Transversal Mathematical Teaching Focus across other Sciences

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Abstract: *Engineering is very important to solve Society problems. Students, in general, do not feel attraction for engineering careers, as they find them very difficult and bored].*
In order students feel more interested, the current teaching habits and procedures must be changed, particularly for mathematics teaching. New focus related to experiments and models of real - life problems must be introduced.
In order to turn over the present situation, the authors propose a new methodology to teach mathematics as transversal subject with other sciences. In this work the authors show how to experiment with new technical resources and solve a real-life problem through a model presentation and its predictions.
The example is the study of a light falling body (coffee filter) within the atmospheric air. Ultrasonic radar coupled to an interface records both the body displacement and velocity. The limit speed is registered. Two physical models are presented, one, the friction force acting on the body is proportional to velocity, and the other, proportional to the velocity squared. The corresponding algorithm’s solutions are obtained using a Computer Algebra System. The corresponding” models” predictions are displayed in the same graphics and compared with the experimental velocity. The conclusion is that the first assumption is the correct one.

1. Introduction

We know about the world’s needs whose solutions depend upon mainly in solving engineering problems, from the poverty reduction to the climatic change attenuation. But we face the reality which shows us the scarcity of future engineers. It is understood the fact that students, in general, do not feel attraction for engineering careers, as they find them very difficult and bored.
Consequently, there is a need that engineering must be understood by young people in an easier and interesting way. For this purpose, mathematics teaching methodologies must be changed, particularly through experimentation and solving real life problems [1].
The gap between how mathematics is taught and real - life mathematical problem solving, is very outsized today. We need to teach mathematics as a transversal science across other sciences, integrating theory and practice [2], [3], [4].

2. Fundamentals

We introduce an experiment in order to show how a certain physical system behaves. Accordingly, the student may describe this behavior by means of a hypothesis, which is translated into a mathematical algorithm.
The aim is to show how problems from Sciences and Engineering can be understood through Mathematical Models, as a first approach. Then solve the differential equations by using software tools. The model predictions must be validated by experiments.
In order to follow these actions we need to change the teaching environments, strategies, methodologies, contents and practices, as it was done at the National Institute of Education, Singapore [5].

3. Results

We studied the displacement of a body falling down within a fluid due to the gravitational attraction. In this case there exists a drag force pulling up the body against the gravitational force.
In the experimental set up, we choose a coffee filter as a body and the atmospheric air as a fluid.

We use an automatic system to determine the body's velocity and displacement. Ultrasonic radar is coupled to an interface connected with a computer. A corresponding program is used to determine the variation of the abovementioned parameters as function of time, such as a table or graphics. Fig. 1 shows the experimental arrangement. Fig. 2 illustrates the velocity graphics as a function of time showing the velocity limit reached by the body. At he beginning, the velocity varies linearly with time, and later on changes the linearity, approaching the velocity limit

$$v_\ell = 2,97\ \frac{m}{s}$$

First assumption

We developed a mathematical model assuming that the drag force acting on the body is proportional to its velocity. That is to say, the total force **F** acting on the body has two components.

The resultant force is expressed by

$$\mathbf{F}(t) = m \cdot a \cdot \mathbf{e_y} = m \cdot g \cdot \cdot \mathbf{e_y} - k\ m \cdot v \cdot \mathbf{e_y}. \qquad (1)$$

where $\mathbf{e_y}$ is a unitary vector pointing down to the earth's center.

From eq. (1) a differential equation is derived

$$\frac{dv}{(g - kv)} = dt, \qquad (2)$$

with the initial condition for t = 0 is $v = v_0$; $y = y_0 = 0$. From the experiment is $v_0 = 0$, then

$$v = \frac{g}{k}(1 - e^{-k \cdot t}). \qquad (3)$$

The velocity limit is $v_\ell = \frac{g}{k}$. (4)

Replacing the experimental values one gets

$$v(t) = 2,97 \cdot (1 - e^{-3,3 \cdot t}), \qquad (5)$$

Second assumption

We developed other mathematical model assuming that the drag force acting on the body is proportional to the square of its velocity. That is to say, the total force **F** acting on the body has two components.

The resultant force is expressed by

$$\mathbf{F}(t) = m \cdot a \cdot \mathbf{e_y} = m \cdot g \cdot \mathbf{e_y} - k \cdot m \cdot v^2 \cdot \mathbf{e_y}.$$

giving rise to the differential equation

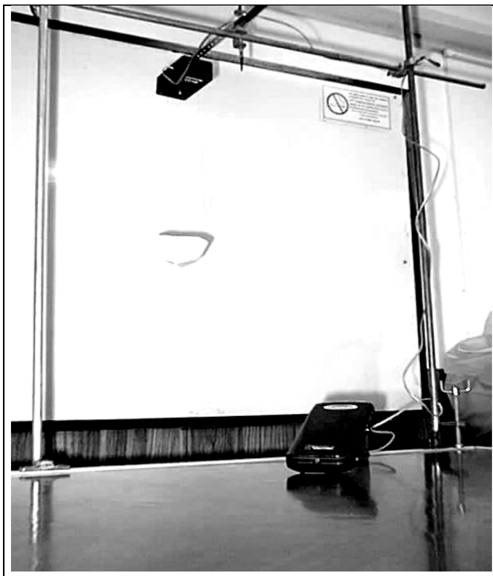

Figure 1. Experimental arrangement for the body's velocity and displacement registration.

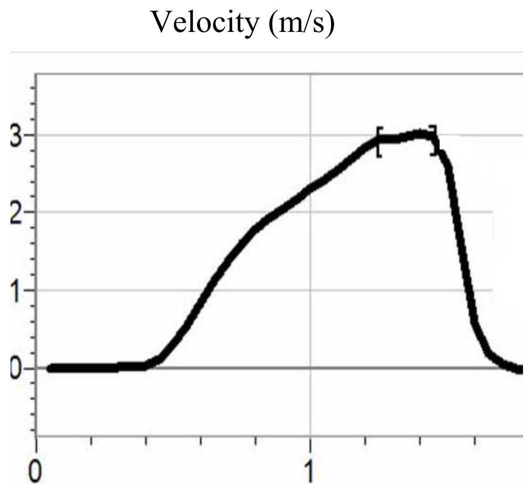

Figure 2. Body's velocity registration as function of time falling.

$$\frac{dv}{(g-k\cdot v^2)}=dt \qquad (6)$$

In order to homogenize the forces acting on the body, the *k* constant is expressed as function of two parameters g y α^2 so that $k=\frac{g}{\alpha^2}$ .The constant α must have a velocity dimension; we choose its value being the velocity limit $\alpha = 2.97\ \frac{m}{s}$. Then the differential equation is set

$$\frac{dv}{\left(1-\frac{v^2}{\alpha^2}\right)}=g\cdot dt\ . \qquad (7)$$

with the initial condition for t = 0 is $v = v_0 = 0$.
The solution of eq. (7) is obtained through the use of the Computer Algebra System Mathematica™ [6] given by

$$\left[\alpha\cdot tanh^{-1}\left(\frac{v}{\alpha}\right)-\alpha\cdot tanh^{-1}\left(\frac{0}{\alpha}\right)=\frac{g}{t}\cdot\right] \qquad (8)$$

$$v(t) = 2{,}97\cdot tanh(3{,}3\cdot t). \qquad (9)$$

Two hypotheses have been assumed, one, the drag force acting on the body is proportional to its velocity, and the other, to the velocity squared. The graphics from Fig. 3 shows the representation of both functions (5) and (9) and the experimental one from Fig. 2, in the time interval where the drag force is acting clearly on the body, that is to say in the time interval [1,1 ; 1,5],

According to these graphics we arrive to the conclusion that the most plausible approximation is that the drag force is proportional to the body's velocity, at least according to the experimental arrangement of Fig. 1.

4. Conclusions

The abovementioned differential equations are treated in many mathematical textbooks without reference to any real life problem and experiment. They just describe how to integrate them and no decision is taken concerning which is the best approach to the real motion within the fluid. Instead, according to our description, what the student is really doing is to find out which is the best approach to describe the falling body's velocity within a fluid. We have demonstrated that the interdisciplinary approach with mathematics across other sciences is the real way to teach mathematics. Is the way to demonstrate the utility of mathematics when we try to solve a real - life problem. If we show to engineering students this new panorama, they may be interested in pursuing engineering studies without being bored [7].

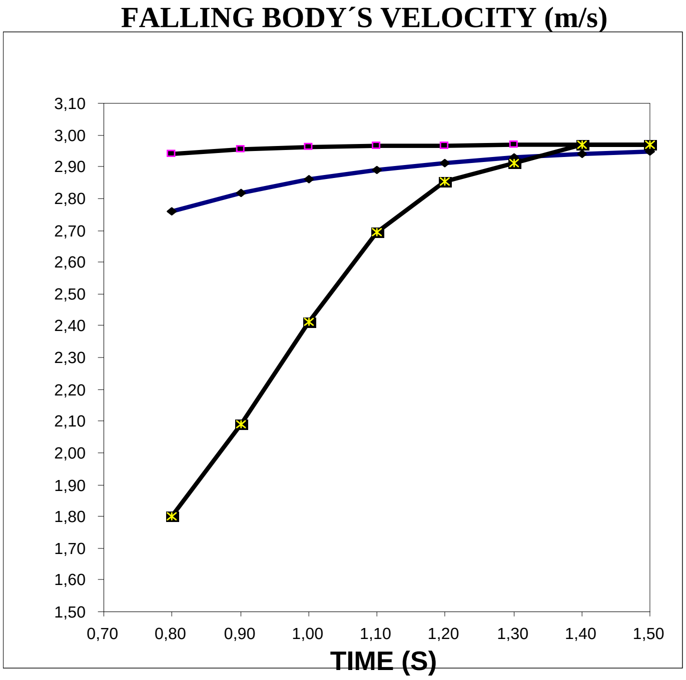

Figure 3. Falling body's velocity graphs according to experimental results and functions (5) and (9). In the time interval between 1.1 and 1.5 s one observes that function (4) approaches more to the experimental function than function (9).

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