Handwriting Tool For Note Takers
In Mathematics Classes

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Abstract: We designed a handwriting input system as a dictation tool to support hearing-impaired students in class. As numerous references to formulae and figures even during conversations in a mathematics class, a hand-writing input system is very useful. As these tasks require promptness, we designed some special edit functions to input content using pre-made ones. Moreover, to adjust to individual usability, we designed a function to suggest improving the system parameters.

1. Introduction

Note-taking is a standard service for the support of hearing-impaired students. As note-takers transcribe what the teacher speaks in class, one can do this support using only paper and pens. There are several software tools [1], [2] that facilitate this service. These are widely used for note-taking support and are very convenient for the following reasons.

1. The contents is reusable.
2. The system and data can be shared among several people.
3. The data are easily arranged.

Users use key board to input information to these systems.

In a class, teachers speak considerable number of words, then note-takers must process comprehensive information very quickly. With regard to usability, computer-driven tools hardly compete with a pen and a sheet of paper. Especially in Mathematics classes, there are numerous references to formulae and figures, then we can hardly input information by key board. Then, we designed a handwriting software tool for note-taking in mathematics classes. This system can enhance by employing an estimation function using Choquet integral of fuzzy measure. In this paper, we describe this software and an improve method.

We use an ultrasonic pen (Pegasus Scientific Inc.) in this system to input hand written sentences. The ultrasonic pen is a ball-point pen with a small ultrasonic generator and a sensor attached to the tip where the paper receives information from the pen. A user writes words, formulae, and figures as usual, and these are available in our software. These lines are reusable and are easily changed with its position. The details of this process are explained in section 2.

Some systems require the use of an ultrasonic pen with tactile display to support visually impaired persons [3], [4]. In contrast, our system views a hand-written sentence as a sequence of two-dimensional points. The data collected by our system is available with tactile display, and by means of some extensions, our system will help visually impaired students.

Several parameters contribute to controlling the usability of the system. While using the system, users have to perform various tasks very quickly, and the speed of the
process depends on these parameters. In general, there may be some configuration file(s) and some interfaces for arranging the values of these configuration files. In our system, users are also able to control the parameters by changing the configuration file directly. However, it is very difficult to determine optimal parameters. Hence our system has a function suggesting improvements. To determine the estimation function we use generalized Choquet integral [5].

Choquet Integral is defined on a fuzzy measure space, and the fuzzy measure can be replaced by a more general set function. In this analysis, the total set is the set of all parameters, and each measure value signifies the importance of corresponding parameter set. We assume that their operating times for a series of tasks can be expressed by a Choquet integral with respect to an unknown set function. Each value of integrand represents the actual value of corresponding parameter. We approximate the set function, which is a generalization of a fuzzy measure, using several samples of parameters and the corresponding operating time. We can improve these parameters by calculating the gradient of the integral value.

To define a general set function on a set of \( n \) points, we must select \( 2^n \) real value parameters. To reduce the number of parameters, we assume that the set function is a two-additive measure [5]. A two-additive measure is defined by \( n(n+1)/2 \) parameters. We explain this concept and its properties in section 4.

2. Outline of the System

Note-taking service requires promptness and a key board is not convenient enough to input mathematical formulae, the use of a pen and paper can not be avoided. Thus, we use ultrasonic pen (Pegasus Scientific Inc.) in this system to input drawing lines. The ultrasonic pen is a ball-point pen with a small ultrasonic generator, and it automatically inputs handwritten lines on a paper into the system. A user requires only this pen while drawing lines on the paper; he/she does not have to verify a monitor before the completion of a block of lines such as a character or some part of a formula. These input lines in the system are text data that are adequate for reuse, communication, and data sharing.

2.1 Data Format

All data are stored in a text file. An input line is an arranged sequence of two-dimensional vectors, and each line is defined by two attributes: color and width. Color is an RGB value that has the BMP file format. Width is a real constant represented by numbers with a fixed decimal point. Each page consists of some lines, and an entire document consists of some pages. The following figure is an example of a part of a document.

```
DRP:line Num=53 (0.000000,0.000000)-(816.000000,1169.000000)...................
DRL:12 pts,WD=3.000000,COL=[0,0,0]::(168.952206,1014.419035)-(168.2........
DRL:17 pts,WD=3.000000,COL=[0,0,0]::(188.911493,929.313445)-(188.2,.........
931.268761)-(190.864684,980.710339)-(198.6,......
DRL:11 pts,WD=3.000000,COL=[0,0,0]::(258.249791,978.475691)-(260.2,......
DRL:6 pts,WD=3.000000,COL=[0,0,0]::(233.509365,954.732561)-(237.41,......
```

Figure 2.1: Sample Document
2.2 Layout

We assume that two users use this system at a time. One user writes with a pen or edits the content, and the other user reads the pages and occasionally he also edits the content when the other user does not use the system. Hence, there are two view areas for the document. Drawn lines are input in the right-hand area and users can edit content in both areas. There are some buttons into the top area, and the pop-up menu is listed when the right mouse button is clicked. Users edit the content using these buttons.

2.3 Edit functions

Several functions have been designed for the effective use of the input content. The drawn lines are selected by a click or drag of the mouse, and standard edit functions, cut, copy, and paste are employed on them. Moreover, there are 5 memory areas (CB1, ..., CB5, at the center of the upper part of the area) for temporarily saving of the content. The selected content are stored in the area by clicking the left mouse button. The contents in the selected memory area is displayed when the mouse cursor moves on the corresponding button, and the content can be pasted anywhere in either of the view areas. The content is stored in the clipboard as a text document, and it can be accessed through other Windows applications. Users can save a part of the document in a text file using a text editor, and these text documents are also available in this system.
3. Parameters.

The following table lists parameters. The usability or efficiency of the system largely depends on these parameters. It is difficult to determine their optimal values as they differ from person to person. In this system, we have incorporated a function to provide a suggestion for the improvement of the parameters.

**Table 3.1: Parameters**

<table>
<thead>
<tr>
<th>Time:</th>
<th>shortTimeMD, enoughTimeMD, sortTimeCB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length:</td>
<td>smallMoseMove, nbhdDist, smallBend,</td>
</tr>
<tr>
<td></td>
<td>smallPenDist, hugePenDist, keyMoveStep</td>
</tr>
</tbody>
</table>

3.1 Time parameters

Some functions are called automatically depending on the movement of a mouse or pen or some other hardware interruption. If a function is executed for every interruption, the executions are too frequent, and this adversely affects the performance of the system. Thus, to ignore some interrupt events, these three parameters control the working of the functions.

The parameters "shortTimeMD" and "shortTimeCB" represent the waiting times for the mouse-dragging event and the mouse-moving event, respectively. During the mouse dragging event, the user determines a rectangular area if there are no selected objects. All drawn lines which intersect the rectangular area are selected as targets for move and copy functions. In the case where some objects are selected, the user determines the move position for the selected objects during the mouse move. In the case where the mouse cursor is on the button "CB01", "CB02", "CB03", "CB04", "CB05" (temporal memory areas), the stored content is displayed in the top-left position of the right-hand view area. These drawing functions are called when the mouse moves; and the parameter "shortTimeCB" represents a waiting time for the above situation.

If the mouse cursor moves quickly, mouse drag may end before the waiting time passes. A substantial amount of time("enoughTimeMD") is allowed to pass after dragging begins, the system calls a function for defining the move of the selected lines or the selection rectangle. Sometimes, if the move of the selected lines is very delicate, then these tasks are controlled not by length but by time. The function of defining the rectangle also controlled by length (smallMoseMove).

3.2 Length parameters

As discussed in the previous subsection, the parameter "smallMoseMove" controls the function that defines the rectangle for line selection. A user can also select one line by clicking the left mouse button. Then, the system finds the line nearest to the point where the left mouse button is clicked. In the case where the distance to the nearest line is less than "nbhdDist" (short for "neighborhood distance"), the system decides that no line is selected. If no line has been selected by the left button click, all other selections are initialized. In other words, a user can cancel all selections by clicking a vacant area.
To determine the move position of selected objects, a user can also use direction keys on the keyboard. "keyMoveStep" is a move length that is equivalent to one key press. In the case where a user need to finely adjust the position with the direction keys, this value should be small.

The parameters "smallBend", "smallPenDist", and "hugePenDist" control the number of data points of the drawn lines. Data points are reduced using these parameters, smoother curves can be reconstructed.

4. Choquet integral with respect to Two-additive measure.

Let \( A \) be an \( n \)-points set (finite set) and \( \mu \) be a fuzzy measure or a general set function defined on power set of \( A \) with \( \mu(\emptyset)=0 \). If \( \mu \) is an additive measure, all measure values determined from \( \mu(\{x\}) \) \( (x \in A) \), that is, \( \mu \) is defined by \( n \) values. However, a general set function is defined by \( 2^n \) values. Then, to reduce the number of parameters, we consider a two-additive measure as the first generalization of a non-additive measure analysis.

A two additive measure is given by
\[
\mu(A) = \sum_{x \in A} \mu_x + \sum_{\{x,y\} \subset A} \nu_{x,y},
\]
using real constants \( \mu_x, \nu_{x,y} \) \( (x, y \in A) \). Then two-additive measure on an \( n \)-point set is defined by \( n+n(n-1)/2 \) values.

The Choquet integral of a function \( f : A \to [0,1] \) is given by
\[
\int_A f \, d\mu = \int_0^1 \mu(\{x : f(x) > r\}) \, dr
\]
and in the case where \( \mu \) is two-additive, it is represented by
\[
\int_A f \, d\mu = \sum_{x \in A} f(x) \mu_x + \sum_{\{x,y\} \subset A} f(x) \land f(y) \nu_{x,y}
\]
Let us assume that a calculated value is represented by the above integral. We can approximate the measure values using a least mean square approximation. For this analysis, we need several sets of functions and calculated values. Functions are originally \( n \)-dimensional vectors. These functions can be extended to \( n+n(n-1)/2 \) dimensional vectors using the operator \( (x, y) \to x \land y \). This binary operator is so simple that degeneration sometimes gives bad influence. Hence, we use a general T-Norm \( \circ \) to generalize the above integral as follows [5].

\[
\int_A f \, d\mu = \sum_{x \in A} f(x) \mu_x + \sum_{\{x,y\} \subset A} f(x) \circ f(y) \nu_{x,y}.
\]

T-Norm [6] \( \circ \) is a binary operator on \([0,1] \times [0,1]\) satisfying.

1. \( 0 \circ x = 0 \), \( 1 \circ x = x \) for any \( x \in [0,1] \),
2. \( x \circ z \leq y \circ z \) when \( x \leq y \),
3. \( x \circ y = y \circ x \) for any \( x, y \in [0,1] \),
4. \( x \circ (y \circ z) = (x \circ y) \circ z \).

Let us assume that \( x \) and \( y \) are values of membership functions of two fuzzy sets \( A \) and \( B \), \( x \circ y \) is one extension of (a value of the membership function of) the intersection \( A \cap B \).
Some T-norms are defined with one real parameter, Dombi, Dubois (et al.), Yager, Hamacher (et al.) and Schweizer (et al.) ([6] ~ [10]) defined their own T-norms. We can optimize the approximation performance. Then, we verified the ability of the above approximation and chose Dombi's T-Norm for this system. This T-Norm is defined as follows.

\[ T_\lambda(x, y) = \frac{1}{((1/x-1)^\lambda + (1/y-1)^\lambda)^{1/\lambda}} \]

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5. Improvement of Parameters

In section 3, we discussed three time parameters and six length parameters. Three length parameters (smallBend, smallPenDist, and hugePenDist) and one time parameter (sortTimeCB) do not have a considerable effect on the usability or operating time. To improve the usability of the system, let us consider a five-parameter set and assume that the operating time can be expressed by the Choquet integral with respect to some two-additive measure. For the generalized Choquet integral using the T-Norm defined in Section 4, the integrand functions take the value in \([0,1]\). Hence, we have to normalize all parameters such that they take a value in \([0,1]\). Before obtaining a suggestion for any change in the parameters, we have to test the system and acquire several trial operating times. These values must be acquired using different parameters and the same target object. For each parameter \(p\), let \(m_p\) and \(M_p\) be the minimum and maximum values of \(p\).

Then the normalized values \(\tilde{p}\) are given by

\[ \tilde{p} = \frac{p - m_p}{2M_p - m_p}/2 \]

In this analysis we have \(5+5(5-1)/2=15\) coefficients to define one two-additive measure. Then a matrix made in a standard least mean square approximation must degenerate if the number of sample data is less than 15. However, the task to check the operating time is a kind of bother and 15 times trial may be many enough to give up to get suggestions.

Integrand functions, which are 5-dimensional vectors, are extended to 15-dimensional vector using Dombi's T-Norm. The parameter \(\lambda\) of this T-Norm is usually fixed throughout the analysis. When we choose different parameter \(\lambda'\), the extended vector is different even if the first 5 coordinate is same. Then varying the parameter randomly we obtain enough data for least mean approximation. Of course the larger the data size is the more reliable the result is. Assume that the Choquet integral with respect to approximated two additive measure and the T-Norm with some unknown parameter expresses the usability or the operating time, the each partial derivative with respect to normalized parameters denotes the suggestion on increase or decrease of corresponding parameter. Figure 5.1 is an example of this suggestion.
6. Conclusion

We designed a handwriting input system that can be used by note-takers to support hearing-impaired students in a mathematics class. There are nine parameters in some functions of the system. For five of these parameters the system provides some suggestion to improve its usability. The following are topics that could be addressed in future studies.

1. Design of a suggestion function for other parameters with no correspondence with operating time.
2. Estimation and improvement of the suggestion function.
3. Provision of support for visually impaired persons.

7. References


