Level 2 activity.

Framework Component	**“Mouldy Bread”**
1. WHICH Level of Learning Experience?	Level 2 – two lessons, separated by about 10 days of data collection.
2. WHAT is the Skill/Competency?	Knowledge of the logistic growth model, and ability to use the Solver Tool in Excel to find parameter value.
3. WHERE is the Mathematics?	a) Using curve fitting techniques to fit a set of data to a function of the form, $$x(t) = \frac{N}{1+Ae^{-kt}}.$$ b) Differential equation representing the logistic growth model of the form, $$\frac{dx}{dt} = ax\,(1-bx).$$
4. HOW to Solve the problem/model?	Use of *Excel*’s Solver Tool to find the best value of *k* that minimizes error between data and model, or values of *a* and *b* in the differential equation.
5. WHY is this experience a Success?	Students examine and apply a model in a specific situation, and are able to collect data to carry out empirical modelling.

Example 3: Accident at the MRT

In the April of 2011, one 14-year-old student from Thailand was hit by a Mass Rapid Transit (MRT) train in Singapore. She lost both her legs. According to one news report (Figure 3), the girl had fallen onto the tracks because "she had a dizzy spell while waiting on the platform". This incident took place at around 11am on a Monday, and the station where it happened was one where many people were expected to be waiting for the train at that time. One then wonders if it is possible for the girl to be walking around randomly (as a result of a dizzy spell) without being noticed. One could also examine how many steps such a random walker would take on the average before she falls off the platform?

A simulation model may be set up to examine these questions. This is a problem that involves a substantial amount of complexity, requiring a number of mathematical and modelling skills, as well as technological knowledge. This problem fits the bill for a Level 3 modelling task.



Singapore

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Apr 4, 2011

Thai teen loses both legs after being hit by MRT train

By Jalelah Abu Baker

Peneakchanasak Nitcharee (above), 14, who came here three weeks ago to learn English, being carried out of Ang Mo Kio MRT station by officers from the Singapore Civil Defence Force on Sunday. -- PHOTO: LIANHE WANBAO

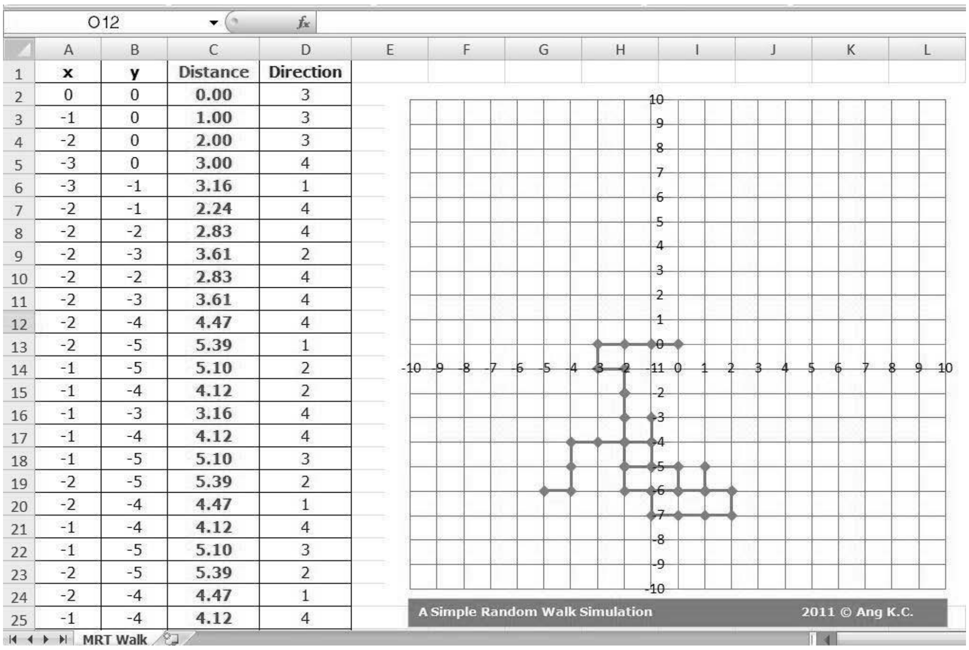
A 14-YEAR-OLD Thai girl who fell onto an MRT track at the Ang Mo Kio station has lost both her legs.

One was severed by a train as it came into the station, and the other was so badly mangled that it had to be amputated by doctors.

It is not known why Peneakchanasak Nitcharee fell onto the track in the above-ground station at about 11am yesterday, but according to Shin Min Daily News, she had a dizzy spell while waiting on the platform.

Figure 3. News report on an accident at an MRT station (The Straits Times, 2011)

The simulation may be constructed and implemented using Excel. For simplicity, we assume that the random walker can move in only four directions, suitably represented by integers 1, 2, 3 and 4. We use Excel's "**`randbetween()`**" function to generate random numbers from a uniform distribution to simulate the random walk on a plane. We could construct the path taken (Figure 4) and find some average value of the number of steps the walker needs before falling off the platform.



	A	B	C	D
1	x	y	Distance	Direction
2	0	0	0.00	3
3	-1	0	1.00	3
4	-2	0	2.00	3
5	-3	0	3.00	4
6	-3	-1	3.16	1
7	-2	-1	2.24	4
8	-2	-2	2.83	4
9	-2	-3	3.61	2
10	-2	-2	2.83	4
11	-2	-3	3.61	4
12	-2	-4	4.47	4
13	-2	-5	5.39	1
14	-1	-5	5.10	2
15	-1	-4	4.12	2
16	-1	-3	3.16	4
17	-1	-4	4.12	4
18	-1	-5	5.10	3
19	-2	-5	5.39	2
20	-2	-4	4.47	1
21	-1	-4	4.12	4
22	-1	-5	5.10	3
23	-2	-5	5.39	2
24	-2	-4	4.47	1
25	-1	-4	4.12	4

Figure 4. A typical run of the simulation on random walk implemented on MS Excel

This problem may be framed as a Level 3 modelling task for students who have been introduced to probability and the idea of experimenting with chance. The framework helps us check the various components as shown below.

Framework Component	**"MRT Accident"**
1. WHICH Level of Learning Experience?	Level 3 – over at least two sessions, possibly with students working in small groups
2. WHAT is the Skill/Competency?	Listing variables or factors in a model, Making assumptions about real physical situation and simplifying problem, Designing and carrying out a simulation
3. WHERE is the Mathematics?	Probability, random numbers and the use of a random number generator, Coordinate geometry
4. HOW to Solve the problem/model?	• Decide on the dimensions of the platform • Let the walker begin random walk from origin • Generate random numbers 1, 2, 3 and 4 to represent a step in four different directions • Compute distance from origin at each step • Find average number of steps taken to step off
5. WHY is this experience a Success?	As a small-group modelling activity, this task allows students to work as a group to discuss ideas, list suitable and reasonable assumptions, consolidate conceptual understanding of experimental and theoretical probability (and random numbers), practise generating random numbers, design a plausible simulation model

4. Conclusion

There are several examples of mathematical modelling tasks and activities that relevant and can possibly be carried out in the Singapore classrooms. Some of these applying the logistic equation to model the 2003 outbreak of the Severe Acute Respiratory Syndrome, or SARS, in Singapore (Ang, 2004), and studying how equations can be used to construct a model for traffic flow (Ang and Neo, 2005). Using technology to support the teaching of mathematical modelling has also be discussed (Ang, 2006; Ang, 2007; Ang, 2010b), and resources for teachers in the form of booklets containing collections of modelling and application activities have been made available (Ang, 2009; Dindyal 2009).

Somehow, despite the availability of resources, some teachers still find it a challenge to include mathematical modelling as part of their classroom learning activities. Moreover, as noted by Smith (1996), even if these were carried out, they may be done consciously and systematically. Mathematical modelling is considered cognitively demanding, both for the student and the teacher (Blum and Borromeo Ferri, 2009). It is therefore important that one approaches this aspect of mathematics teaching with much care and planning. The teacher may have to learn alongside the student, and a first step would be to "become conversant with the tools of the trade" (Warwick, 2007, pp. 34).

The framework discussed, which advocates a systematic and organized way of planning learning experiences involving mathematical modelling, provides a set of guidelines for the teacher. It is hoped that with regular use, asking the questions in the framework when planning mathematical modelling activities will become second nature to the teacher.

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