# Acquiring Declarative and Procedural Knowledge with the Aid of the ClassPad Calculator

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**Abstract:** In this paper, some findings from the use of the ClassPad calculator in the classroom are reported. Observations were made of Year 8 students who were introduced to ClassPad calculators in a classroom equipped with a tablet computer connected to a digital projector. The observations related to the use of an algebra error locator which was installed in the calculator and to the use of the interactive geometry application of the calculator. The advantages of using the technology were considered with reference to two main types of knowledge known as declarative knowledge and procedural knowledge as described in the Dimensions of Learning framework. The paper focuses on the benefits that the use of technology can bring to the shaping phase of acquiring procedural knowledge.

### 1. Background

The use of technology in mathematics classrooms has become more than just a widely accepted practice. In the contemporary mathematics classroom, the use of technology has become an expectation. Consequently, there are ever-growing demands on mathematics teachers to develop the skills that are necessary to integrate the use of technology into their pedagogy (see [3]). Incorporating the use of technology into lessons has brought benefits to the students. Through the medium of technology, multiple strategies can be implemented to solve problems. The use of colour and movement aids students' conceptual development. Exploratory approaches may also be adopted more readily hence encouraging experimentation and the probing of conjectures. In this way, technology may support discourse that leads to greater depth of understanding (see [2]).

Benefits to mathematics teaching and learning do not flow automatically, however, from the mere presence of technology in the classroom. Glover, Miller, Averis and Dorr (see [4]) do well to remind us that "technology alone cannot support educationally effective change." (p.7). In this paper I seek to demonstrate ways in which technology can be introduced in order to aid the acquisition of knowledge. This process is enhanced by connecting ideas from learning theory to technology-based teaching practices in the classroom. By drawing on a model of learning known as Dimensions of Learning (see [6]), different types of knowledge can be outlined along with the processes through which they are acquired. Links can then be made between these processes and suitable ways of using technology in the classroom in order to help students gain new knowledge.

Dimensions of Learning is a learning framework that originated from research at the Association for Supervision and Curriculum Development in Alexandria in the United States. It is widely used in Queensland, Australia in both public and private schools across all subject areas. It is also used as the main model for learning at Central Queensland University. The five dimensions are: (1) attitudes and perceptions; (2) acquiring and integrating knowledge; (3) extending and refining knowledge; (4) using knowledge meaningfully; and (5) habits of mind. In the second dimension, knowledge is divided into two categories, *procedural knowledge* and *declarative knowledge*.

Procedural knowledge relates to the skills and processes that are relevant to a particular content area whereas declarative knowledge is concerned with concepts and facts. There are three phases associated with acquiring procedural knowledge. The first phase is called *model construction*. In this phase, the learner develops an idea of the steps involved. The second phase is called *shaping*. This phase involves the elimination of errors and the identification of efficient techniques. The third phase is called *internalizing*. In this phase the learner engages in practice until the procedure can be performed with relative ease. Declarative knowledge also has three phases. The first phase of acquiring declarative knowledge is called *constructing meaning*. In this phase the learner uses existing knowledge to build a new mental framework. The second phase is *organizing*. In this phase, the learner summarises and generalizes the new information. The third phase of acquiring declarative knowledge involves *storing* information in such a way that it can be easily retrieved.

## 2. Aims

The aims of this paper are to illustrate ways in which the ClassPad calculator can be used to enhance the process of acquiring knowledge, both procedural and declarative. By describing actual classroom examples, the intention is to show how the ClassPad can be used to assist learners in the shaping phase of acquiring procedural knowledge and in the organizing phase of acquiring declarative knowledge.

## 3. Methods

Twenty year 8 students of mathematics at Ormiston College in Queensland, Australia were involved. The students were identified and selected as students who were capable of staying a lesson ahead in their normal class and who would benefit from being given extra challenges. The students were divided into two groups and extracted from their normal classes for one lesson per week over a four month period.

Each student had access to a "ClassPad" calculator. These calculators have a touch sensitive screen and are operated by a stylus. They incorporate a variety of applications which can be accessed through a computer-like interface. The applications include a computer algebra system and an interactive geometry facility. In addition, one tablet laptop computer connected to a digital projector was used in the classroom. A ClassPad emulator (computerised version of the calculator) was installed on the laptop. By combining technologies in this way, it was possible to produce a teaching and learning environment that amalgamated effective instruction from the teacher with active participation from the students.

A series of geometry activities was devised and used in conjunction with other problem solving activities. The students were also introduced to algebra error locator software called Algy which was installed as an add-in application to the ClassPad calculators. Data was collected in the form of teacher reflections, classroom observations, audio-tapes of lessons, and audio-tapes of semi-structured interviews. Some students were also recorded working on computers. These recordings were made through the use of screen capture software which had an accompanying audio recording.

# 4. Results

#### Acquiring procedural knowledge

Interesting observations were made when the students were working on algebra. The procedural knowledge in this case involved solving linear equations. As is often the case, the students had difficulty in finding their errors. If their final answer did not agree with the answer in the textbook then they would have to wade through their steps to find the error or wait for the teacher to help them. I introduced the students to an application in the calculator called Algy (see [1]). Algy is designed to help students carry out the steps correctly when they are solving equations (simple linear equations in this case). Algy allows the students to enter into the calculator the standard algorithmic steps that are required to solve the equation. Using Algy, the students are able to check their working in two ways. One method produces a response from Algy which indicates whether or not their working is correct. This method makes no allowance for working that follows from the previous line. In the example shown in Figure 1, one line of working has been entered and checked using this method. Algy indicates that there is an error by displaying the symbol ×.

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Figure 1. Algy indicates an error

In this example, the student has entered four lines of working before checking the answer. Notice that although several lines of working follow from the previous lines, each line of working has been marked with the symbol  $\times$ .

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Figure 2. Algy marks all lines as incorrect

In the example shown in Figure 3, the algebra has been done correctly throughout. The calculator indicates that a particular line of algebra is correct by displaying the symbol "->". In the recording, the student in this case was heard to conclude by saying "And I got that one all right!"

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-> 5+ x=7	
-> x= 5	

Figure 3. All correct

The students could choose another method of checking their working, however. Algy can display the symbol  $\times$  where there is an error, but display the symbol "- >" when a line follows from a previous line even if the final answer is incorrect. In the example below, there is one line of working that is incorrect. This is indicated by the symbol " $\times$ ". Taking this error into account, however, the subsequent lines are marked as correctly following from the previous line.

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-> /• x=9	
-> ×= 7	

Figure 4. Algy takes errors into account

Essentially, therefore, the calculator could tell the students where they had gone wrong. In this example, the student understood that only one mistake had been made in the working. This mistake had been identified by the calculator. The students can relate this type of feedback to the feedback they would receive normally from their teachers. In a test, for example, they might expect their teacher to give partial marks rather than full marks or no marks at all. The students can obtain the feedback from the calculator in an instant, however, and in this sense the feedback from the calculator is more immediate than the teacher's feedback.

The students found Algy to be helpful and responded positively to using it as exemplified below.

Me: So what's the difference then when you're using Algy? S2: It helps you more S3: It tells you where you've gone wrong. Like it still says if you've done this instead of that then it's still right what you've done [Lesson transcription, 11<sup>th</sup> August, 2009]

The observations on the use of Algy in the classroom indicate that Algy was helpful in the shaping phase of acquiring procedural knowledge. An important aspect of this shaping phase is the elimination of errors and Algy helped in two main ways. First, Algy could accurately pin-point errors. This made learning more efficient because the students were able to focus on where they had gone wrong and make a correction. Second, Algy provided immediate feedback either in locating an error or by affirming that the work was correct. In these ways, therefore, Algy provided the students with an improved way of correcting their work. They were able to access specific and immediate feedback instead of relying on just a final answer from the back of a textbook or having to wait for the teacher.

A note of caution needs to be expressed at this point, however. It is possible for students to perform procedures successfully but in a shallow way that lacks conceptual understanding (see [6]). I received a salutary reminder of this when interviewing some of the students.

- Me: How do you usually know if you've got an algebra equation correct?
- S3: Emm, well you only have *x* and a number left
- S2: And it's usually a whole number
- S3: Yeah it's usually a whole number or a fraction [Lesson transcription, 11<sup>th</sup> August, 2009]

Clearly I needed to reflect on the depth of understanding that the students had acquired despite their demonstrated ability to perform procedures correctly. A difference in conceptual understanding amongst the students came to light when a student who had successfully been using Algy had asked me to show him how to use the computer algebra system (CAS) that is built into the ClassPad.

Whereas Algy is an application that has been specially designed to support students as they carry out algebraic procedures, a CAS can provide the student with an answer without displaying any intermediate steps. This distinction is illustrated below using the equation 3(x+2) + 4 = 4x as an example.



When using Algy, the user inputs the solution to the question which can then be checked by Algy. When using the CAS, the calculator displays the answer

Figure 5. The difference between using Algy and using a CAS

I showed him how to use the CAS and after he had used it to solve an equation I asked him questions about what he had done.

Teacher: How do you know that you have got the right answer? Student1: I've done the equation and it's right? Teacher: You can check it by hand? Student1: Yeah Teacher: How do you know that is the right answer to that equation...x=2? Student1: I did it using Algy Teacher: Alright, how else could you do it if you look at the original equation? How could you tell that that is the correct answer? Student1: I could replace *x* with 2 Teacher: And does that work? Student1: Give me a sec. That side is 6 and that would be 6. Me: So it works out? Student1: Yeah [Lesson transcription, 11<sup>th</sup> August 2009]

This student displayed a better understanding than some of the other students of what it means to solve an equation. He understood the concept of balance in an equation and used this to check his answer by substitution. He was able to display this understanding despite the fact that he did not perform the steps to solve the equation (the CAS did this for him). On the other hand, some students who performed all the procedural steps successfully still only had a shallow understanding of what it means to solve an equation. All of this served to remind me that a successful display of procedural knowledge does not guarantee a high level of conceptual understanding.

### Acquiring declarative knowledge

In another session I introduced a group of 12 Year 8 students to the geometry application of the ClassPad. I allowed the students time to experiment with the geometry tools. During this time they learned how to draw points, lines, and circles, and how to make composite shapes of their own devising. They also picked up how to clear the screen and delete parts of their drawings.

Using the emulator displayed through the digital projector, I demonstrated how to display grid points on the screen and draw a randomly shaped polygon. The area of the polygon could be calculated by the calculator and displayed at the top of the screen as illustrated in Figure 6.



Figure 6. Calculating the area of a polygon

The students then drew polygons in the way that I had shown them and they all recorded the number of grid points in the interior of the polygon, the number of grid points on the boundary of the polygon, and the area of the polygon. A table was made on the tablet computer with students coming forward to contribute their results. The table was displayed through the digital projector. The start of the table is shown below. Could there be a pattern that connected the number of interior points, the number of boundary points and the area?

Interi	or Boundar	-> Area	
20	14	2.6	
28	12	33	
9	8	12	
20	G	27	
	10	S.	

Figure 7. Students contribute results

In the class sharing discussion that took place, students made suggestions as to what the rule for the pattern might be. The first two suggestions that were made were discounted because they only fitted some of the results and not all. Eventually a suggestion was made that the rule could be "subtract one from the number of boundary points, divide by two, add the number of interior points and then take away a half". The class tested this rule against the results and found it to be satisfactory. A simpler version was also offered which was "the number of interior points add half the number of boundary points then take away one". I congratulated the class on coming up with a convincing rule and encouraged them to think about how they might prove that this rule would always work.

By the close of the session, the students had made significant progress in the organizing phase of acquiring declarative knowledge. The declarative knowledge in this case was a rule to calculate the area of polygon drawn on a grid. The results contributed by the students onto the tablet computer screen can be described in the language of the Dimensions of Learning framework as a *graphic organizer* from which a *generalized pattern* was developed (see [6]).

The advantage of using the ClassPad in this situation was that the students were able to use the ClassPad to calculate the areas of the polygons. In this way, the students were relieved from the drudgery of calculating areas and could instead focus their attention on finding a generalized

pattern. The technology aided in the organizing phase of acquiring declarative knowledge by alleviating the use of procedural knowledge that was not directly relevant to the main objective.

In yet another session using the geometry application of the ClassPad, my objective was for the students to discover the relationship between the centroid, the orthocentre and the circumcentre of a triangle. I hoped that they would be able to find that these three centres lie on a straight line and that the distance from the orthocentre to the centroid is twice the distance from the centroid to the circumcentre. I demonstrated how each centre was constructed. I was then impressed by how quickly the students were able to acquire the necessary skills through the use of the technology. Screen capture recordings made by students using the ClassPad emulator on the tablet computer showed that they were able to construct all three centres in less than four minutes. The fact that the students were able to make these constructions so quickly meant that they could spend more time exploring the relationship between the three centres shown in Figure 8. The interactive geometry application of the ClassPad then became an example of a dynamic geometry system which provides the students with scope for exploration and experimentation (see [5]).



Figure 8. The three centres

The students were able to use the ClassPad to measure the distances between the centres and quickly spotted the ratio involved. Although it seemed clear that the centres lay on a straight line, some of the students checked this by measuring the angle at the centroid to be 180 degrees. The interactive power of the technology was also found to be helpful. The effect of changing the position of vertices was investigated as illustrated below. This meant that students could effectively reconstruct three centres of another triangle and still find the same relationship between the three centres as illustrated in Figure 9.



Figure 9. Using the interactive power of the technology

As in the previous example, the technology assisted the students in acquiring declarative knowledge by alleviating the drudgery of the procedural work. In this case, the technology helped the students reach the internalizing phase of acquiring procedural knowledge in an efficient way. More time could then be devoted to the organizing phase of acquiring declarative knowledge.

# 5. Conclusion

Two main types of knowledge identified in the Dimensions of Learning framework as declarative knowledge and procedural knowledge were considered. Actual examples from the classroom were used to illustrate ways in which acquiring both of these types of knowledge can be assisted through the use of the ClassPad calculator. In particular, it was found that the ClassPad can assist students in the shaping phase of acquiring procedural knowledge by pin-pointing errors in algebraic processes and providing immediate feedback. It was noted that procedural knowledge should also be accompanied by conceptual understanding. Benefits were also described in helping students in the organizing phase of acquiring declarative knowledge. This was achieved by using the ClassPad to alleviate students from the need to do procedural work or by making it quick and efficient.

## References

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