TEACHING TEACHERS WITH MATHEMATICAL ACTIVITIES USING THE GRAPHIC CALCULATOR

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Abstract. *This paper explains the teaching method used in introducing graphic calculator to teachers and pre-service teachers in a master's level course on mathematics and technology at the School of Mathematical Sciences, USM, Penang, Malaysia. This result is obtained after 5 weeks of lab exploration done in the first semester of the 2009/2010 Academic Session. Topics were mostly from linear algebra and calculus and activities were done based on materials found in the Texas Instrument (TI)'s websites and mathematics books. In this paper, we present 3 out of the 5 explorations done in the lab sessions. The conduct of each lab, which explored a specific topic of mathematics, is presented briefly. At the end of the 5-week exploration, students were asked to answer survey questions regarding their experiences with the hand-held tool. Analysis of the student survey is presented briefly to discuss the students' perception towards the graphic calculator technology before and after the exploration.*

1   Introduction

Mathematics and Technology is one of the courses offered as an option for students to take towards obtaining the degree of Masters of Science (Teaching of Mathematics) by coursework at the School of Mathematical Sciences, USM, Penang, Malaysia. The purpose of this course is to educate the students, who are mostly in-service teachers from schools around Malaysia, on mathematical software and hand-held tools as helpful aids in teaching mathematics.

Some public schools in Malaysia have been provided with graphic calculators by the government; however, without proper exposure and guidance, these teachers are oblivious on how to implement lesson plans for their students on the use of graphic calculators. According to a study by Dewey, Singletary and Kinzel (2009), even though technology has been proven to be a critical component for conceptual development and problem-solving as well for exploring patterns and relationships in mathematics, teachers are still unsure of how to reconcile technology's capabilities with mathematics curriculum. It is for this reason that the Mathematics and Technology course were offered so that teachers who are doing the MSc program can benefit from knowledge given in this course and hopefully they will be able to prepare appropriate lesson plans based on what they have learnt in this course. However, Balling (2004) concluded from her project that sufficient resources must be devoted to teacher training and development to see the full potential of graphic calculator as a teaching tool. Therefore, it is hoped that the Malaysian government will put an effort to provide mathematics teachers, especially at the secondary level, with opportunities and encouragement to learn more on how to use graphic calculator in the classroom by short courses or workshops.

Graphic calculator is more than just a tool for computation like the scientific calculator. Nonetheless, graphic calculator does help the students on higher level computation for example calculating matrix multiplication for which the scientific calculator cannot do. According to a study

done by Sulaiman (2003), with the graphic calculator, students could leave out all the difficult calculations and concentrate more on the interpretation of the results. This also gave them time to visualize and explore the underlying concepts of the specific topics. So lab activities must be appropriately chosen to meet these objectives. Efforts should be focused on the use of technology to enhance learning, and not only to learn about technology. Although both are worthy of attention, it is important to distinguish between technology as a subject area and the use of technology to facilitate learning about any subject area (Ali, Seth, Zainuddin, Kassim, Sulaiman, Haili, 2002).

2   Course Description

For the first half of the semester, the students were exposed to the graphic calculator technology. Each week, an activity was done in a 3-hour lab on specific topics of mathematics at a secondary or high-school level. Each student was loaned either a TI-92 Plus or Voyage 200 graphic calculator at the beginning of the course for usage in the lab and at home. Handouts were given for each activity consisting detailed instruction on the hands-on activity, an exercise and a take-home assignment. The first half-hour was filled by giving necessary background on mathematics needed for the specific activity. Students were given a brief explanation on what will be done in the activity and what is the mathematical explanation behind the activity. Step by step, we walked through the activity with the students with our graphic calculator connected to the projector so the students could compare the result they obtained on their calculator's home screen with the one shown by us. This activity would take approximately one hour to complete depending on how fast the students could follow. The lab time remaining was spent on exercises handed out afterwards. Before the end of the lab, we handed out an assignment for that week to be completed by the students before the next lab session.

3   Lab Session

23 students out of the 27 registered for the course were teachers who were currently teaching at either a primary school or secondary school in Malaysia. Most of them had taken mathematics at a bachelor's degree level. However they had forgotten some background knowledge needed for the mathematical activities. Thus, we would start a lab session with some mathematical background significant to the activity that they would be doing. For example, in the activity on coding secret messages, modular arithmetic was used in the conversion of characters and numbers representing a secret message(described further in 3.5). We showed the students that, for example 7 is equivalent to 0 modulo 7(denoted $7 \equiv 0 \pmod{7}$) because 7 is divisible by 7, and this would be the same for 14, 21, 28 etc. Then, 8 has remainder 1 when divided by 7 so $8 \equiv 1 \pmod{7}$, and this would be the same for 16, 24, 32 etc. So, all the positive integers are equivalent to 0, 1, 2, 3, 4, 5 or 6 modulo 7. In the hands-on activity, a particular question would be given (in the handout) and we would show the students on how to solve it. The solution usually involved answering several steps where one small question required them to enter one command in the graphic calculator and then they would write down the result. We would wait for all the students to finish one step before going to the next one. In some cases, we might ask the students to complete the steps by themselves. This was to ensure that they do the activity with their graphic calculator and not just copying what they saw on the projector screen. When the activity had finished, we distributed an exercise sheet for the students to do in the remaining hours of the lab. This was when we could interact with the students more effectively and made sure that they understood the objective of that day's exploration. Then, an assignment would be distributed before the session ended.

3.1 Solving Systems of Linear Equations

3.1.1 Preliminaries

The activity is divided into two parts. In the first part, we use the simultaneous linear equations to solve the linear system. We show two methods in particular: elimination-by-addition and substitution. An example of a system with two equations was given and we showed how to find the solution by both methods. The second part of the activity is solving a linear system by augmented matrix and elementary row operations. Of course, this part is more useful for a system with more than 3 equations and 3 unknowns. The students were shown how to solve the system by Gaussian Elimination and Gauss-Jordan methods.

3.1.2 Guided Activity

A real-world mathematical problem was given whose solution could be obtained by converting to a system of simultaneous linear equation. First, the students were taught on how to define the variables x and y for the linear equation $ax + by = c$, where a, b and c are constants. Then, they were shown how to convert the problem into a system of two equations. Thus, the solution to the problem was just the solution to the system. Two equations were obtained and so a unique solution was possible. The first method was elimination-by-addition. The step-by-step instruction using the graphic calculator was shown in implementing the elimination-by-addition method. This was just a sequence of **"solve("** and ⌐ command beside the obvious "+" operations. This is similar to the substitution method. At the end of that, the students were shown that the graphic calculator had a special command **"simult("** that can give the solution immediately but only if the system has *n* equations and *n* unknowns (and of course it must be a consistent system).

In the second part, another real-world problem was given. The problem was more complex and had to be explained slowly to ensure that the students understood the system of equations obtained. We showed the commands in the graphic calculator that correspond to the three elementary row operations. The system was then changed into the augmented matrix form which was then defined in the graphic calculator. Then, following the step-by-step Gauss-Jordan method, we showed the students how to determine the solution to the system. We also showed afterwards that the graphic calculator also enabled them to obtain the row-reduced echelon form of the augmented matrix (the final matrix of the Gauss-Jordan method) immediately by using the **"rref("** command.

3.1.3 Exercises

Plenty of questions were given as exercises: 2 on solving by substitution, 2 on elimination-by-addition, 2 on using the **"simult("** command, 2 on doing back-substitutions with an augmented matrix in row echelon form, 1 on solving by Gaussian Elimination and 1 on Gauss-Jordan Method. All were required to be done using the graphic calculator; however, the exercise sheet was formatted to include guided blanks on how to answer. An example of a question was: "*John has the face amount of $5.60 in his antique coin collection, all in dimes and nickels. He knows he has 80 coins altogether. How many dimes and nickels are in John's coin collection?*" The students were required to solve this using the elimination-by-addition method. We would guide them on how to formulate the problem into a system of two linear equations. For one thing, the word "dime" and "nickel" might be unfamiliar to them since Malaysian monetary unit did not have such terms for 10 cents and 5 cents (changing the question to have "*10 cents*" and "*5 cents*" might not be as fun). The

students who had taught this topic in the secondary or high school could formulate the problem more quickly than the others. However, once they had obtained the system, they only needed to use the graphic calculator to solve. So, these students learnt that although graphic calculator helped them in computing for the answer, they still needed to understand the problem first before they could use the tool.

3.2 Graphing Rational Functions

3.2.1 Preliminaries

This was a little bit hard for the students to follow because it was a topic taught at the high school level. Some of these students had not taught mathematics at this level. For the preliminaries, we gave the definition of a rational function. An example of an improper rational function $f(x) = \dfrac{3x+1}{x}$ was shown then rewritten into a sum of a polynomial and a proper rational function which was $f(x) = 3 + \dfrac{1}{x}$. Then we showed how to find the domain, the zeros, x-intercepts and y-intercept, the vertical and horizontal asymptotes of f. The students were also given a review on **limits** of functions. All these information were used to sketch the graph of f.

3.2.2 Guided Activity

The handout which contains the activity was based on the one published by C. Brueningsen, Turley and E. Brueningsen (1999). We gave a rational function whose numerator has the same degree as the denominator, i.e 2. Using the **"factor("** command, the graphic calculator returned the function as $f(x) = \dfrac{2.(x-1).(x+1)}{(x-2).(x+2)}$. From this factorisation, the zeros of the numerator were 1 and -1 which were the x-intercepts of the function. Then the zeros of the denominator was 2 and -2 which entailed that the vertical asymptotes were at $x = 2$ and $x = -2$. Using the "**limit("** command, we found that $f(x)$ tended to ∞ as x approached -2 from the left and 2 from the right and that $f(x)$ tended to -∞ as x approached -2 from the right and 2 from the left. Also, $\lim\limits_{x \to \pm\infty} f(x) = 2$ so $y = 2$ was the horizontal asymptote. When $x = 0$, $f(x) = \dfrac{1}{2}$ so the y-intercept was at $\dfrac{1}{2}$. The information was used to graph the function by hand. This was then verified by graphing the function on the graphic calculator.
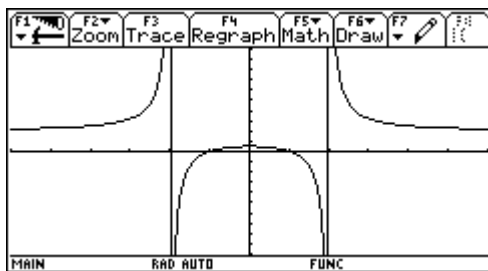

Figure 1: The graph of $y = \dfrac{2.(x-1).(x+1)}{(x-2).(x+2)}$

3.2.3 Exercises

Similar questions as the activity above were given as exercises. The rational functions varied in terms of degrees of the numerator and the denominator. As an example, a function had degree 2 for both the numerator and denominator and using **"factor("** command we obtained $f(x) = \dfrac{-2.(x-3).(x+3)}{x^2+6}$ which gave one horizontal asymptote but no vertical asymptote. This was because the function was defined at all values of x. For the functions with the numerator's degree smaller than the degree of the denominator, $y=0$ was the horizontal asymptote. The reason was that the denominator would get very large very quickly as the value of x increased without limit compared to the numerator and so the function would tend to 0 as x tended to the extremes. For the functions with the numerator's degree larger than the degree of the denominator, there was no horizontal asymptote. This made sense because the function would then increase without limit as x tended to the extremes. For the function which had the same degree for numerator and the denominator, as in the function shown above, the horizontal asymptote was the quotient of the leading coefficients between the numerator and the denominator. So, for $f(x) = \dfrac{-2.(x-3).(x+3)}{x^2+6}$, the horizontal asymptote would be at $y=-2$. The students were required to graph each function using the graphic calculator after evaluating the zeros, intercepts and limits.

3.3 Least Squares

3.3.1 Preliminaries

For a linear system with no solution, we use the least squares method to solve. The students were given an inconsistent system with 3 equations and 2 unknowns. We wrote down the matrix equation $Av = b$ representing the system and defined the matrices A, v and b. The normal equation $A^T A v = A^T b$ was used to find an approximate solution and this equation can be rearranged to give a unique solution $v = \left(A^T A\right)^{-1} A^T b$ .gave a table showing a firm's spending amount on advertising and its total sales.

3.3.2 Guided Activity

We gave a table showing a firm's spending amount on advertising and its total sales for each of the first six months of the year . The x values represent the amounts spent on advertising and the y values are the sales amounts. The students were shown how to put the set of points $(2.4, 33.8)$, $(3.6, 34.7)$, $(3.6, 35.5)$, $(4.1, 36.0)$, $(4.7, 37.5)$ and $(5.3, 38.1)$ in the data editor of the graphic calculator and then plot the scatter plot of the data. The students could see that the points would not form a straight line.

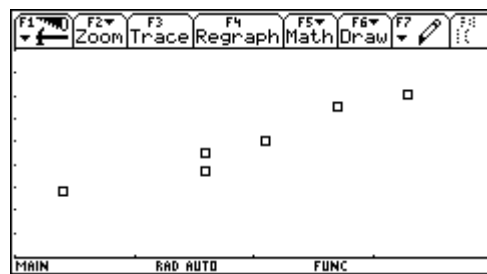**Figure 2: Points entered in the Data Editor**   **Figure 3: From Data Editor, the scatter plot was produced**

So, it followed that a best-fit line must be found using the least squares method. With the general equation of the line, $y = mx + c$, we have a system of 6 equations with unknowns m and c. We identify the matrices A (variable name *adv1*), and b (variable name *yadv*) from the system. The students were shown how to define these matrices using the matrix editor. Then, as shown in Figure 6, we use the formula $v = \left(A^T A \right)^{-1} A^T b$ in the graphic calculator to obtain the approximate solution for (m, c).

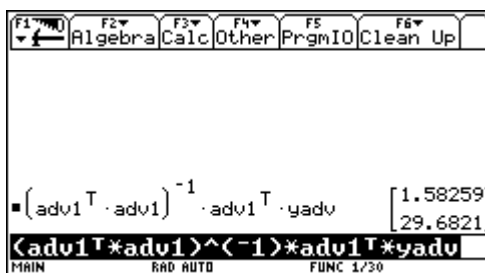The linear least squares equation that models the data was: $y = 1.58259x + 29.6821$.

Figure 4: We calculated $v = \left(A^T A \right)^{-1} A^T b$ **which gave the value**

for the slope $m = 1.58259$ **and the *y*-intercept** $c = 29.6821$

In the graphical view, we define this function on the **Y=** editor (Figure 7). Then we plot a scatter plot of the data and the linear least squares function on the same plot as shown in Figure 8.
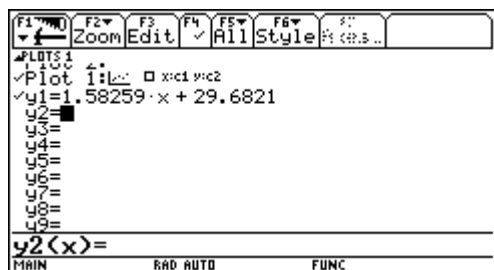

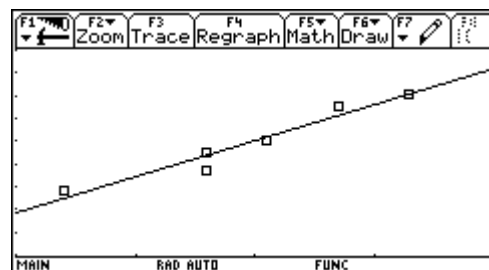**Figure 5: Equation** $y = 1.58259x + 29.6821$ **Figure 6: The line** $y = 1.58259x + 29.6821$

was defined in the Y= editor   **was plotted together with the scatter plot**

From the graph, we observe that the approximate equation we obtained fits very well with the data. The students were then required to predict future sales for the rest of the year if the company plans to spend $6000 on advertising in July keep increasing the amount they spend on advertising by $500 a month. For this, the students would use the model to predict the value of *y* when a value for *x* was given.

3.3.3 *Exercises*

The exercise contained real-life situations where data are collected and the students have to approximate the linear or quadratic equation that models the data. Some questions require them to predict the future value by calculations and also graphically. As an example, one question provided the data on a study done on five tracts of a wildlife preserve to determine the rate of reproduction of deer. A record of the number of females in each tract and the percentage of them having offspring are shown in Table 1:

Table 1. Percentage of female deers having offspring in relation to their number in 5 tracks.

Number *(x)*	80	100	120	140	160
Percent *(y)*	80	75	68	55	30

From the data, students could see that the percentage of females with offspring reduced as their number increased. Using the method of least squares, the quadratic model obtained was $y = -0.0082x^2 + 1.371x + 21.89$. The students then plot the graph over the scatter plot of the data to observe how the curve fits the data. Using the model, they predicted that 18% if of the females would have offspring if there are 170 females in a tract.

4   Assignments

An assignment was given after the students have completed the exercises in each lab session. The assignment must be completed and submitted before the next lab session. The assignment contained more difficult questions and some require the students to apply other mathematical methods to solve. For example, in the activity on linear equation systems, a system of two equations and two unknowns (which has a unique solution) were given and the students were required to reduce it using the Gauss-Jordan method; however, with each elementary row operation(ERO) done, they had to describe what happened to the lines graphically and how it relates to the solution.

In the assignment regarding least squares, for questions where they have to choose which polynomial equation that best modelled the data, they have to form their own opinion based on the graphs of the equations. This is an example where the graphic calculator helps in producing accurately drawn graphs so that the students can make a more convincing judgement. In the rational function assignment, students were also encouraged to present their work to include displays from the graphic calculator. So they were taught on how to install the appropriate softwares into their computer, one in particular was the *TI-connect*. The software was found in the TI-resource CD provided by the lab and at this opportunity we also encouraged the students to try the available *Flash* softwares in the TI-resource CD.

5   Evaluation

As mentioned above, an assignment sheet was distributed before the end of each lab. The sheet contained at most 3 questions related to what they had done in the lab. We gave assignment questions which would require the students to do further exploration by themselves and not just answer similar questions as in the exercises. The students were given one week's time to complete each assignment. The allocation marks for the assignments were 30% out of the 100% coursework marks and the total marks for each assignment varied. A test was conducted one week after they had their last lab session. Questions were set up in a way that the students must answer on the question sheet itself. They were not allowed to look at their notes. We tested the mathematical knowledge that the students have achieved and also the skill that they acquired in using the graphic calculator technology. The test was divided into two parts. Part A involved filling in the blanks (where they were required to write out the commands entered in the graphic calculator and the results of that action). Part B was where they were required to solve mathematical problems. So, they must remember the steps that were shown in the labs. This test contributes to 20% of the coursework marks. The remaining 50% would be accumulated after the second half of the course which involved learning mathematical softwares and constructing WEB sites. This part would be handled by another instructor.

6   Survey Summary and Analysis

In the final lab session, a survey was given out to the students. The summary given in Table 1 is the report of findings following the survey.
1. Before taking this course, did you feel that you had a good understanding of the theories and concepts of (i) f*inding a linear model from two points*, (ii) *methods of solving system of linear equations*, (iii) *evaluating behaviour of rational functions*, (iv) *method of least squares*, (v) *encrypting messages by matrix multiplication.*
2. Before taking this course, did you have a high level of interest in using a graphic calculator?
3. Before taking this course, did you have a high anticipation to teach mathematics with the aid of graphic calculators?
4. Before taking this course, did you feel that learning and using the graphic calculator could be fun?
5. Before taking this course, did you appreciate the usage of graphic calculators in solving real world problems?
6. After attending this course for the last 5 weeks, has your understanding or appreciation of mathematics changed?
7. Has the graphic calculator enhanced your understanding of the topics in mathematics explored?
8. Has your experience working with the graphic calculator during the past 5 weeks given you an insight into the value or limitations of integrating technology into the classroom?

Table 2. Percentage respondents to graphic calculator survey
Number of respondents: 27 out of 27 (100%)

Item	1^{i}(%)	1^{ii}(%)	1^{iii}(%)	1^{iv}(%)	1^{v}(%)	2(%)	3(%)	4(%)	5(%)	6(%)	7(%)	8(%)
SA/Agree	88.9	92.6	85.2	63.0	59.3	96.3	88.9	92.6	96.3	96.3	92.6	81.5
Not Sure	11.1	7.4	11.1	22.2	11.1	3.7	7.4	3.7	3.7	3.7	7.4	18.5
SD/Disagree	0.0	0.0	3.7	14.8	29.6	0.0	3.7	3.7	0.0	0.0	0.0	0.0

The results show that before going through the graphic calculator exploration, the students' understanding on the topics of mathematics were good in the beginning (except the least squares method and secret codes) and this is comprehensible as most of them were in-service mathematics teachers. Nonetheless, 96.3% of the students agreed that after using the graphic calculator for 5 weeks they better understood the topics of mathematics explored and 92.6% felt that the graphic calculator has enhanced this understanding. Furthermore, 81.5% realized the value and limitations of integrating technology in the classrooms.

Other prevailing views are:

❖ Students felt that the exploration with the graphic calculator were exciting as they were able to solve difficult problems easily and the solutions were convincing.

❖ Students felt that they acquired better understanding about the behaviour of rational functions after being able to visualize them graphically.

❖ Students who were teachers at the high school level felt that the graphic calculator technology is applicable in teaching and learning since it helps to promote the understanding of difficult concepts.

❖ Students enjoyed the special commands they could use with the graphic calculator which enabled them to get the answers easily for example the **"expand("**, **"factor("**, **"rref("** and **"solve("** commands.

❖ Students felt that the activity on secret codes were a new and meaningful experience to them.

❖ Students felt that they were ready to introduce this technology in their own classrooms.

7   Concluding remarks

The most common view from the survey was that the students felt 5 weeks were not enough for them. They wished that more time were allocated for the graphic calculator exploration. However, they were told that while they still have the graphic calculator, they could visit the websites which contain similar activities on other topics of mathematics and do their own exploration. Some students even thought of buying the graphic calculator Voyage 200 (although it is expensive) because they wanted to continue learning the tool after finishing the course. There were also some students who decided to do their master's projects regarding the use of graphic calculator at the schools they were teaching. This showed that the students embraced this technology and would like to spread their knowledge to other people. With these views, we felt that the objectives of the course (for the graphic calculator part) were achieved and we hope that the course will continue to influence the use of this technology to more teachers in Malaysia.

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