Mathematical Simulation Experiment by Using Graphing Calculator（HP39gs）

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Abstract: *A 'Mathematical experiment' is an experiment by using a combination of thinking and operating. Graphing calculators bring the possibility of a 'Mathematical experiment' to reality. In practice, students can construct mathematics concept and solve mathematics problems by operating, observing, experimenting and argumentation independently and positively as a team. Breaking subject's boundary in order to come up with a multi-subject knowledge point integrative creation and innovation to accomplish independent learning and research. In this paper, the author will discuss on 'simulation mathematical experiment' by using HP39gs, also combine 'fair-coin-tossing experiment' and 'normal distribution experiment' to explain the practical importance of the mathematical experiment.*

1. New curriculum reform's requirements to 'simulation mathematical experiment' and problems need to faced.

'Monte Carlo' is proposed in chapter 3 of Common High School Course Standard Compulsory Mathematics. In China It requires students to use information techniques fully to produce random numbers to simulate coin-flipping using HP39g. It also requires schools to let students draw statistical line chart and histogram using Statistical software.' (quoted from Compulsory Mathematics 3 Teacher's Book.) In this chapter, it contains several layers of meaning. First of all, it shows that we need to use information technique while learning particular topics. Secondly, information technique is needed to complete a mathematical simulation experiment. Finally, if conditions are allowed, students must do the experiments personally. It shows that using information techniques as an assisting tool during education is extremely normal and the information techniques is considered as an essential teaching tool as well.

But how do we explain the 'condition' mentioned in the third layer of meaning? Should we consider it as the students being able to perform experiments using computers in order to have a further understanding on subject's knowledge and concepts through personal experiences? It is a matter of fact, that it is hard to accomplish this in the China classroom. Therefore, we have to face the problem which that we depend on computers while combining Information Technique and High School mathematics together, it greatly limits the experiment sites. On the contrary, HP39gs could solve these kinds of problems. In this paper, the author would like to show how students are going to use their HP39gs to calculate a random number, abstractive simulation of the fair-coin-tossing

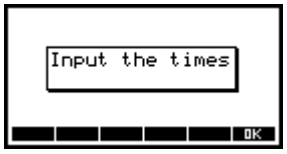
experiment and imaginary simulation of coin-flipping using HP39g. The students are able to design the frequency distribution table according to the data collected from experiments, and understanding the concepts of ‘frequency’, ‘frequently’ and ‘probability’. It can be said, computer’s software function surely is better than HP39gs’s APLET function, but HP39gs is much portable than computers. In nowadays, with the continuous development of the information technology, it would be best for us to focus on HP39gs.

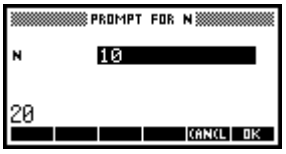
2. HP39gs simulation experiment

Experiment I : Simulate coin-flipping using HP39g

While learning about the probability of random events in the field of Statistics, we need to let the students experience the instability and uncertainty of the random events through experiments, then induce the definition of probability. The teachers’ manual requires the experiment of coin-flipping. Due to the limited time, the students are asked to do the coin-flipping experiments for only 10 flips. According to the teachers’ manual, there are several problems to be considered while the students carry out experiments. Firstly, some emergent cases occur when students conduct the experiments, for example, the coins are easy to slide and roll on the floor so that the students have to leave the seats to look for them. Secondly, the students can not repeat many flips due to the limitation of time and space, therefore, it is difficult to compare the results of different flips. In order to overcome these problems, I have designed the following program..

Experiment:



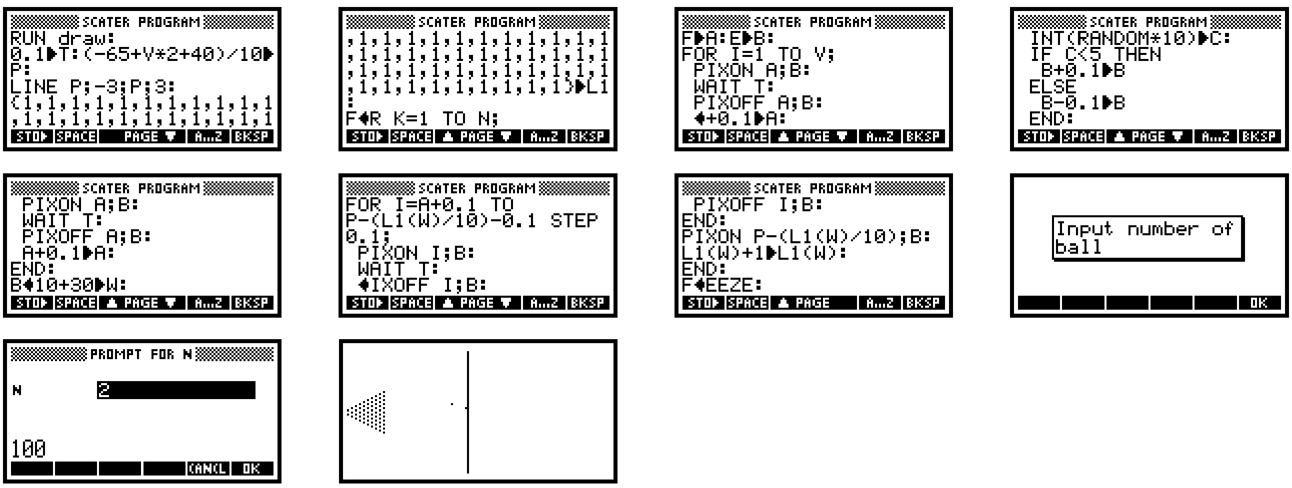


The program is designed to simulate the ‘coin-flipping experiments’. The coin-flipping is accomplished by pressing the ‘enter’ key. The ‘+’ sign on the screen indicates that the flip shows head side coin while the ‘-’ sign indicates the tail side. Thus, the program allows students to operate the program, summarize the results of experiments and conclude the rules by themselves. It is a good opportunity for them to enjoy the authenticity and pleasure of mathematics. The program has a number of advantages over the traditional methods. The traditional coin-flipping experiments are

usually accomplish through oral description and require the students to understanding them through imagination, therefore, it can not show the application of mathematics in the real life. I have ever designed a dynamic coin-flipping experiments using the FLASH software. Although it can approximately show the process of experiments, the students still have to understand it through the vision and imagination. Regardless of its several improvement compared to traditional method, the simulation using FLASH software is still not satisfactory because it rely too much on the senses. This is why I have further designed another program using HP39gs by which every student with HP39gs can easily finish the experiments through pressing the ‘enter’ key. Thus, all the students can conduct the experiments and experience the process by themselves. Futhermore, this simulation program not only effectively reduces the time spent on the experiments and increases the flips in the experiment, but also avoids the trouble of using physical experimental instruments. All in all, this simulation program allows the students to conclude the rules, understand the relative concepts, such as frequency and probability. It is also worth mentioning here that another simulation using EXCEL can be set up. During this simulation, EXCEL is used to provide random numbers: the number ‘0’ represent the heads and the number ‘1’ represent the tails. The flip is represented by the counts of random numbers and the frequency of the occurrence of head can be obtained through counting the number of ‘1’s’ provided. Although it is also a good method, the computers accomplish the simulation by EXCEL so that we are unable to let every student conduct the simulation in person. In addition, the random number ‘1’ and ‘0’ can only represent the results of coins flipping abstractly and thus, the vivid simulation by HP39gs is advantageous to some extent.

Experiment Ⅱ: ‘HP39gs simulate normal distribution’

In the experiment of Galton board, the ball will collide with many wood blocks when moving down the board. This will lead to the ball randomly falling to the left or right. Therefore, the ‘x’ coordinate of the ball when it touches the Galton board for the fist time is resulted from many random collisions and approximately follows the normal distribution. In order to observe the normal distribution curve through the experiment of ‘Galton Board’, I have designed the following program:



This program may repeat the entire process of ‘Galton Board’ perfectly and the normal

distribution curve can be accurately demonstrated through the results of the experiment. The students can experience the pleasure of mathematical knowledge and the establishment of a mathematical model, which will help them understand the essence of mathematics and enhances their enthusiasm towards mathematics. This experiment, however, will still suffer from several weaknesses. For example, the experiment frame is too small; the ball is too small due to the limited pixels of HP39gs; the speed of the experiment is too slow due to the limited memory of HP39gs. Therefore, in the real teaching process, I assign the students to observe and explore the phenomenon after class and submit the reports on their findings. The following questions are designed to help the students explore these problems:

Q1: What factors will affect the validity of the results of this experment?

Q2: Given the function of normal distribution: $\varphi_{\mu,\sigma}(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}},x\in(-\infty,+\infty)$, is it possible to investigate how the shape of curve change with the μ and σ?

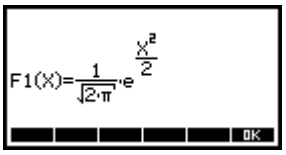
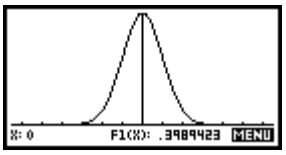
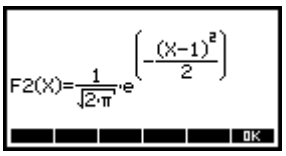
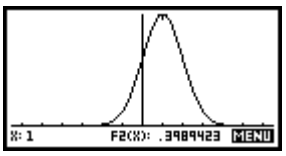
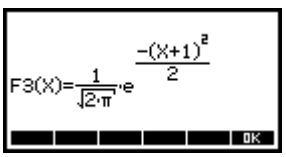
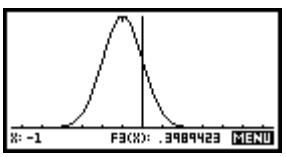
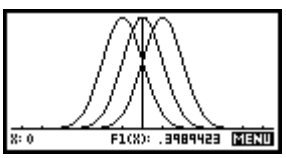
Q3: Can you tell me the probability that the ball will fall from the third barrel to the fifth barrel? Show the method to calculate the probability.

Q4: Is there any other phenomenon that you can observe from this experiment?

The following is part of the report from a student , Lin Kunhua:

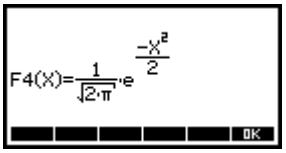
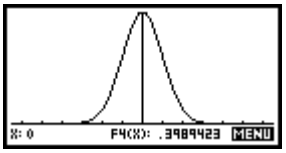
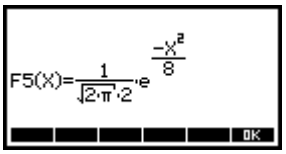
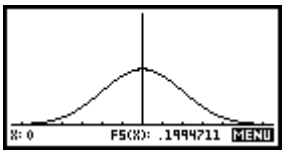
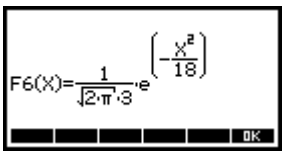
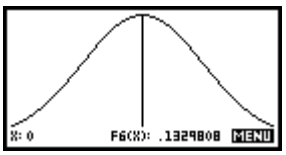
（1）The number of balls used in the experiment have significant effect on the results of experiments. The more the ball is used, the accurater the curve is obtained.

（2）Using the picture drawing functions of HP39gs , the curve shape of different μ and σ and be compared. With σ fixed at 1, changing the value of μ, the contrast graphics are shown as follows:

μ value	Fuction	Figure
$\mu=0$		
$\mu=1$		
$\mu=-1$		
Cluster figure		

Conclusion	The value of μ affect the position of the curve，the value of μ should be the population mean; 2、From the symmetry axes of the function, $\varphi_{\mu,\sigma}(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x\in(-\infty,+\infty)$, $(x-\mu)$ move horizontally toward right or left.

（3）Using the picture drawing functions of HP39gs , the curve shape of different μ and σ and be compared. With μ fixed at 0, changing the value of σ, the contrast graphics are shown as follows:

σ value	Fuction	Figure
$\sigma=1$		
$\sigma=2$		
$\sigma=3$		
Cluster Figure		
Conclusion and question	1、σ controls the width of the curve and the height of peak point; 2、From the point of function： $\varphi_{\mu,\sigma}(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x\in(-\infty,+\infty)$,how to analyze the value of σ ?	

（4）According to the geometry, the probability should be calculated as the proportion of region area to the total area. But the c the area can not calculated.

（5）There are two questions unsolved.: First, the statistics meaning of σ, which is the overall standard deviation according to the textbook. Second, how the area is calculated. Third, how to theoretically analyse the relation between the value of σ and the width of curve and peak point.

Taking the advantage of HP39gs, students can learn by themselves under the guidance of the teachers during the exploration process. The power of HP39gs can be seen through the program. The students are able to learn what we expect them to learn independently and discover further

questions that lead to further exploration. According to the reports from the students, I can answer questions or provide the clues for solution during or after the class. I can also provide the direction for further exploration, which help enhance the enthusiasm toward mathematics and motivate the students to think about questions deeply.

3. Appreciation of students' work

Work I: 'HP39gs simulate clock of time zones', by Wu Yousong from Senior Department, Quanzhou No.7 Middle School.

（1）To simulate static graphics

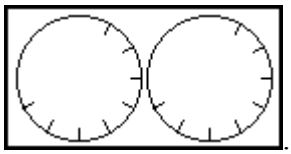
Create two circles on both left and right sides; 

, we need to divide the circle into 12 parts and indicate the scale. The HP39gs only has a function of line drawing. So we calculate the coordinate of these 12 points of parts with the following formula：

$\begin{cases} x = 2.5 + 2\sin(\frac{A\pi}{6}) \\ y = 5 + 4\cos(\frac{A\pi}{6}) \end{cases} \Rightarrow A(2.5 + 2\sin(\frac{A\pi}{6}), 5 + 4\cos(\frac{A\pi}{6}))$. Since there is only one point, we need to draw another concentric circle then divide it into 12 parts and find the coordinates of the 12 points of the parts:

$\begin{cases} x = 2.5 + 2.4\sin(\frac{A\pi}{6}) \\ y = 5 + 4.8\cos(\frac{A\pi}{6}) \end{cases} \Rightarrow B(2.5 + 2.4\sin(\frac{A\pi}{6}), 5 + 4.8\cos(\frac{A\pi}{6}))$

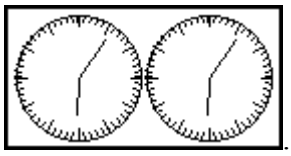
，Connect up these 2 points to a line segment that looks like'scale'. The editing order is 

and thesolution is  .

Likewise we shall draw the hour hand, the minute hand and the second hand in the same way, but use the center of circle as one of the end points, while the point is part as the other end. Because distance between them and their lengths are different, the formulas used are also slightly different. For example, the formula for the minute hand is：

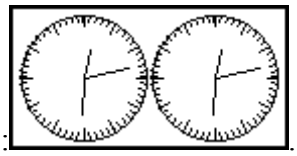
$$\begin{cases} x = 2.5 + 2.2\sin(\dfrac{A\pi}{30}) \\ y = 5 + 4.4\cos(\dfrac{A\pi}{6}) \end{cases} \Rightarrow A_1(2.5 + 2.2\sin(\frac{A\pi}{6}), 5 + 4.4\cos(\frac{A\pi}{6}))$$

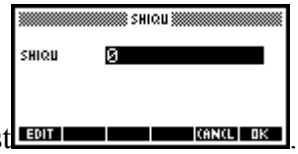
，To connect the center of circle and Point A_1, we can get the graphics of the minute hand. The editing order is

and the solution is  . Get the scale of second hand in the same way as attached.

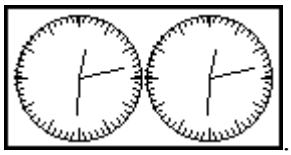
（2）To utilize the function of screen wiping to simulate dynamic graphics

Use “TLINE” to wipe the beeline as well as the original beeline. Therefore, the point wiped is coordinate of original beeline. The editing order is

(graphics of minute hand is wiped). It is worth a mention that the initial value $\dfrac{M\pi}{60}$ is added so as to input a beginning time. In the same way, we can get graphics of the second hand and hour hand. As shown :  .

（3）The formula used to convert time zone is very easy. According to the rule of 1 hour different between neighboring time zone, set T as the present time in Chinese Mainland (East 8th time zone) at first 

. T+J would be the time in east time zone you want to look up, if J is a positive integer. Inversely, T+J would be the time in the west time zone if J is a negative integer. It would be just OK to assign a value to the clock at the beginning with the HP39gs programming. (Time zones vary by hour).



Work Ⅱ: 'MCL detect manufacturer of white noise', by Kunhua Lin and Kunzeng Lin from Senior Department, Quanzhou No.7 Middle School.

(1) List those appliances which may make noise: electric hair dryers, electric kettles, computer cases (speakers off),lampblack machines, refrigerators.

(2) Use the 'Sound' tool in MCL and analyze the data.

Item	Brand	Power	Noise measurements	Making noise or not
electric hair dryers	Philips	1600w		

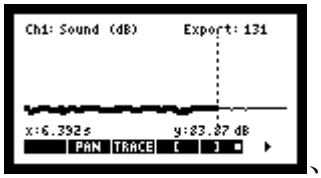
(3) Steps:

1 Start: 

2. Data collecting:

As soon as the 'Sound' tool starts, there will be a 'strip curve': 

, appearing on the screen of HP39gs with little variation, though, which is due to the excessively-dense data collection. Now we need some 'finer' graphics. So we can press the key 'zoom' ,together with the direction keys '+, -'for the amplification and narrowing of the graphics (when the 'zoom' key works,'ZOOM'will turn to'ZOOM'. There will be a direction for the special function of the 'pan' key in the following table.)

Use the keys 'exprt' and '【'and'】'to intercept the 

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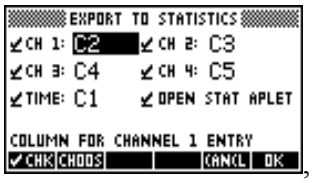
parts of the curve, notice that the keys '+'and '-' on the keyboard which can be used for the fine-tuning of the number of those selected data, which will be reflected in the right top corner of the screen, for example, in the above two pictures, the elected data is 131, if we need to reduce the

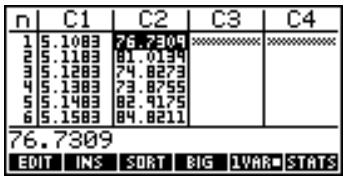
data, press '-' : 

,the data is intercepted, press the 'ok':



, and then click 'ok' again： 

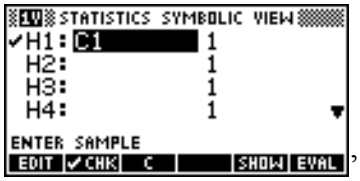
. Thus, we accomplish 'streamsmart' operation (that is the data acquisition), APLET will transmit automatically the collected data to the 'statistics'(statistical APLET).

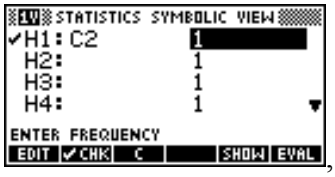
3. Statistics converting： 

， there are two columns of data, the C1 data shows the time data and the C2 data represents the loudness of the sound (decibels).

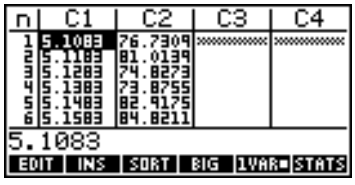
4. Data processing:

We should pay special attention to APLET statistics which is only for two-dimensional design, therefore, we need to convert '2VAR' into '1VAR'. Besides, we need to test the loudness of the electric hair dryers, so the statistical data is only valid in the C2 column.

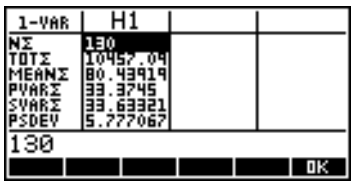
Therefore, firstly press the Symb key, we will see: 

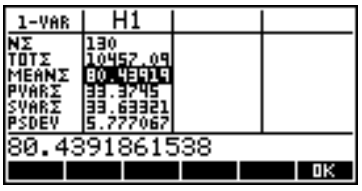
' then press the Edit key to change it to： 

, thirdly press the Number key and back to ：



(In particular, we can use the Sort key on the keyboard to select 'ascending' or 'descending' to know the scope of the data in C2.） We are able to get the average data through the real value of the approximate average, by clicking 'stats' :



and finding ‘meanΣ’ :


, the lighted number 80.439 is the result.

Table 1

Item	Brand	Power	Noise measurements	Making Noise or not
electric hair dryer	Philips	1600w	80.439	yes

（4）**Table 2** Statistics Table of Some Domestic Noises Sources

Item	Brand	Power	Noise measurements	Making Noise or not 40 dB
electric hair dryers	Philips	1600w	80.439	yes
Lampblack Machine	weilin kitchenware	130w	69.790	yes
Electric kettle	“Tea king” tea-set	1200w	59.866	yes
Computer case (speakers off)	DIY computer (used for 2 years),	300w	53.295	yes
Refrigerator	Midea	580w	46.431	yes

4. Thinking

(1)From a macro point of view, using graphing calculator to construct a model in mathematics, will not only show the focus on new curriculum ideas in real terms, but also the students are able to explore and investigate the problems. The students construct ‘questions’ during the study, mathematical models through abstraction and modeling; use the known knowledge or life experience to get inspiration; solve and conjecture problems, and finally explain all those problems by using HP39gs. With the help of HP39gs, this kind of the ‘find-solve-conclude-find’ cycle mode greatly stimulates students’ enthusiasm for exploring the nature of mathematics. it will not only help students to gain a sense of worth, but also let them understand how to cooperate and communicate; Thus learning mathematics can become a morw direct, interesting experience. It will also stimulate the students’ interest in learning mathematics, develop a habit of thinking independently and exploring actively.

(2) From the microscopic point of view, on one hand, HP3gs’s simulation experiment could not only avoid a large number of repeated experiments, but also offer the students chances to experiment personally, reproduce the real background of mathematical problems, experience the whole process from finding mathematical problems to solving them and truly transfer ‘virtual study’ to ‘actual study’. For example, in section two: ‘MCL detect manufacturer of white noise’, it simplifies the process of a model by turning the invisible sound waves into visual curve according

to noise data collected by MCL. It restores the sound wave, makes the process of constructing easy, fast and accurate. It provides a full of meaningful, interesting process. In another example ‘ HP39gs simulation normal contribution’, the ‘ normal curve’ was constructed after a ball was dropped, it allowed the students to understand what a ‘normal curve’ is intuitively, help the students construct ‘normal distribution’ by using actual image instead of virtual image and using dynamic experiment to described a static experiment. In this way, mathematics problems are solved while investigating life issues. On the other hand, it gives the students a real idea of the concept and definition What is required is to lead the students to transfer from inducting and conjecturing of objective phenomenon to using their knowledge to demonstrate the conjecture. Through thinking and operating the students study the nature of mathematics systematically and scientifically and discover the laws of mathematics.

Annotation

1、《普通高中课程标准实验教科书数学》必修 3.2.2 随机数的产生。
2、蒙特卡罗法：利用计算机或计算器模拟试验的方法称为随机模拟法或蒙特卡罗（Monte Carlo）法。
3、建立 0 或 1 与硬币正反面的一一对应关系，通过产生 0 或 1 的随机数，抽象模拟掷硬币实验。
4、利用编程绘制硬币图案，电子模拟硬币抛掷画面，形象模拟掷硬币实验。
5、《图形计算器：不可替代的“数学工具”》——王长沛。
6、《图形计算器在数学建模中的应用》——钟强 王光明。

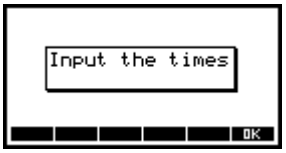
References

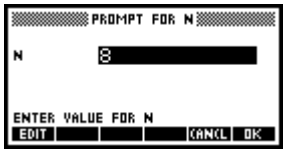
【1】《普通高中课程标准实验教科书数学必修 3》
【2】《普通高中课程标准实验教科书数学必修 3——教师用书》
【3】王长沛 《图形计算器：不可替代的“数学工具”》
【4】钱扬义 《手执技术在理科试验中的应用研究》
【5】凌艺国 ATCM 文集《新课标下信息技术与数学课程的整合点》
【6】张余婷 《在游戏中体会算法》
【7】《IT 图形计算器教学应用研究——解决问题》
【8】ATCM 文集学生作品：《运用手持计算器通过掷硬币概率模型进行建模预测——戴维》、《HP39gs 模拟抛硬币实验——吴友松》

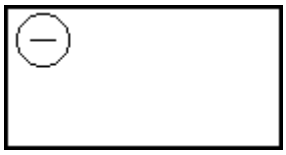
Attachment：Simulate coin-flipping using HP39g by Liu Kunzeng Student of Senior Department , No.7 High School

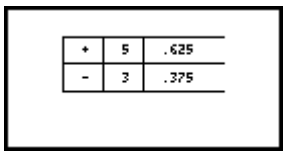
【source program】

【program process】









+	5	.625
-	3	.375