An Analysis of Students' Research on Model Lessons That Integrate GeoGebra into School Mathematics

Jack A. Carter
jack.carter@csueastbay.edu
California State University East Bay

Beverly J. Ferrucci
bferrucc@keene.edu
Keene State College

Abstract: *After participating in a GeoGebra orientation, prospective elementary and secondary school mathematics teachers completed a senior-level course project in which they researched and prepared model lessons that incorporated GeoGebra into the curriculum they were preparing to teach. The future teachers then concluded the project by presenting their model lessons to classmates in the course. Presentations were videotaped and analyzed for content and categories of tools and activities that were used in the lesson. All model lessons emphasized multiple geometric constructions and measurement activities. Project presentations showed that the prospective teachers exhibited considerable expertise in using GeoGebra's coordinate axes, sidebar display of measures, and a various measurement tools in presenting the results of the research. Results showed that over 80% of the future teachers used some form of dynamic geometry in their presentations, although there were often missed opportunities wherein dynamic geometry could have provided demonstrations that were more extensively and illustratively presented. Student presentations also used dynamic geometry as aids in constructing figures for the presentations, as a means for adjusting lengths to integer values, and as possible extension activities for the model lessons. Conclusions include suggestions for increasing the effectiveness of dynamic geometry and the use of sliders in future student research projects.*

1. Introduction

As Falconer and Holcomb [2] note, undergraduate research experience is a powerful complement to conventional classroom learning that gives students valuable preparation for graduate education or careers. In mathematics teacher education, student research can provide scholarly experiences where students apply their knowledge to address novel problems and applications. Today, a major application of technology in mathematics education is the integration of mathematical software into teaching practices.

Fahlberg-Stojanovska and Stojanovski [1] examined technology's current contributions in mathematical education and concluded that the open source software or freeware, GeoGebra, can be of "tremendous value to students in their understanding of mathematics from the smallest ages on up". As its name suggests, the GeoGebra freeware aims to provide a bidirectional combination and connective link between the visualization capabilities of computer algebra systems and the dynamic changeability of dynamic geometry systems. Little [3] reviewed the development of dynamic geometry and found barriers to its implementation related to curricular scope and accessibility, while Lu [4] identified stages of teachers' progress from starting to learn GeoGebra to teaching by utilizing it in the classroom. The current study sought to focus on easing the barriers to implementation identified by Little [3] by involving pre-service teachers in the first stages of developing expertise with GeoGebra within the framework of an undergraduate research project.

2. Methodology

The prospective teachers who participated in this study were enrolled in a senior-level seminar in mathematics education, and the research project was a graded component of their coursework. The seminar participants were five prospective secondary school teachers, one pre-service middle school teacher, and five future elementary teachers. The authors of this report presented a GeoGebra orientation consisting of two hour-long activities that provided participants with opportunities to construct geometric figures and perform mathematical investigations. During these activities, students worked individually on a classroom set of laptop computers, each with a wireless Internet connection to the GeoGebra website.

The orientation's first part, the construction activities, enabled students to complete the following constructions: a segment bisector, a square, the circumscribed circle of a triangle, a parallelogram with its vertex angle measures, a tool for drawing figures with line symmetry, and a pentagon and its image under a rotation. In these activities, students used GeoGebra to construct, measure, and transform geometric figures. Students worked independently in class and were required to show an instructor a completed construction and to demonstrate dynamically the soundness of the construction before moving on to the next construction. In the segment bisector construction, for example, instructors asked students to drag one of the endpoints of the segment to confirm dynamically that the construction was correctly and accurately executed. Each student completed the six constructions successfully within an hour and each student was then given a two-page evaluation form to rate the difficulty levels of the activities and GeoGebra tools on a Likert scale, as well as to elaborate on their likes and dislikes related to GeoGebra.

After students completed the evaluation form, they moved on to investigative activities in which they used GeoGebra to investigate midpoint polygons and the slope-intercept form of a linear equation. In the activities on midpoint polygons, students first constructed a quadrilateral and the midpoint of its sides. Then they connected the midpoints to form the midpoint quadrilateral and measured the areas of the two quadrilaterals. Students were then invited to drag a vertex of the original quadrilateral, compare the areas, and make a conjecture about how the areas were related. Students were next asked to measure the vertex angles and side lengths of the quadrilateral, to make a conjecture about the midpoint quadrilateral's type, and to explain their reasoning.

The investigation of the slope-intercept form introduced students to GeoGebra sketches and sliders that allowed them to manipulate manually the values of variables and simultaneously observe the impact on the sketches. After opening a sketch of the graph of the linear equation $y = 2x + 1$ on a coordinate system, students were invited to change the values of m and b in this $y = mx + b$ form by dragging the sliders for these variables. Students were then asked to describe the effect of adjusting m's slider by detailing the differences on the linear graph resulting from values of m that were positive, negative, and zero. Students then answered a corresponding set of questions about adjustments to b's slider and the effect these had on the line. After practice with activating and deactivating a feature to trace graphically any movements of the line, students were next invited to activate the trace, to adjust a slider, and to answer questions concerning the family of lines that resulted from varying the m and b values. The last part of the investigative activities asked students to write the equation of a line in slope-intercept form given specific information about the line. The information provided in this part of the activities included a given value for the slope along with either an *x*- or *y*-intercept or some other point, or alternatively, two given points. Students were also reminded that they could check their answers by adjusting the *m* and *b* sliders.

At the next class meeting students were advised of their assignment to research GeoGebra model lessons and to make a presentation to the class of a model GeoGebra lesson appropriate for the grade level that they were preparing to teach. Students were reminded that the activities they completed during the last class session were an introduction to the application and that, in addition to standard educational research sites, specific information about aspects of the application were available on the GeoGebra website and at a companion website that served as a user's forum.

3. Results

Both qualitative and quantitative data resulted from the prospective teachers' participation in the GeoGebra research project. Qualitative data included their GeoGebra constructions during the orientation and demonstrations of their model lessons research to the class. Quantitative data included students' evaluations of the initial construction activities and tools, and their responses to, and scores on the investigative items during their GeoGebra orientation, GeoGebra research presentations, and course scores.

3.1 Qualitative Results

Qualitative data resulted from students' constructions during the GeoGebra orientation that were saved as files and assessed by the instructors. A second major source of qualitative data came from the students' research and presentations of GeoGebra model lessons. To insure that students researched different model lessons, the seminar leader individually approved each student's selected model lesson. As a result, students' research involved a total of eleven different models: volume of a pyramid, circumference and area of a circle, Pythagorean Theorem, triangle congruence, area of a trapezoid and a rhombus, volume of a prism, tessellations, parallel lines and transversals, reflections and rotations, midpoints and similar triangles, and medians of a triangle. The seminar participants delivered short presentations of how to teach the topics using GeoGebra and provided accompanying written materials such as an outline of the lesson with learning objectives, a description of students' prerequisite knowledge, and website references, as well as a variety of student worksheets, assessment and evaluation forms, and teachers' guides.

3.2 Quantitative Results

After completing the GeoGebra construction activities during the initial orientation, students evaluated the activities and tools they had used. Table A shows the ratings that students assigned the activities in this evaluation with a rating of 1 indicating very easy and a 5 indicating very difficult. In this table the five future secondary school teachers are designated as s1 to s5, the one future middle school teacher as m1, and the five future elementary school teachers as e1 to e5.
Although the overall average of students' ratings for the six activities was 2.2, further analysis showed that the average difficulty levels ranged from 1.3 to 2.9 for the activities and that the average difficulty levels increased from easiest to most difficult in the order shown, from top to bottom, in Table A. For the pre-service elementary school teachers the overall average difficulty level for the activities was 2.4, while for the prospective secondary school teachers this average was 1.9.

Table A Students' Ratings of the Difficulty Levels of the Construction Activities in the GeoGebra Orientation

ACTIVITY	s1	s2	s3	s4	s5	m1	e1	e2	e3	e4	e5
Segment Bisector	2	1	1	1	1	1	2	1	1	2	1
Square	2	1	2	2	2	2	2	2	2	4	1
Circumscribed Circle	2	1	2	2	1	2	2	2	3	3	2
Parallelogram	3	1	1	2	2	2	2	3	3	3	2
Line Symmetry Tool	3	2	3	2	2	3	2	2	3	4	2
Rotated Pentagon	3	3	3	3	1	3	3	3	4	4	2

Table B Students' Ratings of the Difficulty Levels of the Construction Tools Used in the GeoGebra Orientation

TOOL	s1	s2	s3	s4	s5	m1	e1	e2	e3	e4	e5
Segment from points	2	1	2	1	1	2	1	2	1	2	2
Circle	2	1	1	2	1	2	1	2	1	2	2
Intersect two objects	2	1	2	2	1	2	1	2	2	3	2
Move	2	1	1	1	1	1	1	2	1	2	2
Polygon	3	2	1	3	1	2	2	2	1	2	2
Line bisector	3	2	1	2	1	2	2	2	2	2	2
Moving drawing pad	3	1	1	2	1	1	2	2	1	2	2
Zoom out	2	1	2	1	1	1	2	2	1	1	2
Perpendicular line	2	2	2	2	1	2	2	2	3	3	2
Parallel line	2	2	2	2	1	2	2	2	3	4	2
Angle	2	2	2	3	1	2	2	2	3	3	2
Mirror object at line	2	2	2	3	1	1	2	2	4	4	2
New point	2	1	1	1	1	1	2	2	1	1	2
Rotate object	2	3	3	2	1	3	2	2	4	2	2

Table B shows the participants' ratings of the difficulty levels for the tools used in the GeoGebra orientation. Using the same rating scale as for the activities, the overall difficulty level for tools was 1.8, and further analysis revealed that average difficulty levels for the tools ranged from 1.4 to 2.4. For the future elementary school teachers the overall average difficulty level was 2.0 and for their secondary level counterparts this value was 1.7.

Another initial source of quantitative data was students' responses to, and scores on, the investigative items during the GeoGebra orientation. Figure 1 shows each of the students' percent scores on all the investigative items, as well as each student's percent scores on the two components of the investigative items, the midpoint items and the slope-intercept items. Since the results shown in Figure 1 showed a contrast in the percent answered by the participants based on their intended level of teaching, a further analysis of the percent of investigative items was undertaken.

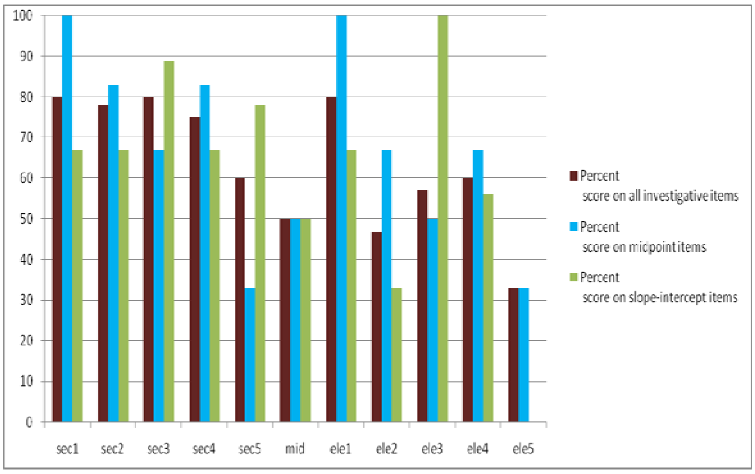

Figure 1 Percent Score on All Investigative Items and on Midpoint and Slope-Intercept Items

Analysis of students' responses showed that the percent of the 15 investigative items answers ranged from 20 to 100. Figure 2 shows the percent of the investigative items answered for each of the five future secondary school teachers (sec1-5), the one future middle school teacher (mid), and the five future elementary school teachers (ele1-5) who participated in the research project.

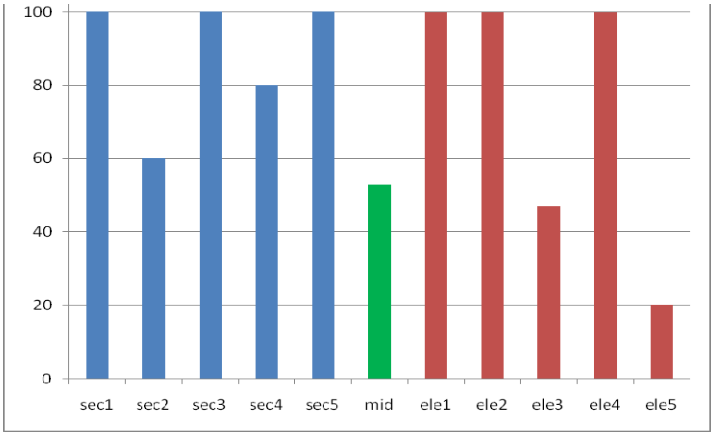

Figure 2 Percent of Investigative Items Answered by the 11 Future Teachers

Further analysis of the students' responses and scores on the investigative items continued to show contrast between the average percent answered and the average percent score when these averages were compared for the prospective secondary school and the prospective elementary school teachers (Figure 3) and this contrast persisted, although was somewhat numerically diminished, when average percent scores were compared for the two groups on midpoint and slope-intercept items (Figure 4).

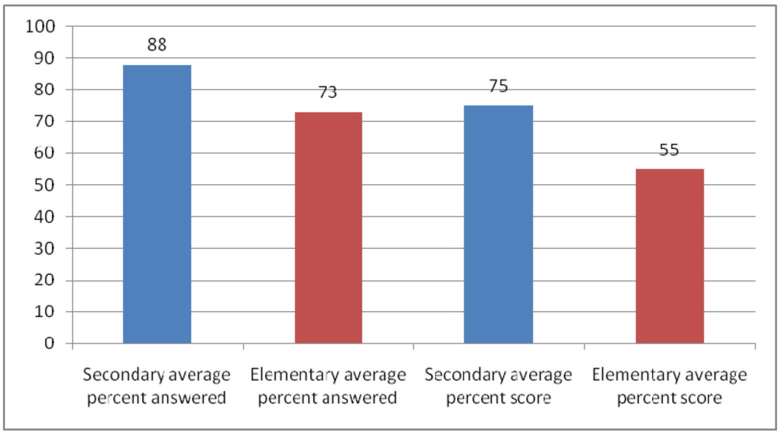

Figure 3 Average Percent Answered and Average Percent Scores for the Pre-Service Secondary and the Pre-Service Elementary School Teachers

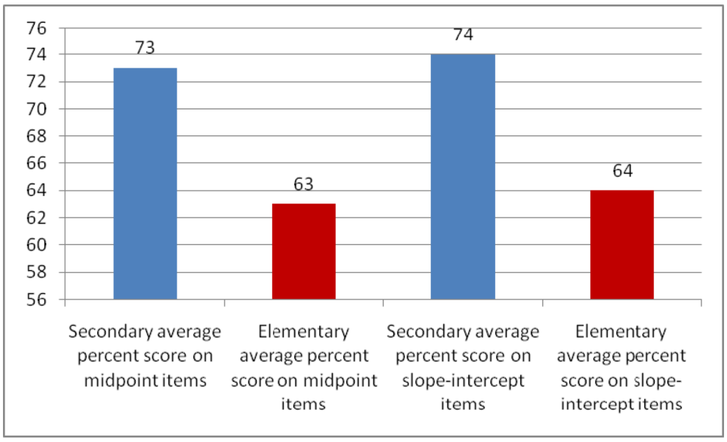

Figure 4 Average Percent Scores on Midpoint and Slope-Intercept Items for the Pre-Service Secondary and the Pre-Service Elementary School Teachers

Another contrast in the quantitative data was evident in a comparison of students' percent scores on the presentation of their GeoGebra research and their percent scores in the course. Particularly, Figure 5 shows the scores on the GeoGebra research, although roughly aligned to the course scores, tended to be somewhat higher for all except one of the pre-service teachers.

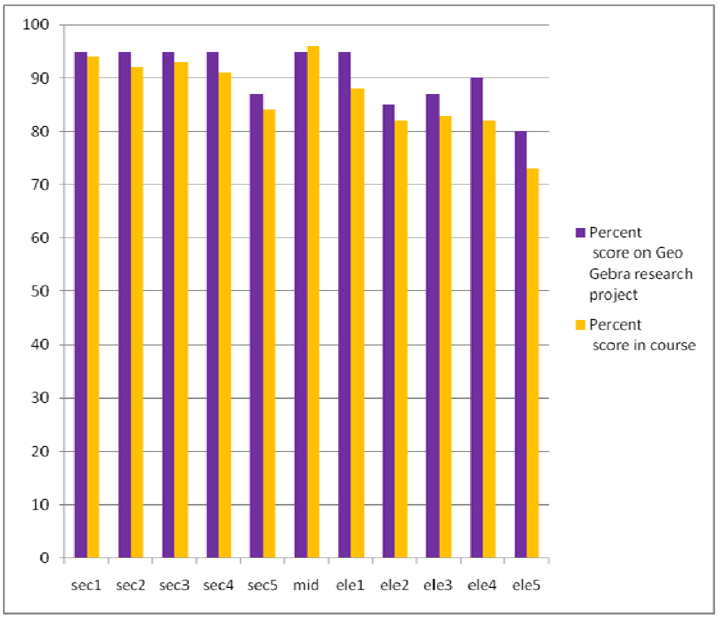

Figure 5 GeoGebra Research Project and Whole Course Average Percent Scores

4. Conclusions

The quantitative results for the prospective secondary school students provide good evidence that the orientation and investigative activities provided adequate background for these students to obtain an initial grasp of the GeoGebra. Tables A and B show that both groups of pre-service teachers rated the construction tools and activities in the orientation to be easy. Overall, the pre-service secondary school teachers answered an average 88% of the investigative items and achieved a mean score of 75% on these items. The pre-service elementary school teachers answered an average of 73% of the investigative items with an overall average score of 55%. These 15 and 20 percentage point differences suggest that the preliminary GeoGebra session was more challenging for the future elementary school teachers than for their pre-service secondary school counterparts. Overall course scores on the research project showed that average course scores for the prospective secondary school participants were 93% versus 87% for the pre-service elementary school participants. Ameliorating these findings, however, is the result that both groups of future teachers scored somewhat higher on the research project than they did in comparison to their overall score in the course (Figure 5). Tables A and B further show that both groups rated GeoGebra activities and tools as easy.

The qualitative review of students' presentations of their research showed that students exhibited considerable expertise in using GeoGebra's coordinate axes, sidebar display of measures, and a variety of measurement tools in presenting the results of the research. Moreover, the research topics selected by the students for their research and class presentation all emphasized multiple geometric constructions and measurement activities. A review of the presentations also showed that 9 of the 11 student presenters used dynamic geometry in some way to illustrate or execute the model lesson that they had selected and researched. Dynamic geometry, when it was used by the students, often served rather utilitarian purposes: as an aid in constructing figures for presentations, as a means for adjusting lengths to integer values, and as a possible extension activity to a model lesson. Although looking at the videos of the presentations can lead to an observation that there were missed opportunities in which dynamic geometry could have provided demonstrations that were more extensively and illustratively presented, it should be kept in mind that these were novice GeoGebra users and that their GeoGebra orientation sessions and subsequent assignment did not explicitly emphasize the importance of dynamic geometry in completing the research project.

Construction and measurement of mathematical objects are prerequisites to using GeoGebra effectively, but the investigative power of this application lies in its facility to make mathematics dynamic. The model lessons researched and presented by these pre-service teachers emphasized the construction and measurement stages of using the application, but generally did not make the transition to a stage where they fully used the available dynamics. In retrospect, the authors believe this an area for improving the student research experience. That is, in future orientations the authors would focus more on the uniquely dynamic feature of such applications and the power of the dynamic geometry to enhance understanding of mathematical concepts. However, at the time of the original orientation, the authors were more attentive to not limiting students' research topics to only geometric ones. Particularly, the authors had hoped that students would exploit the use of GeoGebra sliders more in their research. Although sliders were important components of the investigative activity on the slope-intercept form during the orientation activities, the authors noted that none of the student researchers constructed sliders to illustrate or investigate their own research. Two student research presentations did incorporate sliders, but these were sliders that part of an online demonstration that was used to illustrate what the student presenter hoped to do if more time were available for the research. Since sliders have the potential to expand the range of research topics that students can address, the authors believe that more attention to constructing sliders during the orientation is another area that can improve future GeoGebra student research projects.

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