

The Graphing Calculator (HP39GS) Based Numerical Experiment

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Abstract: *The college mathematics teaching program aims at cultivating undergraduate's capabilities of innovation, mathematical modeling and problem solving by means of the computer. The Graphing Calculator is a professional and highly efficient tool that meets the need of the teaching program. With the help of the Graphing Calculator, we have designed and implemented many mathematical experiments in the past ten years, including calculus, linear algebra, probability and statistics, and so on. Some of these cases are described in detail in the paper.*

1. The role of college mathematical education

Mathematical education plays an important role in cultivating one's abilities of induction, deduction, creation and mathematical modeling. Induction is an ability to describe practical problems in a mathematical way, deduction requires logical reasoning, while the creation and mathematical modeling focus on analyzing, modeling and resolving a problem completely with the help of computer (e.g. graphing calculator).

Generally speaking, higher mathematics education of China focuses on obtaining knowledge of theories, methods, induction and deduction skills, rather than cultivating abilities of creation and mathematical modeling. To modify this situation, we must update our idea of education; introduce some new contents and methods which are supported by modern technologies in teaching program.

It is useful to combine mathematical principles and mathematical experiments, both studying principles and cultivating the abilities of creation and modeling will benefit from a mathematical experiment.

2. The role of mathematical experiment

The mathematical experiment is a product of Chinese educational reformation. It is fortunately a common view taking a mathematical experiment as a procedure of analyzing and resolving problems with the aid of the computer (e.g. graph calculator) by oneself. The function of the mathematical experiment is mainly about math modeling, computer applying and creation, and has been characterized by:

- (1) Presenting an actual problem. Each experiment was designed for a special case. In a mathematical experiment, student must completely resolve an actual problem by themselves.
- (2) Using computer (e.g. graphing calculator) and mathematic software. According with the complexity of the model established, we will find the solution by the aid of computer.
- (3) Student-centered. In a mathematical experiment, student will experience, explore and practice a problem freely and effectively.

3. The scheme of mathematical experiment

3.1 Mathematical experiment categories

Mathematical experiment can be divided into three categories. The first functionality of the graphing calculator is to deal with the simple computational problems of linear algebra, calculus, probability theory and statistics, the second aims at simulative proof, and the third emphasizes the whole procedure of analyzing and resolving some typical problems using the graphing calculator. The first and the third cases could be founded in [1]. The second category will be discussed later. Some experimental items are listed in Table 3.1.1.

Table 3.1.1 Experimental items and aims

Experimental items	Experimental aims	Subjects
approximate the area of a round with regular polygon	understanding the definition of limitation	Calculus
definition of definite integral	understanding method of exhaustion	Calculus
prove Lagrange's Mean Value Theorem	Learning about proof method of math theory using Graphing Calculator	Calculus
prove Cauchy's Mean Value Theorem	Learning about proof method of math theory using Graphing Calculator	Calculus
prove Definite Integral Mean Value Theorem	Learning about proof method of math theory using Graphing Calculator	Calculus
approximate a transcendental function by polynomials	Learning about storing a transcendental function into memory	Calculus
accelerate series' convergence	Finding methods to accelerate series' convergence	Calculus
compute determinants by it's definition	Understanding the role of constructive definition in computing	Algebra
image transformation	Introduction of commonly used image transforming matrices	Algebra
design a clock	Learning about rotating transformation	Algebra
program the Schmidt's Orthogonal Method	Learning about recursive programming method	Algebra
Toss an uniform coin	Interpreting probability using statistical view	Probability theory
studying Bertrand paradox	Understanding uniform hypotheses in geograph probability	Probability theory
approximate the probability by random test	Learning about methods of random test designing	Probability theory
approximate the mathematical expectation of a random variable by random test	Learning about methods computing the mathematical expectation of a random variable	Probability theory
Galton board experiment	Learning about Normal distribution	Probability theory
prove the Bernoulli's Great Number Law simulation	Comparing probability with frequency	Probability theory
drawing histogram of frequencies of sample	Studying probability distribution of a sample	Mathematical statistics
Chi-square testing of normal distribution	Learning about testing methods of distribution	Mathematical statistics

3.2 A clock—an example

(1) Mathematical model

The clock looks like a circle with radius 2 and centered at (0,0), its second hand, minute hand and hour hand are begin at coordinate (0,0), and end at (0,1.7), (0,1.5) and (0,1), respectively. These end point coordinates are separately stored in matrix M_1 , M_2 and M_3 . The second hand always goes around the point (0,0) clockwise with the interval $\frac{\pi}{30}$. After the first rotation with angle $\frac{\pi}{30}$, the begin point's coordinate was not changed, but the end point's coordinate is updated by

$$\begin{bmatrix} \cos \frac{\pi}{30} & \sin \frac{\pi}{30} \\ -\sin \frac{\pi}{30} & \cos \frac{\pi}{30} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1.7 \end{bmatrix}.$$

Let rotating matrix be $M_4 = \begin{bmatrix} \cos \frac{\pi}{30} & \sin \frac{\pi}{30} \\ -\sin \frac{\pi}{30} & \cos \frac{\pi}{30} \end{bmatrix}$, when we rotated the second hand k times, the

end point's coordinate could be represented as $M_4^k * M_1$, where $M_1 = \begin{bmatrix} 0 \\ 1.7 \end{bmatrix}$.

Let k values 1,2,...,60, respectively, we draw lines from (0, 0) to $M_4^k * M_1$ with interval of 1 second, then it will show the animation of the running second hand. Set the counting variables M, F and S, the values of M, F and S correspond to the times the second hand, the minute hand and the hour hand have moved, respectively. Whenever $M \equiv 0 \pmod{60}$, F increases by 1, and we draw the minute hand same as the second hand once, whenever $F \equiv 0 \pmod{60}$, S increases by 1, and so does the hour hand. The clock begins at 0 second, 0 minute and 0 hour.

(2) Source code

```
2 ▶ Angle: // set the angle to radians
// draw the clock face
-6 ▶ Xmin:
6 ▶ Xmax:
3 ▶ Ymin:
3 ▶ Ymax:
[[0], [1.7]] ▶ M1: //the end point coordinate of the second hand
[[0], [1.5]] ▶ M2: //the end point coordinate of the minute hand
[[0], [1]] ▶ M3: //the end point coordinate of the hour hand
0 ▶ M: // count the times the second hand has gone
0 ▶ F: // count the times the minute hand has gone
0 ▶ S: //count the times the hour hand has gone
[[COS(π/30), SIN(π/30)], [-SIN(π/30), COS(π/30)]] ▶ M4:
// rotation matrices of the second hand and the minute hand
[[COS(π/6), SIN(π/6)], [-SIN(π/6), COS(π/6)]] ▶ M5: //rotation matrix of the hand
ARC 0; 0; 2; 0; 2*π: //draw the circle of the clock
```

```

LINE 0; 0; M1(1,1); M1(2,1): //draw the second hand
WAIT 1;
WHILE 1 REPEAT // repeating condition, it is always true here
TLINE 0; 0; M1(1, 1); M1(2, 1): //delete original second hand
M4*M1►M1: //compute new coordinate of the second hand
M+1►M: //increase by 1
LINE 0; 0; M1(1, 1); M1(2, 1): //update the second hand
LINE 0; 0; M2(1, 1); M2(2, 1): //draw the minute hand
IF M==60 THEN //condition of increase F by 1
TLINE 0; 0; M2(1, 1); M2(2, 1)://delete original minute hand
M4*M2►M2: //compute new coordinate of the minute hand
F+1►F: //increase by 1
0►M: //set M=0
LINE 0; 0; M2(1, 1); M2(2, 1): //update the minute hand
END:
LINE 0;0;M3(1, 1);M3(2, 1): //draw the hand
IF F==60 THEN //condition of increase S by 1
TLINE 0;0;M3(1, 1);M3(2, 1): //delete original hand
M5*M3►M3: //compute new coordinate of the hand
S+1►S: //increase by 1
0►F: //set M=0
LINE 0; 0; M3(1, 1); M3(2, 1): //update the hand
END:
//set display format as 2 digits
IF S>9 THEN
DISPXY -6; 3; 1; "Hour="S: //display S directly if S ≥ 10
ELSE
DISPXY -6; 3; 1; "Hour=0"S: //display S after a 0.
END:
IF F>9 THEN
DISPXY -6; 2; 1; "Minu="F: //display F directly if F ≥ 10
ELSE
DISPXY -6; 2; 1; "Minu=0"F: //display F after a 0.
END:
IF M>9 THEN
DISPXY -6; 1; 1; "Seco="M: //display M directly if M ≥ 10
ELSE
DISPXY -6; 1; 1; "Seco=0"M: //display F after a 0.
END:
IF S==24 THEN
0►M:
0►F:
0►S:
END:

```

END:

(3) Running result

We have a true representation of an analog clock. To terminate the program, press buttons **ALPHA** **ON**. The running result of a digital clock is shown in figure 3.2.1.

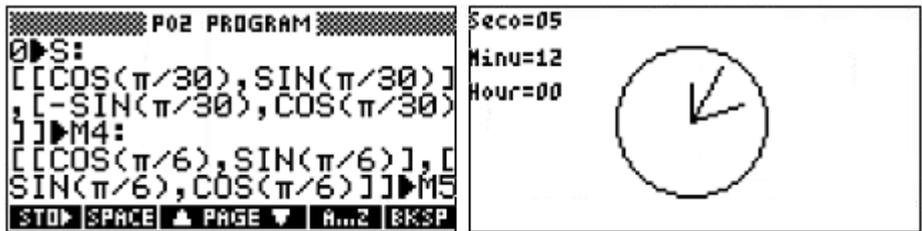


Figure 3.2.1 The running result of a digital clock

3.3 Galton board---another example

A four layered Galton board is shown in figure 3.3.1. There are 10 solid points in the figure, each point represents a nail. The bottom of the board divided into 5 evenly-spaced rectangular slots, numbered by 0 , ..., 4. For n values from 1 to 4, there are n evenly-spaced nails in *n*th layer. These nails are arranged in staggered order. In the middle of the upper edge, there is a funnel into which balls can be poured. The funnel is located precisely above the central nail of the first row so that each ball would fall vertically and directly onto the uppermost point of this nail's surface. Each time a ball hits one of the nails, it can bounce right (or left) with probability 1/2. Each ball will hit only one nail in each layer, and fall into a slot. We have totally 500 balls poured and gave a different number of balls in each slot.

Question 1: Let *X* be the number of slots, find the distribution of balls in slot *X* .

Question 2: With total *n* balls, find the number of the ball in each slot.

Question 3: When *n*=500, find the number of the ball in each slot and draw out the histogram of these heights.

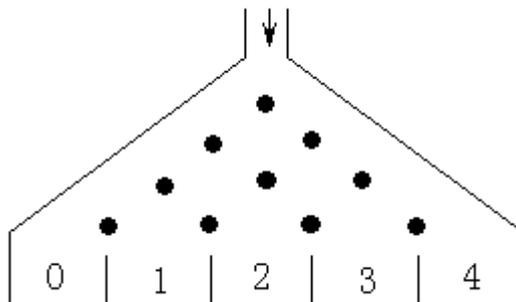


Figure 3.3.1 Four layers Galton board

(1) Designing idea

Each time a ball hits one of the nails, it can bounce right (or left) with probability 1/2. Let bouncing left be 0 and bouncing right is 1. It can simulate the process of falling ball by a random number in the set (0,1). Whenever we received a number less than 0.5, it means the ball is bouncing left, otherwise, it means it is bouncing right. After a ball hit a nail 4 times, we have a binary string containing 4 characters and the sum of the 4 characters will be the number of slot into which the ball will fall.

(2) Mathematical model and solution

Let X be the number of slot. Then X values in the set $\{0,1,2,3,4\}$. If $X=2$, there are two zeros and two ones in the binary string. When the ball hit a nail, 0 and 1 occurs independently, so

$$P\{X = 2\} = C_4^2 \left(\frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right)^2 = C_4^2 \left(\frac{1}{2}\right)^4.$$

Similarly, $P\{X = 0\} = C_4^0 \left(\frac{1}{2}\right)^4$, $P\{X = 3\} = C_4^3 \left(\frac{1}{2}\right)^4$, $P\{X = 4\} = C_4^4 \left(\frac{1}{2}\right)^4$.

When we have poured all balls in, the numbers of ball in each slot will be

$$nC_4^0 \left(\frac{1}{2}\right)^4, \quad nC_4^1 \left(\frac{1}{2}\right)^4, \quad nC_4^2 \left(\frac{1}{2}\right)^4, \quad nC_4^3 \left(\frac{1}{2}\right)^4, \quad nC_4^4 \left(\frac{1}{2}\right)^4.$$

Table 3.3.1 shows the value of each slot with total 500 balls.

Table 3.3.1 The values of each slot with total 500 balls

slot number	0	1	2	3	4
height of ball in each slot	31.25	125	187.5	125	31.25

(3) Programming with the Graphing Calculator

Let array L1 describes the number of ball in each slot and L2 records the binary string which a ball experienced.

```

DISP 1; "PROGRAM RUN, WAIT...": //waiting
500►N: //initiate total balls N=500
{0, 0, 0, 0, 0}►L1: // initiate ball height in each slot with 0
FOR I=1 TO N STEP 1; // pour balls N times
{0, 0, 0, 0}►L2: //initiate binary string with 0000
FOR J=1 TO 4 STEP 1; //simulate process of ball falling
RANDOM►X: //generate a random number values in (0,1)
IF X≥0.5 THEN
1►L2(J): // bounce right
END:
END:
ΣLIST(L2) ►Y: //sum up array L2 and get slot number
L1 (Y+1) +1►L1 (Y+1): //increase ball height in slot Y by 1
END:
DISP 2; L1(1)“, ”L1(2)“, ”L1(3)“, ”L1(4)“, ”L1(5): //display ball height in each slot
//set display field wide and height
0►Xmin:
L1(3)+1►Xmax:
0►Ymin:
7►Ymax:
LINE 0; 0; L1 (3) +1; 0: //draw X axis which denote ball height in a slot
LINE 0; 0; 0; 5: //draw Y axis which denote slot number
FOR J=1 TO 5 STEP 1;
LINE 0; J; L1(J); J: //draw point (j,L1(j))
END:

```

FREEZE:

Figure 3.3.2 shows the result of a Galton board.

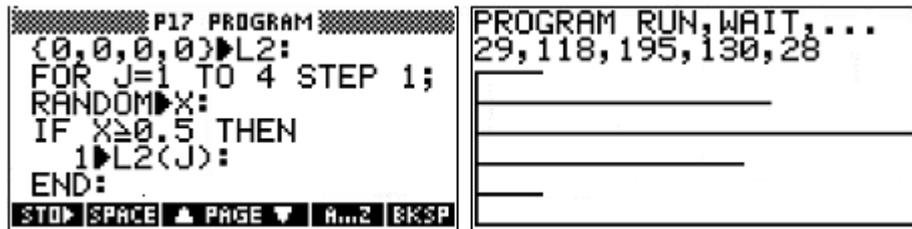


Figure 3.3.2 The running result of Galton board

Experiment result shows that the distribution of the heights of the ball approximates a normal distribution quite well and the histogram looks like a bell.

3.4 computing the area of a curved side trapezoid—the fourth example

Question: Finding the area of a curved side trapezoid which is enclosed by parabola $y = x^2$, lines of $x = 1$ and $y = 0$.

(1) scheme to resolve the problem

Let

$$x_i = 0 + ih = ih, \quad h = \frac{1-0}{n} = \frac{1}{n}, \quad (i = 0, 1, \dots, n)$$

be a partition of the interval $[0,1]$. We draw lines of $x = x_i$ ($i = 0, 1, \dots, n$), these lines then parted the original curved side trapezoid into n smaller trapezoids, the area of each small trapezoid can be approximated by areas of corresponding inner and external connection rectangles. Figure 3.4.1 gives an intuitive illustration of the scheme.

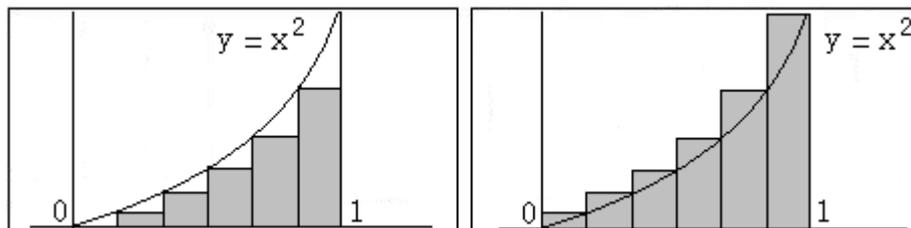


Figure 3.4.1 an illustration of the scheme

(2) approximating by Graphing Calculator

Let $F1(X)$ be formula of curve of original trapezoid, and variable X values in interval $[A,B]$. Let N be maximum number of parts of $[A,B]$, finally, we use H , L and W denote width, areas of inner and external connection rectangles of small trapezoid, respectively.

' X^2 '▶ $F1(X)$

0▶ A :

1▶ B :

INPUT N ; "N INPUT"; "N="; "ENTER DIVIDING NUM"; 10:

ERASE:

```

FOR J=1 TO N STEP 1;
  (B-A)/J ► H;
  A ► X;
  0 ► L;
  0 ► W;
  FOR I=1 TO J STEP 1;
    L+H*F1(X) ► L;
    W+H*F1(X+H) ► W;
    X+H ► X;
  END;
  ROUND(L, 4) ► L;
  ROUND(W, 4) ► W;
  IF INT(J/7) = J/7 THEN
    DISP 7; "N="J ", L = "L", W="W;
    WAIT 5;
    ERASE;
  ELSE
    DISP J-7*INT(J/7); "N="J ", L = "L", W="W;
  END;
END;
FREEZE:

```

The program will output the values of corresponding N, L and W. Figure 3.4.2 is an output with N=77.

N=64, L=.3256, W=.3412	N=71, L=.3263, W=.3404
N=65, L=.3257, W=.3411	N=72, L=.3264, W=.3403
N=66, L=.3258, W=.3409	N=73, L=.3265, W=.3402
N=67, L=.3259, W=.3408	N=74, L=.3266, W=.3401
N=68, L=.326, W=.3407	N=75, L=.3267, W=.34
N=69, L=.3261, W=.3406	N=76, L=.3268, W=.3399
N=70, L=.3262, W=.3405	N=77, L=.3269, W=.3398

Figure 3.4.2 an output with N=77

We can then draw a conclusion from the approximation that L increases with N monotonously and W is reverse. We also know that L and M are around 0.33 from the experiment, and so we get the area of original trapezoid of 0.33.

(3) Mathematical proof

Let A be area of the original trapezoid, let L_n and W_n be total areas of inner and external connection rectangles of small trapezoid with n parts, respectively. Because of

$$L_n = \sum_{i=0}^{n-1} x_i^2 h = \sum_{i=0}^{n-1} \left(\frac{i}{n}\right)^2 \frac{1}{n} = \frac{1}{n^3} \sum_{i=1}^{n-1} i^2 = \frac{1}{n^3} \cdot \frac{1}{6} (n-1)n(2n-1) = \frac{1}{6} \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right),$$

$$W_n = \sum_{i=1}^n x_i^2 h = \sum_{i=1}^n \left(\frac{i}{n}\right)^2 \frac{1}{n} = \frac{1}{n^3} \sum_{i=1}^n i^2 = \frac{1}{n^3} \cdot \frac{1}{6} n(n+1)(2n+1) = \frac{1}{6} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right).$$

Further more, $L_n < A < W_n$, and $\lim_{n \rightarrow \infty} L_n = \lim_{n \rightarrow \infty} W_n = \frac{1}{3}$,

we then have $A = \frac{1}{3}$.

4. Conclusions

The Graphing Calculator is one promising tool to aid mathematical teaching in the light of educational reform. Firstly, it is helpful in learning mathematical theory, cultivating practical consciousness, capability and scientific spirit. Secondly, it is also a useful way to improve the abilities of scientific calculation and computer application by combining mathematics theory with software programming. Finally, because of the portability of a graphical calculator, the method is convenient and interactive everywhere.

References

- [1] Shihong Jiang. *The Graphing Calculator and Mathematical Experiments[M]*. Science Press, Beijing, China, 2008
- [2] Shihong Jiang(2004). *The Graphing Calculator Aided Math Teaching[J]*. Chemical Engineering Education, 2004(4):72-74
- [3] Shihong Jiang. *Research on Modern Technology Aided Math Teaching[J]*. High technology and Industrialization, 2008(11):71-72
- [4] Shihong Jiang. *Research on Modern Technology Aided Applied talents Cultivating[C]*. The 14th Asia Mathematical Technology Annual, 2009, 179-183