

Cases in Technology-Enriched School Mathematics

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1. Introduction

Moyer, Bolyard, and Spikell [1] made a case for a clear distinction between the static from the dynamic digital representation of the concrete materials used for the teaching and learning of mathematics. Unlike the static representations which are essentially pictures, they argued for the dynamic representations which can be manipulated in the same way as concrete manipulatives as true virtual manipulatives. They felt that the ability to manipulative these representations on the computer “connects the user with the real teaching and learning power of virtual manipulatives, that is, the opportunity to make meaning and see relationships as a result of one’s own actions”. Suh, Moyer, and Heo [2] also observed that by allowing students to manipulate “on-screen objects to test hypotheses and experiment with ideas, the virtual manipulatives may more closely model the dynamic nature of thinking”, which may enhance “students’ thinking and creativity”. At the same time, they also noted that it is important to encourage students to make connections between different modes of representation of mathematical concepts so as to develop students’ representational fluency.

Many mathematics educators believe that students in every grade level should be given ample opportunities to develop their mathematical literacy and competency by acquiring an understanding of mathematics as a dynamic, interesting, and ever-evolving subject [3]. Silk, Higashi, Shoop, and Schunn [4] share this belief and feel that mathematics taught within well-designed technology lessons provide students additional opportunities to understand mathematics in differing contexts.

The National Council of Teachers of Mathematics’ *Principles and Standards for School Mathematics* [5] and other national documents (Singapore’s Mathematics Syllabus documents for primary and secondary levels by the Ministry of Education [6], [7]) provide a rich vision for mathematics teachers of all levels on the use of educational technology to enhance the teaching and learning of mathematics for their technology-rich classrooms of the twenty-first century. This paper recognizes both the opportunities and the challenges provided by new technologies

and describes three case studies that showcase how teachers taught mathematical activities as a learning process in a technological environment.

2. Teaching Primary Fraction Concepts Using Virtual Manipulatives

In this first case study, the impact on the use of virtual manipulative to teach the concept of equivalent fractions to Primary Three students in the Singapore School system is investigated. In the Singapore Primary School, the concrete-pictorial-abstract (C-P-A) development of concepts is advocated [7]. When concrete materials are used to cater to the needs of the concrete learners at the Primary levels, Ng [8] pointed out that “great care must be demonstrated to show how the concrete representations are linked to their abstract forms”. She also observed that a range of information and communication technologies (ICT), including virtual manipulatives, is available to help motivate and engage children in the learning process. This is in line with the Ministry of Education’s ICT Masterplan (mp) 1, 2, and 3:

- mp1 – The first Masterplan for ICT in Education (1997 — 2002) laid a strong foundation for schools to harness ICT, particularly in the provision of basic ICT infrastructure and in equipping teachers with a basic level of ICT integration competency, which achieved a widespread acceptance for its use in education.
(<http://www.moe.gov.sg/media/press/2008/08/moe-launches-third-masterplan.php>)
- mp2 – The second Masterplan for ICT in Education (2003 — 2008) built on this foundation to strive for an effective and pervasive use of ICT in education by, for example, strengthening the integration of ICT into the curriculum, establishing baseline ICT standards for students, and seeding innovative use of ICT among schools.
(<http://www.moe.gov.sg/media/press/2008/08/moe-launches-third-masterplan.php>)
- mp3 – The third masterplan for ICT in Education (2009-2014) represents a continuum of the vision of the first and second Masterplans i.e. to enrich and transform the learning environments of our students and equip them with the critical competencies and dispositions to succeed in a knowledge economy.
(<http://www.moe.gov.sg/media/press/2008/08/moe-launches-third-masterplan.php>)

In this study, the teacher concerned sought to “enrich and transform the learning environment” of his Primary Three students in the learning of equivalent fractions using the virtual manipulative Fractions [9]. A screen capture of this virtual manipulative is shown in Figure 1.

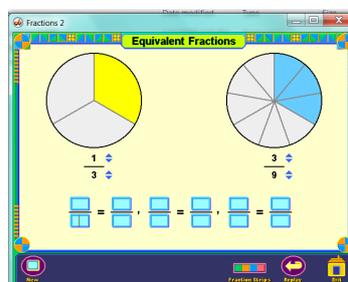


Figure 1. Screen Capture of Virtual Manipulative Fractions

Although the C-P-A approach is advocated, the teacher's past teaching strategy adopted for the introduction of equivalent fractions has been rather deductive and rule-based in nature. The inconvenience of having to transport about 40 sets of the concrete manipulatives to the classroom has resulted in teachers often making use of just a demonstration set. Through the demonstration set, the teacher would then illustrate how two or three fractions can be shown to be equivalent, e.g. using fraction discs to illustrate that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are equal. The rule in recognising how two given fractions are equivalent is then explicitly given to students before students make use of the rule to list the first 8 equivalent fractions of a given fraction with denominator less than 12 – the lesson objective. However, with mp2 in place and appropriate virtual manipulatives are made available through commercially produced textbooks, the teacher was more confident of introducing equivalent fraction to the students through a more inductive approach whereby students could engage in the virtual manipulative to investigate and list equivalent fractions before hypothesising about the rule that governs the relationships between equivalent fractions. This is particularly made possible by the establishment of baseline standards for the students under mp2 which ensure that all students are well equipped with the skills needed to handle the virtual manipulatives on the computers.

For the one-hour lesson, the teacher started by using laminated circle to help students to recap the concept of fraction as parts of a whole which they learnt in Primary Two. The teacher then got the students to work in pairs to represent the fractions $\frac{1}{2}$ and $\frac{2}{4}$ on their respective laminated circle. He then led the students to observe that they both represent the same part of a whole and introduced the term equivalent fractions. The process is repeated using the laminated circles until the students have listed the first eight equivalent fractions of $\frac{1}{2}$. However, instead of the usual practice of next telling the students the rule that governs the numerators and denominators of equivalent fractions, the teacher introduced the virtual manipulative Fractions to the students so that they could see the equivalent of the two representations. Students were then given time to work on the virtual manipulative Fractions or the laminated circle to list the first eight equivalent fractions of $\frac{1}{3}$. The lesson then ended with the teachers getting the students to share the equivalent fractions of $\frac{1}{3}$ in a class discussion.

In order to investigate for a possible difference in impact on different ability students of such an approach, the same lesson was conducted to two different classes, one of higher mathematics ability (Primary 3A) and another of lower mathematics (Primary 3F), as reflected by their mathematics performance in Primary 2. To minimise the teacher factor, the same teacher taught both the classes.

2.1 Results

Table 1 lists the pre-test scores of the two classes on Primary Two fraction concepts to obtain an indication on the achievement levels between the two classes.

	Primary 3A	Primary 3F	Combined
n	40	38	78
Mean	18.83	16.74	17.81
SD	1.71	2.24	2.23

Table 1: Pre-test Scores

The SMD for the pre-test scores is 0.94, which reflects a large effect size between the two classes. In other words, the two classes may not be considered as equivalent classes, and Primary 3A appears to have achieved significantly better than Primary 3F in terms of the Primary Two fraction concepts.

Table 2 provides a breakdown of the number of fractions equivalent to $\frac{1}{3}$ that the students from the two classes listed correctly while Table 3 summarises the mode of representation that the students used to list the equivalent fractions.

	Primary 3A	Primary 3F	Combined
n	37	38	75
Mean	5.64	4.29	4.87
SD	3.21	2.85	3.07

Table 2: Equivalent Fractions Listing Scores

	Primary 3A	Primary 3F	Combined
Laminated Circle (Concrete)	4	0	4
Fraction 2 (Virtual Manipulative)	26	19	45
Did not respond	7	19	26

Table 3: Equivalent Fractions Listing Scores

The SMD for the Equivalent Fractions Listing scores is 0.44, which reflects a small to medium effect size between the two groups. The performance level between the two classes on Fractions appears to have narrowed. Furthermore, it is observed that most of the students used the virtual manipulative (more than 90% of those who responded) to list the equivalent fractions. A large number of the students in Primary 3F who did not indicate the choice of representation used to list the equivalent fractions was due to time constraint; in fact less than five students in each class was observed to be using the laminated circle to list the equivalent fractions. Thus, the use of virtual manipulative appears to have narrowed down the achievement gaps between the two

groups. The teacher also commented that he observed that both groups appeared to be more engaged and motivated, pointing towards an observed overall improvement in mathematics performance in both groups. In fact, a modified version of the PETALS™ Engagement Indicator Questionnaire developed by the Ministry of Education was administered to determine the level of engagement, including the sub-scales on behavioural, cognitive, and affective engagement, of the students in both classes. The Questionnaire was modified to cater to the needs of the Primary 3 students by simplifying the language and reducing the Likert Scale from an 11-point one to a 3-point one. Table 4 shows a breakdown of the Engagement Levels of both groups.

Measure	Primary 3A			Primary 3F			Combined		
	n	Mean	SD	n	Mean	SD	n	Mean	SD
Overall Engagement	38	38.87	0.60	38	40.94	0.50	76	39.91	0.55
Behavioural Engagement	38	12.95	0.60	38	13.28	0.50	76	13.12	0.55
Cognitive Engagement	38	12.08	0.70	38	13.44	0.50	76	12.76	0.61
Affective Engagement	38	13.84	0.50	38	14.21	0.40	76	14.03	0.45

Table 4: Engagement Levels

As there are a total of 5 items for each of the sub-scales, the overall engagement score has a maximum score of 45 while each of the sub-scales has a maximum score of 15. The engagement scores thus did agree with the teacher's observation that the students in both classes are highly engaged. In particular, both classes are not only engaged but highly engaged affectively. The teacher attributed the slightly higher engagement scores of Primary 3F to a possible inflated self-report score due to the use of the virtual manipulative which has allowed for more opportunities for trial and error – an encouragement for these weaker students who often do not succeed at their first trial with problems.

The teacher also observed that the students were engaged in higher order thinking as they go about listing the equivalent fractions using the virtual manipulatives. Students were seen to be hypothesising and observing patterns. The students were only asked to make use of either the concrete or virtual manipulative to list equivalent fractions but not asked to predict how these equivalent fractions may be generated. In fact, the observation of the patterns to predict the rule that governs the numerators and denominators of equivalent fractions has been planned for the following lesson. However, the students who used the virtual manipulative were observed to be studying the patterns and seeking to list the equivalent fractions using their perceived rule. This has been reflected by some of equivalent fractions that these students listed which has denominators that exceed 24; 24 is the largest number of equal pieces that the virtual manipulative could divide the whole into. During the end-of-lesson class discussion, many of the students were also observed to be articulating possible rules that govern the generation of the equivalent fractions without being prompted by the teacher. The virtual manipulative appear to have become vehicles to enhance students' thinking and creativity, as observed by Suh, Moyer, and Heo [2].

3. Prospective Secondary Mathematics Teachers' Varying Technology Lessons on Proportional Reasoning

The second case study investigated various technologies pre-service secondary teachers selected when preparing lessons for their future students on the topic of proportional reasoning. The participants in the study were all senior-level prospective secondary mathematics teachers who were enrolled in a required mathematics education seminar that served as their capstone experience prior to their student teaching practicum. The proportional reasoning technology project was a major requirement for the course. The pre-service student teachers based their lessons on definitions of proportional reasoning as mathematics concepts that involve unit rates, mixtures, or scaling ([10], [11]).

Four weeks prior to the submission and presentation of the project, the prospective teachers were given a detailed description of the project and an accompanying grading rubric. The project description specified that the prospective teachers were to create an original lesson on proportional reasoning that utilized any form of technology that they wished. In addition, the lesson was to be 30 minutes in length and was to include an overview of the lesson, the actual lesson, examples of extension problems, and a reference section. A written report was also to be submitted that included a description of the mathematics standards being addressed in the lesson, supplemental worksheets with assessment guides that could be used for future activities, and a discussion of the advantages and disadvantages of using the particular type of technology.

3.1 Results

The prospective secondary mathematics teachers' presentations were videotaped and analyzed. The presentations were graded with respect to the prospective teachers' ability to present a clear and well-designed lesson, their development of pertinent materials, and on their overall knowledge pertaining to the topic of proportional reasoning.

Analysis of the presentations indicated a range of technologies used. Of the sixteen pre-service teachers involved in the project, twelve used PowerPoint slides to describe and introduce their lessons, three used a Prezi, and one used Geometer's Sketchpad to create the narrative. Table 5 lists the class percentages using each of these technologies for the presentation narratives.

Presentation Narrative Format	Class Percentage
PowerPoint	75.0%
Prezi	18.75%
Geometer's Sketchpad	6.25%

Table 5. Class Percentage of Format of Presentation Narrative

The second most popular choice of technology was used by eight of the pre-service teachers and included the use of web-based activities such as animations, games, or video clips. Topics

discussed within the web-based activities of the presentations of these eight pre-service teachers dealt with topics such as measurement, physics, and biology.

The use of dynamic geometry to illustrate proportional reasoning concepts was the third most popular choice with five of the prospective teachers selecting that type of technology. Of these five presentations, three students utilized GeoGebra and two used Geometer's Sketchpad. In addition, one pre-service teacher utilized a spreadsheet, while two selected a hand-held graphing calculator for their presentation that presented lessons on using proportional reasoning principles in a statistics class. The class percentages for the activity types selected for the presentations are listed in Table 6.

Activities in Presentation	Class Percentage
Web-Based Activity	50.0%
Dynamic Geometry	31.25%
Graphing Calculator	12.5%
Spreadsheet	6.25%

Table 6. Class Percentage of Choice of Technologies

Examples of extension problems within the presentations included areas such as economic analyses, medicine dosages, architectural blueprints, dilation problems, and map making. Advantages of using technology that were listed within the presentations were that technology is an engaging activity for students that is a break from the norm and that technology allows real-life applications to be easily brought into classroom lessons. The disadvantages were numerous and listed things such as possible problems with access to technology for all students, lack of knowledge of the technology or particular software, and that technology is a teaching tool and should be used appropriately when teaching mathematics.

4. Pre-service Secondary Teachers' Lessons using Interactive Whiteboard Technology

As with the second case study, the participants in the third study were also senior-level prospective secondary mathematics teachers who were enrolled in a different section of the required capstone mathematics education seminar. However, this study's project requirements were slightly modified. One month prior to the presentations, the pre-service teachers were given a description of the project, an accompanying rubric, and access to interactive whiteboard software. The prospective teachers were still instructed to create an original proportional reasoning technology lesson of 30 minutes based on definitions of proportional reasoning as mathematics concepts that involve unit rates, mixtures, or scaling ([10], [11]), but the use of interactive whiteboard technology was mandatory for their presentations.

All of the prospective mathematics teachers had a thorough background in mathematics, but no direct experience with the use of interactive whiteboards to teach mathematics. The pre-service teachers were familiar teaching mathematics with didactic activities (e.g. theorems, proofs,

textbook exercises), but not with teaching mathematics with technology and none of the pre-service teachers had ever personally used interactive whiteboard technology.

The prospective secondary mathematics teachers were given rudimentary explanations earlier in the semester to familiarize them with the mechanics of the interactive whiteboard software. After this brief instruction, the professor's role changed from instructor to facilitator for the remainder of the project.

The pre-service teachers' presentations were videotaped for the purpose of grading and further analysis. After the presentations, the pre-service teachers' peers were allowed to ask questions pertaining to the mathematics within their lessons as well as to how they created many of the graphic effects that were incorporated within their lessons. This presented a new dimension in mathematical inquiry for the pre-service mathematics teachers. That is, this activity allowed the pre-service teachers to experience the process of developing mathematics through a new technology that also encouraged them to share their ideas and promote independent thinking.

4.1 Results

The activities that the pre-service mathematics teachers displayed within their presentations covered a variety of instructional techniques and topics. Of the eleven prospective teachers in the class, eight began their presentation by defining a proportion, while only two pre-service teachers discussed ways to represent proportions. All of these eight pre-service teachers included examples of how to solve proportions, but with differing methods. Three future teachers demonstrated how to solve proportions by using cross multiplication, two showed how to use scalars to solve proportions, and a third illustrated how to perform both methods. One other pre-service teacher introduced the notion of using a pictorial representation to teach proportional reasoning, while another asked the class to look for a pattern as a means of finding a proportional relationship. The class percentages pertaining to the method of solving proportional reasoning computational problems are listed in Table 7.

Method of Solving Problems	Class Percentage
Cross Multiplication	50.0%
Scalars	37.5%
Pictorial Representation	12.5%
Finding a Pattern	12.5%

Table 7. Methods of Solving Proportional Reasoning Computational Problems
(from the eight included in the presentations)

Having the class solve proportional reasoning word problems (as opposed to solving computational problems) was the most popular application of proportional reasoning utilized by the pre-service teachers with eight of them choosing to use this method. Examples of these word problems included five recipe examples, a medical dosage problem, two architectural problems, a shadow reckoning problem, and a comparison shopping example.

The second most popular type of problem, selected by six of the pre-service teachers, involved the dilations of similar triangles. Three of the prospective teachers displayed pictures of triangles with lengths of certain sides given and asked the class to find other missing measurements. However, three pre-service teachers just displayed large triangles on the interactive whiteboard with no given measurements and required the class members to come to the whiteboard and use the ruler tool to make actual measurements, while two pre-service teachers extended the activity and had the class members use the protractor tool to make angle measurements as well. The latter two prospective teachers also included the use of the graph paper, ruler, and protractor tools as a preliminary activity to the use of trigonometric functions to solve similar triangle problems.

There were several other interesting questions and applications displayed in the interactive whiteboard presentations. One pre-service teacher asked if a decimal or a percent could be represented as a proportion. This question prompted a lively discussion within the classroom. Another pre-service teacher discussed proportions calculated from Fibonacci numbers and their relationship to Pascal's triangle, while a different pre-service teachers illustrated the use of similar triangles and proportions by Renaissance artists and the use of proportions and trigonometry to solve problem involving light refraction. The class percentages for the components of the presentations are listed in Table 8.

Components of the Presentations	Class Percentage
Definition of Proportional Reasoning	72.7%
Solving Computational Problems	72.7%
Solving Word Problems	72.7%
Dilations of Similar Triangles	54.5%
Ways to Represent Proportions	18.1%

Table 8: Components included in the Presentations

Although interactive whiteboard technology is becoming more accessible, successful classroom use still depends on how the practitioner utilizes it within the classroom. This interactive whiteboard technology project offered the pre-service teachers a totally new experience in creative efforts with an opportunity to formulate, communicate, and present original work. Remarkably, observations and analysis of the video presentations showed a rapid progression from didactic to interactive teaching approaches for these prospective secondary mathematics teachers with the use of the interactive whiteboard technology.

5. Conclusions

As shared by the teacher in the first case study, the key success of the use of the virtual manipulative does not lie solely on the power of the manipulative. The provision of the ICT infrastructure under mp1 and the establishment of the baseline ICT standards under mp2 have both contributed to the right tone of learning environment to tap on the power of the virtual manipulative to enrich and transform the learning environments of the students. This in turn enriches the learning experiences of the students.

Future teachers need to explore and develop lessons with educational technology to enrich the mathematical experiences of their students. This paper presented components of a senior-level mathematics education seminar for prospective secondary mathematics teachers that offered an experience in the creative work of presenting original lessons with technology. The expectation is that the pre-service mathematics teachers who participated in the seminar will enter their future classrooms with a plethora of experiences and with valuable insights of how to use technology to explore and investigate new mathematical situations. After having completed the mathematics education seminar, the prospective secondary mathematics teachers are ready to employ changes in their method of teaching mathematics. They now have both the mathematical understanding and knowledge of technology to implement it.

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