Socratic Method: Understanding Equations of Circles with the Use of a Graphing Calculator

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Abstract: *The modern Socratic Method is a process that employs inductive questioning to lead a person to construct new knowledge through small steps. This knowledge can be a specific concept, training in approaches to apply a process such as the process of problem solving, or leading one to embrace a specific belief. The type of knowledge is not as important with the Modern Socratic Method compared to the fact that the knowledge gained is specifically anticipated by the Socratic questioner. Literature pertinent to oral questioning confirms the many benefits of the Socratic Method. This paper describes a case study of the use of the Socratic Method to develop understanding of the relationship between the standard form and the general algebraic form of the equation of a circle. The study was mediated with the use of the graphing calculator as the primary feedback tool to aid students to arrive at appropriate answers for the questions posed by their lecturer. Twenty-one first year Bachelor of Education, Mathematics major students from a Malaysian teacher training institute participated in this study. The main source of data to describe the effects of the Socratic Method was from the traditional ethnographic technique of "participant observer". The study also revealed the process of thinking of the students, the realization of the importance of a systematic record of data to aid the construction of new knowledge from prior knowledge and the need of an efficient feedback tool to explore students' conjectures which lend to progress in the learning process.*

1. Introduction

Lectures in mathematics that I have delivered, placed me as the authority of the subject matter and my students as ignorant, passive recipients who are to be enlightened by my delivery of new knowledge. My present Bachelor of Education Mathematics students are a real passive lot and the more I lectured, the more I realised that I was answering almost all of my questions. Deciding to find a different means of conducting my lectures, I thought about all the self answered questions and decided to look into the internet about the techniques of questioning. I was fortunate to read about Garlikov's (see [3]), teaching experiment using the Socratic Method. This inspired me to read more about the Socratic Method and I planned a research based lecture to determine the effectiveness of this method based on the claims of the benefits of the Socratic Method.

1.1 The Socratic Method

The Socratic Method is a method that supports discussion as a means of teaching and the impetus of seeking knowledge is the question. The Socratic Method claims that knowledge arises within the individual rather than from an external source (see [4]) and hence, knowledge cannot be transmitted but is generated by man through his own efforts, by building on what he already knows. This is similar to the constructivist approach as stated by von Glasersfeld (see [7]), that 'knowledge cannot simply be transferred ready-made but has to be actively built up by each learner in his or her own mind'.

The Modern Socratic Method, on the other hand, is a process of inductive questioning used to successfully lead a person through a series of questions to specific knowledge (see [2]) and the knowledge gained is anticipated by the questioner.

1.2 Benefits of the Socratic Method

Among the benefits claimed of the use of the Socratic Method, are the following.

(a) Leads to student independence. In the Socratic Method, the teacher, as enabler merely challenges the student to explore on his/her own by questioning. This provokes the student to acquire knowledge autonomously. (see [4]).

(b) Develops critical thinking. The Socratic dialogues allow ideas to be tested critically, finding out the weaknesses and errors. Contrasting points of view are compared and judged for their soundness. Hence, it is ideal for collaborative construction of common understanding.

(c) Provides intrinsic reward. The Socratic Method provides intrinsic reward in that the ultimate reward is gaining meaningful knowledge. Hence, the learning experience is a rewarding motivating experience. (see [5] and [4])

(d) Fosters positive teacher-student relationship. In the Socratic Method teacher and student are interacting to achieve a mutual goal. The process to achieving the goal is guided by the teacher. As Vygotsky (see [8]) argues the Zone of Proximal Development of a child can do with help from others. Hence, here the teacher in a face to face situation provides the scaffolding for the students through a thoughtful line of questioning which enables the student to reach his reward. In return, the teacher obtains benefit of feedback of his method of teaching. He can then modify his approach to become a better teacher. This win-win situation is expected to establish a positive interpersonal relationship between teacher and pupils. (see [4])

2. Method

2.1 Participants

The study involved 21 first year Bachelor of Mathematics Education students (3 boys and 18 girls). I have had several lectures with them and in each one their passiveness had left me wondering if they actually understood anything.

2.2 Methodology

A single, two hours, teaching and learning session was conducted employing the Socratic Method. A basic set of questions were initially constructed which served as guidelines for the teaching and learning session. However, in practice, the line of questioning was allowed to be flexible so that the discussion had continuity. The objective of the lesson was to find the relationship between the standard form and the second degree quadratic equation form of the circle. The discussion was mediated using the graphing calculator as a tool for graphing the circles. The graphing calculator screen was displayed on a big screen using a Liquid Crystal Display projector. The researcher took the role of participant observer and kept field notes of particular changes in behaviours observed during the teaching and learning session. All verbal discussions were recorded on audio tape. All working through out the learning session on four white boards were also used as source of data. Every student was provided with a blank sheet of paper to do calculations and write comments. The papers were collected at the end of the teaching and learning session.

3. Data Analysis: The Classroom Action

During the learning session I started by informing the students that we were going to discuss about circles and that I would be providing guiding questions but no answers.

The following is the transcript of the teaching experiment, using the Socratic Method, with my class. The student responses are regarded as the class response because after every response I requested the rest of the class to state their agreement or refusal to the given response. (Key: R= Researcher; S= Student). The transcript has been analysed for revelation of the benefits of the Socratic Method in text boxes along or below the relevant sections of the transcript. My observation, views and perspectives have been included at the appropriate points. The transcript is not the verbatim version of discussion but the collective agreed outcomes of the discussion. However, for certain sections the verbatim version has been used because it was considered to be relevant to illustrate the actual development while other sections of the discussion considered insignificant have been omitted because of limited space.

R: Who can write the equation of the circle from recall of what you have learnt in school?
Student writes on the white board: $x^2 + y^2 = r^2$
R: How do you draw a circle?
S: Mark centre point and open the compass, to the size of the radius and draw the circumference.
R: O.K. Let us go through this again. You have a centre point and you open your compass. When you initially put the pencil end on the paper what do you get?
S: A dot.
R: A dot? Is there any other word for 'a dot'?
S: Yes, a point.
R: O.K. If the point that you made on the paper is A, and the centre is O, what is OA?
S: Radius. (Researcher marked point O on the board and drew a line to draw the radius from O and marked the other end as A)
R: So now your point A is going to be static – not move. Can you obtain a circle?
S: No
R: Then how can you obtain a circle?
S: Move point A.
R: How will you move point A, up and down?
S: No.
R: Then?
S: At a distance of the radius.
R: From where?
S: The centre of the circle.
R: You have learnt about locus in school, right?

> Challenging students to explore on their own and collaboratively construct a definition of a circle. Scaffolding provided by the teacher by connecting to their prior knowledge. Learning experience is rewarding.

S: Yes
R: Who can tell me about the circle in terms of locus of a moving point?
After several attempts, the students agreed on the following definition.
S: Locus of a circle is a moving point with a fixed distance from the centre point.
All the questions led the students to give their own definition of the locus of a circle. The answers given for each question above were from different students. A sense of achievement was obvious as the class came up with the definition. The definition was amended several times and the above definition was accepted by the class. They looked at me for approval. Instead I went on to the next question. They appeared to perceive this as an indication that their definition was approved by me. Student independence in developing knowledge was eminent but authority of approval of the accuracy of the knowledge was still with the teacher.

R: Can any one draw a circle and mark OA the radius on the board?
A student drew a circle and marked radius OA on the white board (Figure 1)

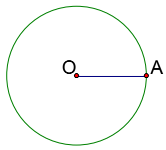

Figure 1: Circle drawn by student

R: Now let me draw the circle on a Cartesian plane, with the centre of the circle at the origin, O. O.K, what is OA?

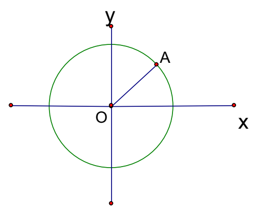

Figure 2: Circle drawn on a Cartesian plane

S: OA equals Distance between the centre point and a point on the circumference equals radius.
R: Where is point A?
S: On the circumference.
R: Can you suggest how you can obtain the equation of the circle from this diagram?
S: Find the gradient of OA.
All students remained silent since I did not state that the response was incorrect. It appeared that they were waiting for a cue from me and since there was no response from me, they believed it was the correct response.

R: O.K let the coordinates of A be (x_2, y_2) and the coordinates of O be (x_1, y_1).
R: So what is the gradient of OA?

Authority has been given to students. Students explore discuss critically, judge for soundness and construct knowledge.

S: The gradient of OA, $m_{OA} = \frac{y_2 - y_1}{x_2 - x_1}$.
R: Can this be written in the form of the equation you wrote as the equation of the circle?
S: No.
R: How do you know?
S: We cannot get the squares of *x* and *y* from this equation.
I had expected reasoning based on the locus definition which involved distance. Instead they viewed the situation in an algebraic relationship between the equations that did not fit with each other.

R: Then, how?
S: Use the triangle and the Pythagoras' theorem.

The student who had suggested the gradient formula realised his error.
He exclaimed: That was silly of me to have suggested the gradient formula.
I smiled and the whole class had a look of surprise. They were surprised that I had made no comments even at this point but had actually written out all that he had suggested on the board. Several whispered to their friends and one student questioned me about my non-judgemental behaviour.
S: Why did you follow his wrong suggestion?
R: For today's lesson it is up to you to tell me if you agree or disagree. I will only ask questions and provide no answers.
More sounds of surprise (Ooh! Aah!) and smiles and shaking of heads in disbelief.

I then led the class to continue with the discussion.
R: Who is willing to explain it on the board and show how to derive the equation of the circle?
A student draws (Figure 3) and explains as follows:

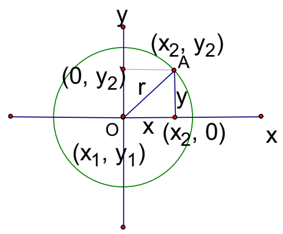

Figure 3: Circle drawn to obtain general equation of circle with centre at the origin

S: From the triangle, using Pythagoras' theorem we obtain $x^2 + y^2 = r^2$
The student volunteered without having to be coaxed. I was surprised at the change in atmosphere in the class. They realised that I was not giving any answers except questions and it was up to them to make progress in the lesson. The fact that I was not judging them appeared to make a difference.

R: How is x related to x_1 and x_2 and y related to y_1 and y_2?
S: $x = x_2 - x_1$ and $y = y_2 - y_1$

R: Alright, you have the equation of the circle as $x^2 + y^2 = r^2$. Can you write it in terms of x_1, x_2, y_1 and y_2?
A student wrote the following on the board: $(x_2 - x_1)^2 + (y_2 - y_1)^2 = r^2$)
R: What are x_1 and y_1?
S: The centre of the circle.
R: If I changed the centre to (h, k) can anyone tell me how to write the equation of the circle in terms of h and k?
S: Yes, replace x_1 and y_1 in the equation with h and k.
R: Can someone write that on the board?
A student wrote $(x_2 - h)^2 + (y_2 - k)^2 = r^2$ on the white board.
R: If the coordinates of point A are given as (x, y) how will the equation written here change?
S: x_2 will be just x and y_2 will be just y.
R: Can someone write that on the board?
A student wrote $(x - h)^2 + (y - k)^2 = r^2$ on the white board.
Researcher showed this equation in the graphing calculator on the screen (Figure 4)

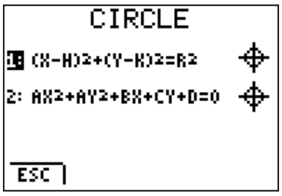

Figure 4: Screen shot of circle equations from the graphing calculator

R: Can anyone suggest values for H, K and R to draw a circle on the screen?
S: 6, 3, 2.
A circle was drawn by the graphing calculator.
R: What is the centre and radius of the circle?
S: (6,3). The radius, 2.
R: Now look at the screen. There are two types of equation that represent the circle. Can you suggest how to show the relationship between the two?
S: Expand the first equation.
R: Why?
S: We can obtain x^2 and y^2.
R: O.K can one of you do it on the board?
The following was shown on the board by a student.

$$(x-h)^2 + (y-k)^2 = r^2 ; (x^2 - 2xh + h^2) + (y^2 - 2yk + k^2) = r^2$$

$$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$$

R: So the calculator has the form $Ax^2 + Ay^2 + Bx + Cy + D = 0$, which we will refer to as the algebraic form of the circle equation. I want you to compare the two equations and tell me what A,B, C and D are equal to in the earlier equation.
S: A = 1, B=-2h, C = -2k and D = $h^2 + k^2 - r^2$

R: Now I have a set of values, A=4, B=4, C=3 and D = 2. Can we draw a circle with these data? Discuss with those at your table.
S: No.
R: Why?
S: Because r is negative.
R: Can you show the class how you obtained your r value?
The following was shown on the board.

$$-2h = 4, h = 2; -2k = 3, k = -\frac{3}{2} = 1.5; 2 = (-2)^2 + (-1.5)^2 - r^2; r = 2.062$$

R: So what is happening? We have a positive r so we must get a circle according to your statement, but the calculator states that parameters are not real. Is the calculator wrong or is something wrong with your calculations? (All students pondered)
R: O.K if A=8, B=4, C = 3 and D=2, should we get the same circle?

Process to achieving the goal is guided by the teacher. Contrasting points of view compared and contrasted demands critical thinking to judge for soundness.

Then suddenly several pupils screamed excitedly and claimed that their calculation was wrong because they had not taken into account the A value. It came from different directions at the same time.
At this point I realised the importance of the use of the graphing calculator to provide immediate feedback.

R: O.K can you show us what you mean by taking A into account?
A student did the following steps and stated that the original equation where A=1 must be multiplied by 4 to obtain the equation below when A=4, that is,
$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$, must be multiplied by 4 and this results in obtain
$4x^2 + 4y^2 - 8hx - 8ky + 4h^2 + 4k^2 - 4r^2 = 0$ and so the B, C and D values are as follows: $B = -8h$, $C=-8k$ and $D=4h^2+4k^2-4r^2$ and from this
$-8h = 4, h = -\frac{1}{2}; -8k = 3, k = -\frac{3}{8}; 2 = 4\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{3}{8}\right)^2 - 4r^2; 4r^2 = -\frac{7}{16}$ and concluded that a circle cannot be drawn because the radius has a negative length and so the calculator is correct.

I also noticed a change in confidence and willingness when the graphing calculator was used to provide feedback. The calculator was non-judgemental but was now the authority to confirm the accuracy of their responses and this motivated them to try their answers. It was obvious that with the feedback that their answer was correct gave them their extrinsic reward (like a praise – you are right) and students felt very comfortable to move on. Hence, the graphing calculator was then the more intelligent partner for the students. It also lent to the achievement of the intrinsic reward because the students claimed that they felt happy about determining the error themselves and that the calculator confirmed their conjecture. The class atmosphere can be described as highly motivated with almost everyone trying to make suggestions and not afraid to make errors any more.

Then I assigned each student a value of *A* and I wanted them to give me values of *B, C* and *D* so that we can draw circles using the graphing calculator with their values. After assigning each student a number from 1 to 21, I called the first student who was assigned $A = 1$ to give me a set of values and she claimed that she had not completed her calculation. After another 5 minutes she gave her answer: A=1, B=4, C= 6 and D=4. Then I challenged the class to give a set of values for *B, C* and *D* very quickly without calculating for A=1. I kept telling them that they do not no need to calculate, can give one set of values and then suddenly the student who was assigned A=1 gave the answer. A=1, B=4, C=3 and $D = 2$. She said that earlier we obtained the *r* values as positive because we had not taken into consideration that the *A* had changed from 1 to 4.
At this point a student suggested drawing a table and keeping record of the values of *A, B, C* and *D*. She explained that in this manner a proper record may show a pattern that would help to give quicker responses by reasoning based on the values recorded. Everyone agreed and they started drawing tables to keep record of the values.
Then I asked the student whom I had assigned A=19 to give me a set of values.
His set of values were A=19, B=38, C=19 and D=16. It did produce a circle.

Then I challenged them to give another set of values very quickly for A=19.
No response. All were discussing busily. So I gave a hint.
R: What if you shifted the centre which eliminated calculating some of the values.
Then several of the students shouted: (0,0). Let the centre be the origin.
R: And so
S: Well B=0, C=0 and D=0.
R: Let us use the graphing calculator and fit in the parameters. Well we don't have a circle. Why?
S: Because r=0
R: Show it on the board.
A student showed the following on the white board.

$$D = 0; D = h^2 + k^2 - r^2; 0 = 0 + 0 - r^2; r = 0$$

R: O.K if A=1 and B=C=0, what is the value of r^2 in terms of D?
A student showed the following on the white board.

$$D = h^2 + k^2 - r^2; D = 0 + 0 - r^2; r^2 = -D$$

R: If A=8, B=C=0 and D=-4 what is the value of *r*? Show it on the board please.
A student showed the following on the white board.

$$r^2 = -D = -(-4) = 4; r = 2$$

R: O.K now write the algebraic equation when A=8, B=C=0 and D=-4
A student showed the following on the white board.

$$8x^2 + 8y^2 = (-4) = 0$$

R: I want it in the form $x^2 + y^2 = r^2$ since B=C=0.
The student writes the following.

$$8x^2 + 8y^2 - 4 = 0; 4(2x^2 + 2y^2 - 1) = 0; 2x^2 + 2y^2 = 1 \; ; x^2 + y^2 = \frac{1}{2}.$$

R: From this equation what is the value of r.
S: $r = \sqrt{\frac{1}{2}} = 0.707$

R: So now you have 2 different values of r for the same value of A and D. Which one is correct and why?
S: r=0.707 because, in the earlier calculation A was not taken into account and so A unknowingly was assigned the value of 1 when it should have been 8.
A student did the following calculations on the board.

$$D = 8h^2 = 8k^2 - 8r^2; -4 = -8r^2; \frac{1}{2} = r^2; r = \sqrt{\frac{1}{2}} = 0.707$$

Again contrasting points of view compared and contrasted demanded critical thinking to judge for soundness. However, here the teacher no longer was needed to provide the scaffolding. The students were capable of functioning independent of the teacher- they had become independent learners capable of acquiring knowledge autonomously but not independent of external source of help. They were still dependent on the graphing calculator.

Students then handed in their respective values of A, B C and D. All had correct parameters.

They were then asked to comment about the lesson.
S1: At first it was a bit scary because you (the researcher) was giving no information except posing questions to which we were at a loss at the beginning. Later we understood and were less aware that you were only asking questions and not giving information or responses.
S2: It was fun, didn't feel sleepy and had learnt a lot which I understood.
S3: The best and most enjoyable mathematics lesson in my life.
S4: I enjoyed so much that I did not realise that two hours had gone by.
R: Why did you enjoy it so much?
S4: I discovered the errors myself ..aah… sort of could do it myself and so I enjoyed.
I take this as an indication of intrinsic reward.

The students were very relaxed. They suggested having all mathematics lectures using this questioning method but they requested that a tool like the graphing calculator for feedback was important.
S5: Love this type of lesson where we understand where we are going in the mathematics lesson. The graphing calculator is important for us to make advancement in the lesson since you were not giving any answers.
I could feel the happiness among the students and I too had not realised that two hours was nearly over. A positive interpersonal relationship between the students and the researcher was established. A passive lot were so vocal. However, there were several side benchers. From my observation there were four girls who only participated when a question was directed at them. As the class grew excited, they also did discuss with their neighbours but were not willing to volunteer answers. When I questioned them about their reaction in class they claimed that they were shy but did enjoy the lesson very much.
S6: I was shy to butt in like the others to say what I wanted to say. My sheet of paper shows that I followed the lesson.
The other two showed their respective sheets which were also full of working and the table. So it appears that the blank sheet of paper did serve to show participation of the students.

4. Conclusion

From the study the creation of a teaching and learning situation that is conducive by using only questions, the Socratic Method, to pave the path of discussion among students is possible. The impetus of seeking knowledge is the question. The study however illustrated that the teacher should not be judgmental. Once this judgmental, authority by the teacher is eliminated the students tend to be more willing to participate in the discussion. When the erroneous response to deriving the equation of the circle was given its due respect for been a suggestion from a student, all the students were willing to provide responses pertaining to the question.

However, as Vygotsky (see [8]) argues that the Zone of Proximal Development of a child can do with help from others. In this study it shows that it is the teacher who can provide the scaffolding for the students through a thoughtful line of questioning to guide the students to reach their ultimate goal. It is also the teacher who can draw the students back to the path to reach the goal when the student goes off on a tangent from the correct line of thought. This help given face to face without been judgemental can lend to establish a positive interpersonal relationship between teacher and students as it did for the researcher and the students in this study.
Another point that came through from the study was that students need sufficient feedback in a classroom situation in which no information can be obtained to confirm student conjectures. Here the role of technology, as in this study the graphing calculator supplied the required feedback for advancement to take place in the learning process.

With the attainment of meaningful knowledge, the learning experience will prove to be a rewarding motivating experience. In this study this was evident several all through the lesson.

Hence, I recommend to teachers and especially lecturers who have a passive class to try using the Socratic Method to have a more active and rewarding lesson.

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