On Construction of Teaching Mode of Mathematics Experiment Based on MCL

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Abstract: *MCL is a new instrument to teach and study mathematics. Its primary characteristics are its portability, low price, easy operation and various functions. A student can get or use it at any time or in any place, so that he/she may be pleased to spend more time in exploratory mathematical activities. Mathematics is not just involved logic reasoning, but is also related to experimenting. Therefore, in a mathematics teaching class, the teacher should fully represent these two sides. Studying mathematics is not just about to learn the deduction or to complete a formal verification, but also involved to learn a mathematical process, or to conduct a series of experimental and conjectural thinking and exploratory activities prior to the formal verification. Besides, the best way to conduct a mathematical exploratory activity is to do Mathematics Experiments. With MCL, the students may construct the teaching mode of a Mathematics Experiment through experimental validation, experimental exploration or experimental construction. While using MCL, the following principles need to be obeyed: grasping appropriate opportunities, adopting appropriate modes, and achieving appropriate goals.*

1. Introduction

In October, 2007, the Center for School Curriculum Research & Development of the Ministry of Education and Beijing Normal University (Key Laboratory affiliated to the Ministry of Education) conducted a research on the topic "Mobile Calculating Laboratory (MCL) – the Combination of Handheld Technology and New Curriculum of Mathematics in Middle School". They totally listed more than 80 middle schools in laboratory areas in 10 prefectural level cities as the laboratory schools. In August, 2008, as one of the laboratory schools, our school participated in this research, and constructed a Teaching Mode in Mathematics Experiment based on MCL after more than 3 years of teaching practice and research.

2. Definition of the Term "Teaching Mode of Mathematics Experiment" and Corresponding Theoretical Basis

2.1 Definition of the Term "Teaching Mode of Mathematics Experiment"

The term "Teaching Mode of Mathematics Experiment" refers to as the process that the teacher, based on requirements for preparing teaching outlines and arrangement for teaching content, uses GC to establish an appropriate scene for a question and designs another series of quiz to support, so that the student may be guided to adopt GC to independently conduct operation, practice or test, or to cooperate, discuss or communicate with each other, or to prove a mathematical conjecture, or to find a verification for a mathematical theory, or to solve a mathematical problem through a mathematical thinking activity, so that he/she may experience the process to construct the mathematical mode in person.

2.2 Theoretical Base for a Teaching Mode of Mathematics Experiment

Based on the constructivism study concept, the students are the primary subject in study. The students can not simply gain knowledge by being taught by teachers or others, but have to initiatively conduct related construction based on their knowledge and experiences already had. Essentially speaking, the mathematics has a duality of both logical and experimental sides, so a mathematical teaching class should fully reflect these two sides. Learning mathematics is not just about to learn the deduction or to complete a formal verification, but also involved to learn a mathematical process, or to conduct a series of experimental and conjectural thinking and exploratory activities prior to the formal verification. As a new study and cognition instrument, with its special characteristics as portability, low price, easy operation and various functions, GC can not only improve the teaching or studying effect of the teacher or the student, but also be integrated deeply into the mathematics teaching structure, target, content and method.

3. Ways to Construct a Teaching Mode of Mathematics Experiment with MCL

3.1 Through Experimental Validation

To construct a mode through experimental validation refers to as a teaching mode that the teacher creates a corresponding scene for the student to form a cognitive conflict, and the student uses GC to experience the forming process of a mathematical concept or a generation process of a mathematical conclusion by experimenting, observing, recording and analyzing or other approaches, so that the student may understand the necessity and rationality to generate a mathematical concept or verify the authenticity of a mathematical conclusion, to achieve the purpose as to acquire a mathematical concept or to verify a mathematical conclusion already knew.

Because of the abstractness of a mathematical concept and the complication to generate a mathematical conclusion, the student may psychologically be averse to accept new knowledge, so the new knowledge may not be integrated into the knowledge structure of the student already has. During the traditional mathematics teaching process, some mathematical concepts were just only being informed. There was no lesson to teach about the rationality and necessity about why this concept should be generated. The correctness of some mathematical conclusions can only be confirmed through logical reasoning and verification, but for the process to form this conclusion, there aren’t many corresponding approaches to verify it. Through experimental validation, the mathematical knowledge can be transferred into details, so that the student may recognize the new knowledge and understand it better.

[Example I] The Tangent Line to a Curve at a Given Point

When learning geometry in middle school, the student is taught by the definition of the term “the Tangent Line of a Circle”. Many students have a concept that the tangent line is just a straight line that has one and the only one meeting point with the curve. Obviously, this cognition is wrong. However, what is the tangent line to a curve at a given point?

Activity I: the student used GC to draw graphs given by equations of one option as follows in one coordinate system respectively, counted how many meeting points the straight line in one coordinate system had with the corresponding curve, and judged if this straight line in this system was the tangent line to the corresponding curve by observing each coordinate system based on his/her recognition of the tangent line.

(1) $y=3-x$， $y=\sqrt{x+1}$;

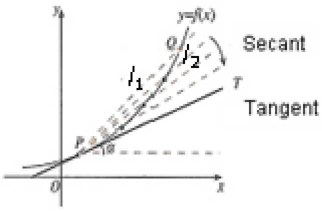
(2) $y=-x-6$， $y=x^4+x^3-7x^2-x-6$;

(3) $y=-3x+1$， $y=x^3-3x+1$.

The student input the analytic formula of the function through GC to gain the corresponding graph. Through observing images of 3 options of equations, he/she instinctively figured out the straight line in image (1) was not the tangent line to the corresponding curve, and straight lines in image (2) and (3) were tangent lines to corresponding curves. Besides, he/she also found that the tangent line in image (2) had 3 meeting points with the corresponding curve, while the tangent line in image (3) came cross the curve. The student's cognition of the tangent line was confused by the identification of these tangent lines, so a conflict was formed, and the student's desire for new knowledge was stimulated. The teacher then asked the student questions like "How to figure out whether a straight line is a tangent line to a curve at a given point?" or "How to define the tangent line to a curve at a given point?", in order to guide the student from perceptual awareness to rational thinking.

Activity II: In order to accurately represented the changing trend of a curve at a given point, the student used GC to draw a curve "C" given by an equation "$y=x^2$" and selected a point "*P*" with the coordinate of "(1, 1)" in curve "C". When magnified the part of curve at the point "*P*", he/she found out that this part of curve near point "*P*" was seemed like almost a straight line. When kept on magnifying this part, he/she found out that this part of curve was closer and closer to an already existed straight line "*l*". Finally, this line "*l*" became the straight line coming across the point "*P*" stayed closest to the curve.

As the Graph1 showed, if "l_1" and "l_2" were two straight lines coming across the Point "*P*", the



Graph1

student might:

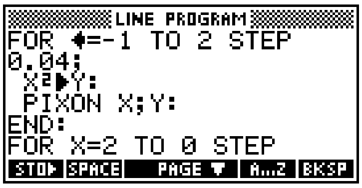
(1) Try to figure out which straight line is closer to the curve at the point "*P*";

(2) Try to figure out if it is possible draw a straight line "l_3", which stays closer to the curve at the point "*P*" compared with "l_1" and "l_2";

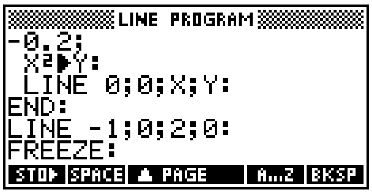
(3) Try to figure out if it is possible draw a straight line "l_4", which is closer to the curve at the point "*P*" compared with "l_1", "l_2" and "l_3";

Through observing the graph, experimenting, comparing and confirming, the student might slowly find out a straight line "l_4", "l_5", "l_6" and etc, which all stayed closer to the curve at the point "*P*" compared with "l_1", "l_2" and "l_3". After experienced the approaching process of a secant to the tangent line, the student perceptually figured out that the tangent line was the closest secant to the curve.

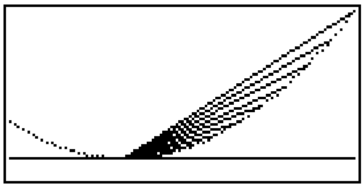
Activity III: The student used the "Editing" function of GC to make a dynamic graph to show that the location of the tangent line to the curve given by the equation "$y = x^2$" at point "*P*" with the coordinate of "(1, 1)" was the extreme position for the secant coming across the point "*P*". See as the Graph2 to the Graph4 shows.



Graph2



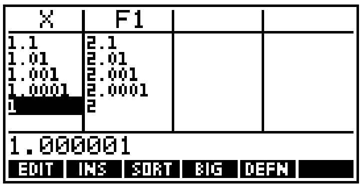
Graph3



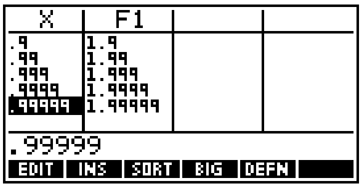
Graph4

The student has been taught with the algorithmic language and already primarily understood corresponding basic concept of algorithm, but to operate a computer through writing procedures still seems to be a challenge for some of the students. The teacher may gather students to form several groups. Within the group, through fully discussions, they may complement each other's disadvantage and achieve the communications between student and student, student and teacher, and student and computer. Students may try their best to complete the dynamic graph, and further experience the process of a secant approaching closer and closer to a tangent line during programming period through observing visual graph and terminal dynamic display. This will not only reflect the spiral movement of the knowledge, but also stimulate the student's interest and explore the student's potential in studying. They will also experience the joy of studying together and share their things they learned.

Activity IV: The student used GC to conduct numerical experiment. He/she selected a point "*Q*" with the coordinate of "(x, x^2)" & "$(x \neq 1)$" near the point "*P* (1, 1)" at the curve "$y = x^2$". The student approached the point "*Q*" closer and closer to point "*P*", and observed the change of "k_{PQ}" value. See as the Graph5 to the Graph 6 shows.



Graph5



Graph6

The student found out that wherever the point "*Q*" was on the left or right side of point "*P*", the closer the point "*Q*" approached point "*P*", the more approximative the "x" value to be "1" and the "k_{PQ}" value to be "2". So the slope of the tangent line at point "*P*" was "2". The teacher would then guide the student to conclude a nonfigurative definition of the tangent line to the curve at a given point, so that to lay groundwork to study the next chapter "The Definition of Derivative".

3.2 Through Experimental Exploration

To construct the mode through experimental exploration refers to as the teaching mode to gain a unknown mathematical conclusion that the teacher arranges appropriate study tasks based on the student's cognitive rules and teaching content and the student uses GC to design and implement the experimental plan and concludes a mathematical conclusion through processing data by experimenting, observing, comparing, analyzing or other means. Actually, this mode will offer the student an approach to gain mathematical knowledge through his/her exploration, and the student may learn and explore mathematics in a way similar with the mathematician. This mode concentrates on gaining mathematical knowledge and understanding mathematics in an exploring process.

[Example II] Graph of Function given by Equation "$y = A\sin(\omega x + \phi)$"

The student already had an initial cognition about the graph and the characteristic of function "$y = \sin x$", so in this class, the key task is to figure out the relation between graphs of function "$y = \sin x$" and function "$y = A\sin(\omega x + \phi)$". In a traditional teaching class, the student may spend more time and energy in drawing graphs by their hands, or ask the teacher to operate and demonstrate. They just accept the knowledge passively. Due to this teaching mode, the student may have no chance to do independent exploration, or to experience the mathematics exploration process and the passion of creation. By using GC, the student may have the possibility to independently explore mathematics in the class.

The students were divided into several groups. Members of each group designed their own experimental plan (an integrated plan after communications in class) of a question as follows.

(1) What is the relation between graphs given by function "$y = \sin(x + \varphi)(\varphi \neq 0)$" and function "$y = \sin x$"?

(2) What is the relation between graphs given by function "$y = A\sin x(A > 0 \,\&\, A \neq 1)$" and

function “ $y = \sin x$”?

(3) What is the relation between graphs given by function “ $y = \sin \omega x (\omega > 0 \,\&\, \omega \neq 1)$ ” and function “ $y = \sin x$”?

(4) What is the relation between graphs given by function “ $y = \sin(\omega x + \varphi)(\omega > 0,\ \varphi \neq 0)$ ” and function “ $y = \sin \omega x (\omega > 0)$ ”?

(5) What is the relation between graphs given by function “ $y = A\sin(\omega x + \varphi)(A > 0 \,\&\, A \neq 1)$ ” and function “ $y = \sin(\omega x + \varphi)$ ”?

(6) What is the relation between graphs given by function “ $y = \sin(\omega x + \varphi)(\omega > 0,\ \varphi \neq 0)$ ” and function “ $y = \sin(x + \varphi)(\varphi \neq 0)$ ”?

In this class, the student opened an exploratory road based on the experimental topic. The road was placed with road signs (6 exploratory questions). The distance between two signs was neither too wide nor too narrow. The parameters involved obeyed the rule “from easy to hard” and the quantity of parameters involved was increased gradually. The student determined the value of each parameter in each exploratory question randomly for adequate observation. He/she used GC to draw his/her own graph of a function, made his/her own conclusion through experimenting, observing, comparing and analyzing, and exchanged it with each other within a group in class. During the teaching process, the teacher and the student as well as the students themselves completed many communications and discussions with each other, reflecting the dominant role of the teacher among the students.

For some students with outstanding ability, the teacher may chose to give a more challenging task like “How to prove it?”, “Is this conclusion available for development and promotion?” or “Can you prove it?”. The most excited part of a Mathematics Experiment is to make a conclusion of a conjecture raised, but it doesn’t mean the experiment is already finished. To verify the conjecture is to represent a scientific spirit. It is also a method to identify whether a Mathematics Experiment is successful. It is necessary for the teacher to guide the student to verify a conjecture or to raise a counter example to deny it, so the student may understand that only those conclusions made through theoretical validation are credible. To expand a conclusion is to make the student to re-consider the conjecture concluded, so that he/she may form a better study habit.

3.3 Through Experimental Construction

To construct a mode through experimental construction refers to as the teaching mode that the teacher (or the student) raises a question and the student collects data to make an appropriate assumption, uses GC to process and analyze the data to construct a reasonable mathematical model and applies the model for a new scene to verify its correctness. The result of this model is not exclusive. It allows the student to make innovations in data interpretation and model construction, so that he/she may understand mathematics and gain knowledge during the model constructing process.

[Example III] Linear Regression Equation

Actually, the relations of variants are usually being divided into the following two types: one is definite relation, while the other is correlative relation. This lesson focused on the research of the common influence on two variants with a correlative relation when values of variants changed, so that the student might experience the whole process to study and research a correlative relation through a statistical method.

Activity I: Created a Scene to Stimulate Interest

A student calculated his/her height based on corresponding formulas: (1) Boy's Height = 59.699 + 0.419 × Father's Height + 0.265 × Mother's Height; (2) Girl's Height = 43.089 + 0.306 ×Father's Height + 0.431 × Mother's Height. When this calculated value was not equaled to the actual value, he/she would have the doubt as "Is this formula correct?" or "Where did this formula come from?" or other questions. The student would figure out the relation between our heights and our parents' was not definite, but only correlative. Well then, how to represent the relation between different variants with a mathematical model?

Activity II: Collected Data and Compared through Experiments

In order to construct a model, the simplification should be done. The height of the daughter and the height of the mother were corrective. How to represent the correlative relation between two variants with a mathematical model? Randomly selected 8 pairs of mothers and daughters, and recorded their heights in the table1 below. (These 8 pairs of mothers and daughters were selected randomly from the data the student collected before class.)

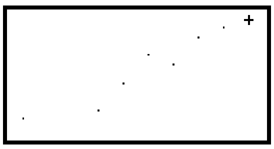
Height of Mother (*x*) *cm*	154	157	158	159	160	161	162	163
Height of Daughter (*y*) *cm*	155	156	159	162	161	164	165	166

Table1

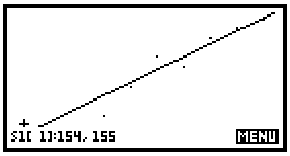
Was there a function can approximatively represent this correlative relation? Which kind of function could approximatively represent this correlative relation?

If the mother's height was 172cm and the daughter's height was 178cm, then how to select a function to represent?

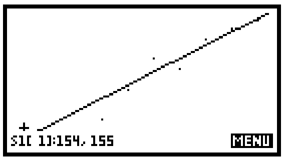
Students used GC to input data. Two students cooperated with each other. One read; one input. When the input was finished, the two should cooperated with each other and used "Send" and "Receive" functions of GC to completely finish the input of data. A scatter diagram was then generated (see the graph below). Through observing this scatter diagram, the students could exchange their opinions and tried to select their function through GC to run data fitting.



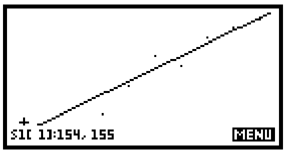
Scatter Diagram



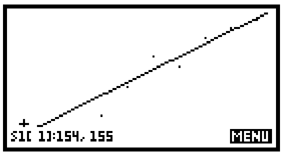
Linear Function



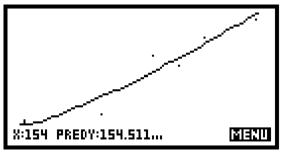
Logarithmic Function



Exponential Function

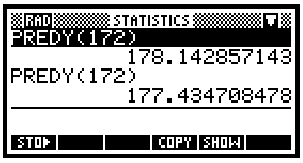


Power Function

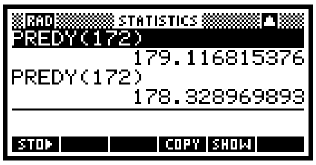


Quadratic Function

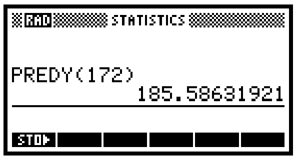
The students used the "Calculation" function of GC to forecast the result based on their function, and compared it with the experimental result.



Graph7



Graph8



Graph9

We hoped the value we conclude stays as approximative as the actual result, so the difference would be the minimum. Through comparing results, we figured out that the linear function's difference was the minimum, so the fitting by the linear function "$\hat{y} \approx 1.345x - 53.118$" was the best. We named this equation as the linear regression equation to fit these 8 pairs of data (Introducing the topic).

Activity III: Raised Questions Bravely and Traced the Cause

Where did you get this coefficient to this equation? Is the predicted result reliable? Why did you choose the linear function "$\hat{y} = bx + a$" for the fitting?

The students designed plans as follows: (1) Selected two points that could maximally reflect the changes on the straight line and connected them; (2) Selected a straight line in where the quantities of points on both sides of the line were equaled; (3) Selected more points to determine several straight lines. Calculated the slope and intercept of each line respectively, and adopted the average as the slope and intercept of the line required.

Although these methods were all applicable, the students still felt unauthentic. Actually, we hoped this straight line might stay as close as the points in scatter diagram. How to determine the close level of the straight line given by the equation "$\hat{y} = bx + a$" with points in the scatter diagram? How to conclude a linear function with the smallest deviation?

The students designed plans as follows: (1) Calculated the sum of the squared values of distances from each data point to the straight line; (2) Calculated the sum of all squared values of distances from each data point to corresponding point having the same abscissa.

The students already learned the basic knowledge of statistics, and experienced the process to determine the most ideal approximation by summing up the squared values of deviations, namely the idea of "estimating the average through all". For plan (1), calculating the oblique distance was not that easy, so the students considered replacing with the longitudinal distance, which brought plan (2). After that, the students calculated the coefficient of the linear regression function with the least square method.

This lesson was about to process the correlative relation by a statistical approach. This referred to as the process to select samples randomly (or obtain sample data), to process the sample data, to construct a mathematical model and to make forecasting and estimation by this model. The students would have to complete the calculation and processing of a large amount of data. In order to released the students from this heavy, complicated and machinery calculation, so that they could focus on constructing the model, GC was adopted for mapping, calculating and fitting as a technical support.

4. Issues Needed to be Considered when Implementing Mathematics Experiment with GC

The adoption of GC makes it possible to implement Mathematics Experiment in class, but some appropriate principles in using need to be followed. Details are as follows.

4.1 Grasping Appropriate Opportunities

Mathematics is abstract. This abstractness makes it difficult for some students to learn math. The “Multiple Representation” function of GC allows the teacher to represent some abstract concepts in many angles, so that the students may understand them easier. However, the abstractness of mathematics is not invariable. Different mathematical concepts’ abstractness is also different. For students with different ages, their abstract thinking abilities are also not the equaled. For those are not very abstract and have no necessity to use GC, the restriction shall be implemented to prevent functional dissimilation. The use of GC shall be consistent with the improving speed of the students. It shall not fall behind the knowledge level or the intelligent development speed of a student.

4.2 Adopting Appropriate Modes

The new teaching structure with "Leading Party and Primary Subject" can not only reflect the teacher’s leading role but also represent the student’s dominant position. The teachers shall only arrange appropriate quantity of mathematical tasks to the students, so they may have enough time and space, use GC independently under the teaching guidance or cooperate with each other and exchange ideas within the group. The teacher shall control the students from time to time. The students shall not be less or over controlled.

Although GC has advantages like low price and portability, it still has its own shortcomings, such as its interface display and dynamic graph are not as good as other software, like Geometer's Sketchpad. The teachers can use other technologies to supplement to achieve best effect in teaching based on actual needs.

4.3 Achieving Appropriate Goals

Mathematics is a cogitative science, and the central concept to teach mathematics is to improve student’s thinking ability. By adopting GC in the class, the students may understand and grasp the essence of mathematics and the relations better, so that they may figure out some general rules from specials, and really understand the spirit, ideas and methods of mathematics. The entire teaching process is to train student’s mathematical thinking ability and to use GC as an instrument to understand and gain mathematics knowledge and thinking methods. Otherwise, even with GC, students’ awareness and understanding of mathematics may still remain in the shallow level.

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