The Role of the Graphing Calculator in the Qualitative Analysis of Function

Li Hong

lihong2209@sina.com

Beijing No.22 High School

China

Abstract: *This paper focuses on several cases of student's qualitative analysis of function under the support of graphing calculator and other forms of technology. This paper proposed the following points. With graphing calculator, students can get abundant material for qualitative analysis, a process of revealing the essence of the research object. Since study of function has very important effects on students' understanding of the characteristics of mathematics and the mathematical research methods and the qualitative analysis is to emphasize the holistic understanding of the pattern of variation expressed by the dependency between the two variables of function, graphing calculator as a technology indeed helps students understand the essence of mathematics, apply mathematics in a better way and even has a great impact on students' development.*

1. Introduction

Information technology greatly influences people's life by not only changing people's lifestyle but also bringing great transformation on people's concept of life and psychology of behavior. Education tightly relates to our life, the effects of technology on our life are permeating into the domain of education. The continuous updates of technology provide mathematical education with a brand-new platform on which teachers can design creative teaching activity, bringing great impacts on students and producing progresses on the methods and results of student's research.

The study of function accompanies with students from grade 7 to grade 12 without interruptions. So it reflects students' levels of mathematical study and characteristics of thinking on different stages in some degrees, becoming the epitome of students' mathematical study. The study of function has very important effects on students' understanding of the characteristics of mathematics and the mathematical research methods.

Qualitative analysis is the process of revealing the essence of the research object. Concretely speaking, qualitative analysis is the process of using methods of induction and deduction, analysis and synthesis, abstraction and generalization to dissect the acquired material mentally so as to comprehend the essence and reveal the intrinsic patterns. Qualitative analysis can be divided into three steps: 1) observation and comparison, 2) analysis and synthesis, 3) abstraction and generalization.

The core of the concept of function is variability, or more precisely, covariablility. Students in the past often neglected the relationship between two variables. They paid too much attention on concrete number values but lack the qualitative understanding of the dependency in function. The qualitative analysis is to emphasize the holistic understanding of the patterns of variation expressed by the dependency between the

two variables of function.

There are some cases extracted from my mathematical teaching to prove my points above.

2. The Convenience for Observation and Research

Case: the qualitative analysis of the properties of linear function

After learning the concept of linear function, students can analyze the properties and graphs of the function qualitatively by observe and research the effects of parameter k, b on the properties of the function $y = kx + b$.

Step 1: Make k constant and b variable such that $b > 0$ or $b = 0$ or $b < 0$.

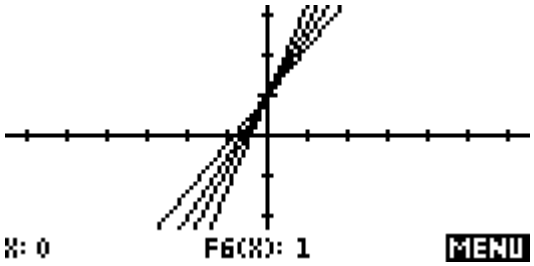


Figure 2.1 Graphs of $y = kx + b$ such that $b > 0$ or $b = 0$ or $b < 0$.

Let students input the functions with different b and the same k into the graphing calculator and then graph what they input in the same coordinate.

Step 2: Make b constant and k variable such that $k > 0$ or $k = 0$ or $k < 0$.

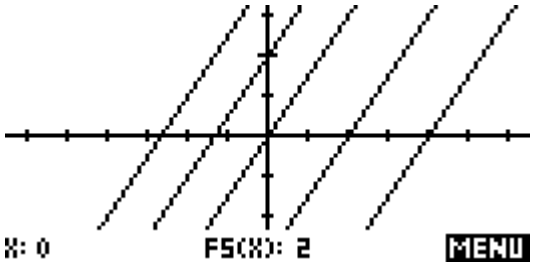


Figure 2.2 Graphs of $y = kx + b$ such that $k > 0$ or $k = 0$ or $k < 0$

Let students input the functions with different k and the same b into the graphing calculator and then graph what they input in the same coordinate.

Using graphing calculator students can graph different functions in the same coordinate. With the power of technology, the observation and research is much easier for students and the aims of mathematical teaching are easier to realize. We can guide students to think: Why are they researching it? Why are they researching it in this way? Therefore, they can understand the covariablility of the essence of function perceptually. They can know that function is a kind mathematical model to describe the change in the nature. They can realize the characteristic of mathematics when they understand that mathematics is the study of quantity, structure, space, and change and experience the research methods of mathematics.

3. Analysis and Synthesis

Case 1: the qualitative analysis of the properties of power function

After learning the concept of power function, students can analyze the properties and graphs of the function qualitatively by using graphing calculator, applying the concept of power function for further study.

Pre: Graph some power functions freely

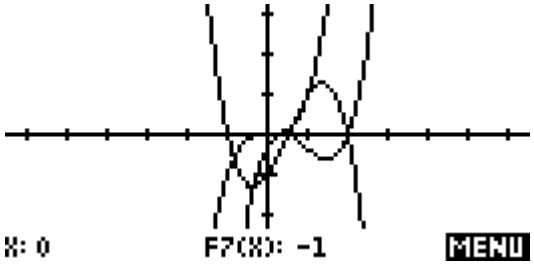


Figure 3.1 Graphs of Power Functions

Step 1: Graph $y = x^{\alpha}(\alpha < 0)$

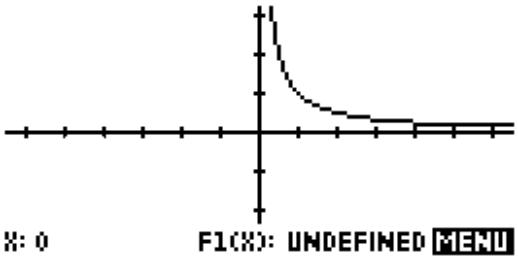


Figure 3.2 A Graph of $y = x^{\alpha}(\alpha < 0)$

Step 2: Graph $y = x^{\alpha}(\alpha > 0)$

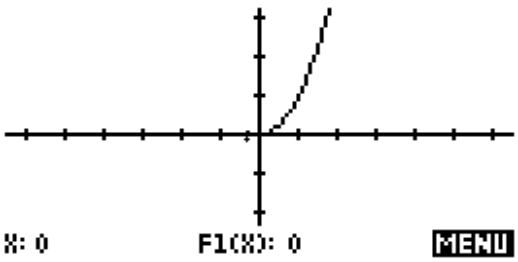


Figure 3.3 A Graph of $y = x^{\alpha}(\alpha > 0)$

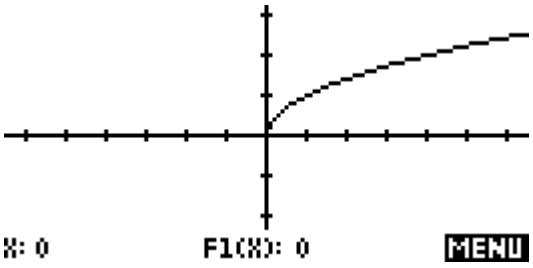


Figure 3.4 A Graph of $y = x^{\alpha}(\alpha > 0)$

Step 3: Graph $y = x^{\alpha}(0 < \alpha < 1)$

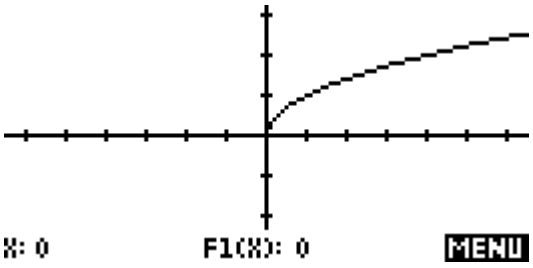


Figure 3.5 A Graph of $y = x^{\alpha}(0 < \alpha < 1)$

Step 4: Graph $y = x^{\alpha}(\alpha > 1)$

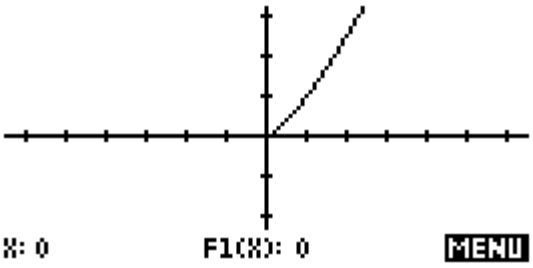


Figure 3.6 A Graph of $y = x^{\alpha}(\alpha > 1)$

Step 5: Let students find the graphs of functions above in other quadrants using the parity of function.

The technology of graphing calculator provides students with a platform where abundant material for analysis and synthesis is easy to get. Hence, students can understand properties of specific functions during the process of analysis and synthesis.

Case 2: the qualitative analysis of the properties of $y = ax + \frac{b}{x}$

Having learnt the properties and graph of function $y = x + \frac{1}{x}$, students were eager to develop the characteristics of the function. So they began the research of the

properties of function $y = ax + \frac{b}{x}$, a more generalized form than that of $y = x + \frac{1}{x}$. They controlled the variability of a and b to analyze the effects of a and b, respectively, on the graph of the function. Remembering the characteristics of the vertex of the graph of quadric function, they used the mean value theorem to express the change of the vertex of the graph of function $y = ax + \frac{b}{x}$ and used graphing calculator to substantiate their analogy.

Step 1: Graph $y = ax + \frac{b}{x}$ such that a is variable and b is constant

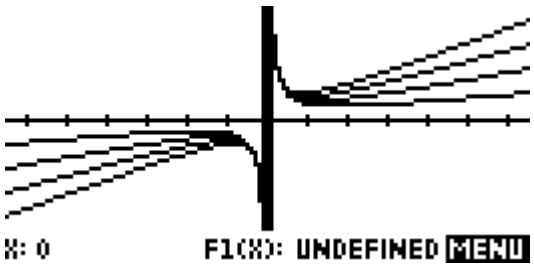


Figure 3.7 Graphs of $y = ax + \frac{b}{x}$ such that a is variable and b is constant

Step 2: Graph $y = ax + \frac{b}{x}$ such that b is variable and a is constant

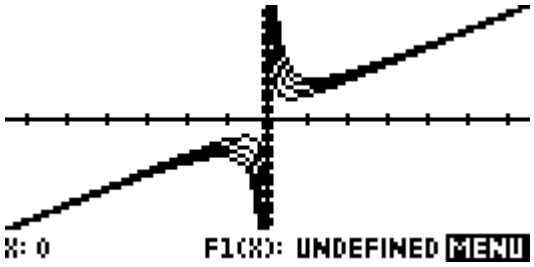


Figure 3.8 Graphs of $y = ax + \frac{b}{x}$ such that b is variable and a is constant

Step 3: Graph $y = ax + \frac{b}{x}$ such that $ab(a > 0, b > 0)$ remains constant.

According to the mean value theorem, its vertex coordinate is $\left(\pm\sqrt{\frac{b}{a}}, \pm 2\sqrt{ab} \right)$.

Let $ab(a > 0, b > 0)$ be constant. So $2\sqrt{ab}$ is constant.

For example, let $ab = 2$.

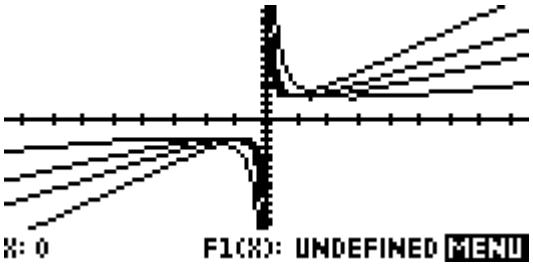


Figure 3.9 Graphs of $y = ax + \frac{b}{x}$ such that $ab(a > 0, b > 0)$ remains constant

4. Abstraction and Generalization

Case 1: the qualitative analysis of the properties of cubic function

Having learnt the concept of linear function and quadratic function, students were asked to graph these kinds of functions on their graphing calculators when they were told to pay extra attention on the intersections of the graphs and x-axis. Then they were asked to make an educated guess on the graph of a cubic function and check their thoughts on their graphing calculators. Having known the basic patterns of cubic function, they were asked to graph other cubic functions to understand the properties of cubic functions better.

Step 1: Graph $y = x + 1$

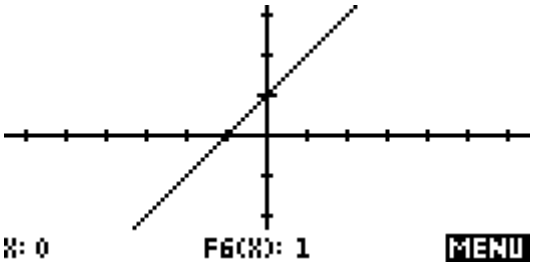


Figure 4.1 The Graph of $y = x + 1$

Step 2: Graph $y = (x + 1)(x - 2)$

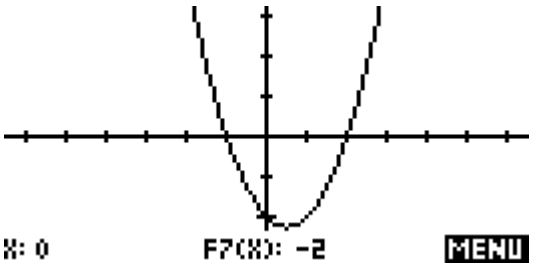


Figure 4.2 The Graph of $y = (x + 1)(x - 2)$

Step 3: Graph $y=(x+1)(x-2)(x-3)$

After graphing the two functions, students will make an educated guess on the graph of a cubic function and check their thoughts on their graphing calculators.

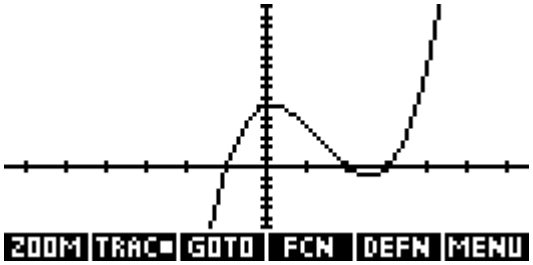
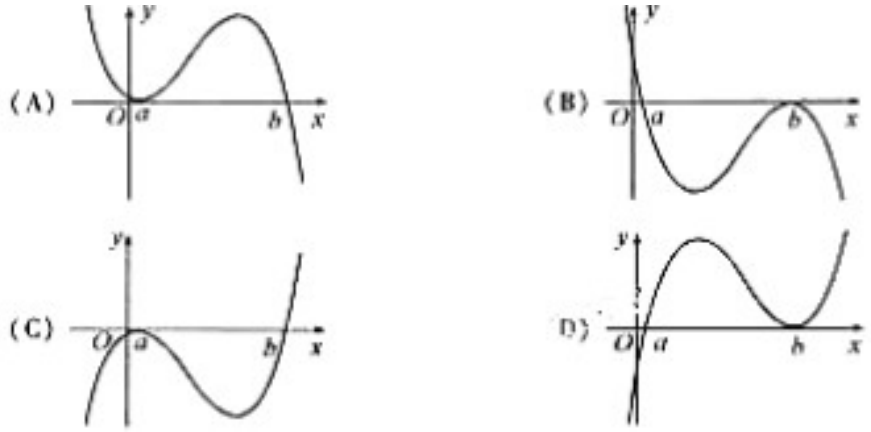


Figure 4.3 The Graph of $y=(x+1)(x-2)(x-3)$

Step 4: Graph other cubic functions

After accomplish the steps above, students can get rational knowledge of the geometric properties of cubic function from perpetual knowledge by observation, generalization, analysis and reasoning.

Exercise: Let $a<b$, the graph of function $y=(x-a)^2(x-b)$ probably is:



The correct answer is (C). From the equation we can know that the monotonicity of the function should be like (A) and (B) and there should be a vertex at $(a,0)$

Case 2: the qualitative analysis of the monotonicity of function through derivative

Having learnt the concept of derivative, students can use graphing calculator for the research of the monotonicity of function by using derivative.

Step 1: Graph $y = x^2$

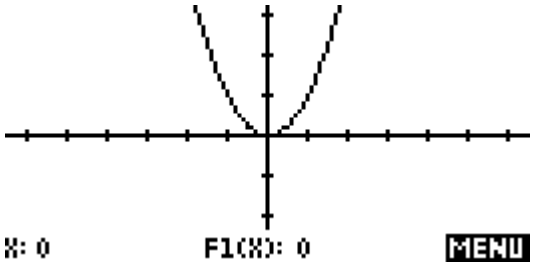


Figure 4.4 The Graph of $y = x^2$

Step 2: Observe the slopes of the tangents of the curves

Step 3: Graph $y = x^3$

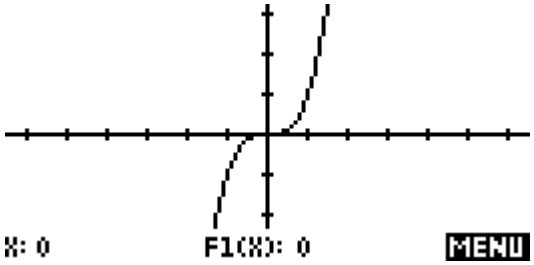


Figure 4.5 The Graph of $y = x^3$

Step 4: Observe the slopes of the tangents of the curves

Step 5: Graph $y = (x+1)(x-2)(x-3)$

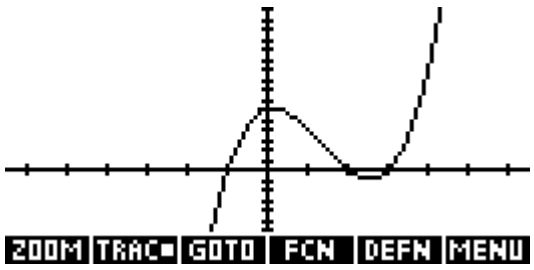


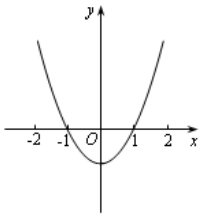
Figure 4.6 The Graph of $y = (x+1)(x-2)(x-3)$

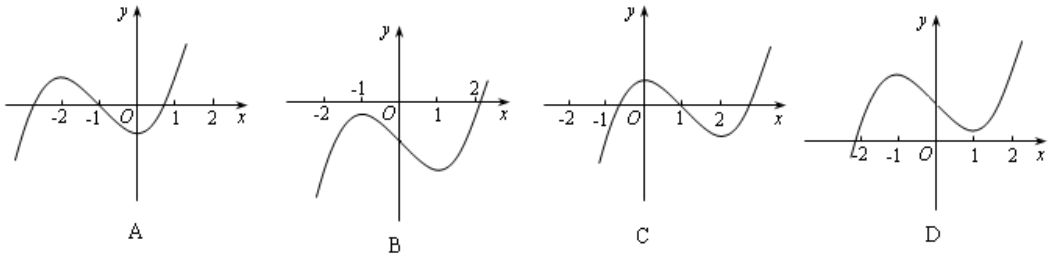
Step 6: Observe the slopes of the tangents of the curves

After accomplish the steps above, students can get rational knowledge of the relationship between $y = f(x)$ and $y = f'(x)$ from perpetual knowledge by observation, generalization, analysis and reasoning.

Exercise 1

The partial graph of $f'(x)$, the derivative of $f(x)$ is shown in the figure, so the graph of function $f(x+1)$ probably is:

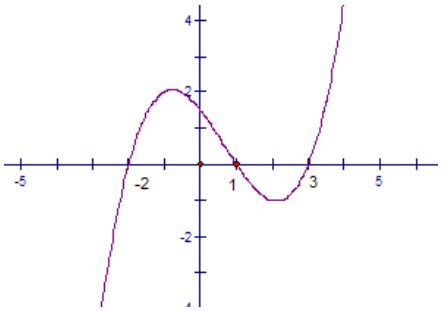




The correct answer is (A).

Exercise 2

Consider $f(x)$ a cubic function such that $f'(x)$ is its derivative. The partial graph of $y=(x-1)f'(x)$ is shown in the figure on the right. So the local maximum point of $f(x)$ is_____ and the minimum point of $f(x)$ is _____.



The correct answer is 3, -2.

With graphing calculator, students can get abundant material for qualitative analysis, a process of revealing the essence of the research object. Since study of function has very important effects on students' understanding of the characteristics of mathematics and the mathematical research methods and the qualitative analysis is to emphasize the holistic understanding of the pattern of variation expressed by the dependency between the two variables of function, graphing calculator as a technology indeed helps students understand the essence of mathematics, apply mathematics in a better way and even has a great impact on students' development.

References

1. Ji Ping. What to Teach. Educational Science Publishing House (China)
2. Xin Lei. Mathematics Education Reform and the Research
3. Mathematics Standard Courses Clarified. Beijing Normal University Press
4. On Math Thinking. Beijing Education Press
5. Mathematics Methodology. Beijing Education Press