Problem Posing and its Environment with Technology

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Abstract: *The OECD reported a "mathematising ability" to extract mathematics from a problem situation as a mathematical literacy, is important. In this research we gave students the task of problem posing and analyzed their perspectives in creating problems. Their posed problems were almost all very similar and the students' passive attitudes toward mathematics were shown in an environment with paper and pencil. In order for students to be able to pose various kinds of own problems, they must change from a passive to an active approach. We will show the new possibility in the learning environment with technology as an appropriate method for these activities. An ideal environment to search various problems from one situation is one in which various expressions can be considered at the same time. Such environments include graphs, tables, mathematical expressions, geometric figures, and etc., and students can manipulate them in a problem situation dynamically. A leaning environment with technology encouraged students to find two or more methods to propose a problem and to confirm the forecasted solution through experiment. We will show a task which includes various ways for posing new problems and exploring them through the use of technology.*

1. Background

In a math class, we occasionally give students a task to make a short-story concerning a point of mathematics and to present it. Students do not develop their own idea by themselves and get various topics on the Internet and present them in a class like they think by themselves. When we gave a task of problem posing in an environment with paper and pencil, it was dealt by students with similar themes and similar problems in the situation were posed. The students' passive attitudes toward mathematics were shown in these activities.

In [1], there is the general strategy used by mathematicians, which the mathematics framework will refer to as "mathematising". An individual who is to engage successfully in mathematisation needs to possess a number of mathematical competencies. To identify and examine these competencies, PISA has decided to make use of eight characteristic mathematical competencies. One of them is "*Problem posing and solving*".

1. This involves posing, formulating and defining different kinds of mathematical problems (for example: "pure", "applied", "open-ended" and "closed"),
2. Solving different kinds of mathematical problems in a variety of ways.

The report cautions about mathematics education which only focuses on answering questions. The recommended approach for a math lesson is not to answer the given questions, but to mathematise the situation (to extract mathematics from a problem situation). There is an emphasis on the importance of the preliminary mathematising step before answering the questions. We think students' passive attitudes are related to a traditional mathematics education in which we have not emphasis this step. We planed activities for problem posing and solving posed problems to mathematise a situation.

2. Purpose

(1) To evaluate how students mathematise the situation through activities for problem posing and solving friends' posed problems.
(2) To analyze how to support the technology as an appropriate method for these activities.

3. Methods

<u>Subject: </u>17 students (12 males, 5 females), university students, 21 or 22 years old.

Problem: Turn the page horizontally to the "landscape" style. Take the top left corner (the point A) and fold it down to meet the bottom long side CD. Point A becomes point E on CD and can move along CD (see Fig.1) (from [6]). What problem can you construct from this figure?

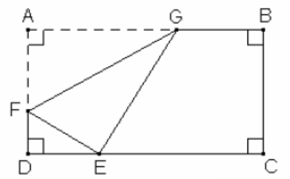

Fig.1

Process:
1) Students pose a problem using paper and pencil.
2) Their posed problems are exchanged and evaluated by each other.
3) We show how to use technology for different approaches in this task, then students try to explore a new problem again and confirm their new approach by using technology.
4) Analyze the influence of using technology.

4. Problem posing by paper and pencil

The following problems were posed.

Problem 1:

In this figure (see Fig.2), some pairs of triangles are congruent to each other and other triangles are similar to each other. Identify all of them.

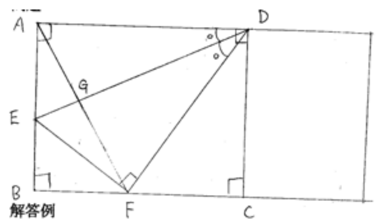

Fig.2

This kind of problem was posed by some students. When they constructed a figure on this situation, it was happened that in their figures some parts were incorrect. For example their triangles DAE and DFE were non-symmetrical. Maybe it comes that they drew the segment DE at first.

Problem 2:

In rectangle ABCD, take the top left corner (point B) and fold it down to meet the bottom long side CD. Point P is on CD, and point E is on BC. PC=x, PD=y. (see Fig.3)

1) *Prove "triangle EPC is similar to triangle APD"*
2) *Calculate the value of x and y*

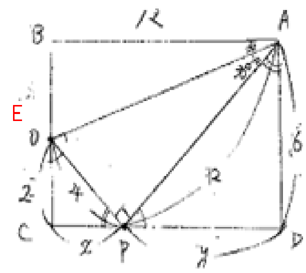

Fig.3

Many students posed these kinds of calculating problems. Sometimes, in their problem, there were too many contradictory assumptions to construct the figure. Therefore friends who answered this problem could not construct the figure, then they had to change the value of the length of the segments for constructing it.

Problem 3:

Point P moves on side AB, 1cm per second. Point Q moves on side BC, 2cm per second. Calculate the area of the triangle PBQ under this condition.

1) *After 4 seconds.*
2) *When does the maximum area occur?*

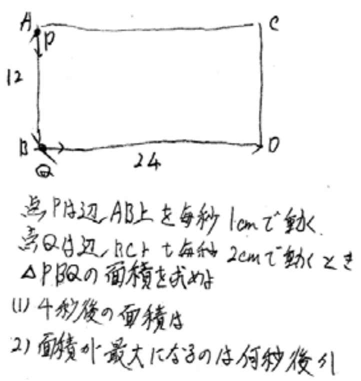

Fig.4

One student posed this problem which included the idea of functions. He referenced a problem he had learned in middle school. But in this problem, the assumptions "take the top left corner and fold it down to meet the bottom long side CD" were not satisfied.

Through their activities in the paper and pencil environment, we got the following results,

1) Students were interested in solving and evaluating problems posed by friends and during these

activities, they discussed actively.

2) There were many calculating problems with fixed numerical values. There were few problems with symbolic.
3) The students were stuck in the drawn figures by hand. And they could not see any symmetrical relation in their figure, or they did not notice the contradictory parts in their figure.
4) Students did not have any dynamic viewpoint with which they could move the figure under a special condition.
5) Some of the problems were not coherent. Because in these problems, too many assumptions were set to construct the figure.
6) A few students tried to make generalizations by using specific values or conditions.
7) There were few posed problems from a new aspect. Most problems were similar to others.

5. Problem posing with technological tools

In the recent technological environments, when the condition is changed in a problem, we can simulate the influence of it and our conjecture is confirmed at once. Especially in multi-representative environments (tables, graphs, expressions, geometrical figures and etc.), we can approach the task in various ways. We showed students various ways for approach in this task by using technology. Then students tried to pose new problems again and to confirm them using technology.

<u>To approach geometrically</u>

Two different construction problems are posed in case of giving the point E and giving the point G. But some students gave too many assumptions or gave contradictory assumptions to construct this problem, then they found their figure could be not constructed correctly using Cabri-Geometry (see [a]), though they had thought their figure was correct in the paper and pencil environment. When students moved some points in the figure constructed by a friend, the figure was broken. Their figure was not satisfied by some given assumptions though it looked correct. They could understand necessary and sufficient condition in the construction problem. When students construct a figure by using technology they can confirm the figures' validity because that the figure is not broken when they move any points in it.

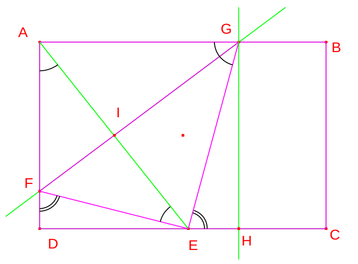

Fig.5

<u>To approximate the relation using a scatter diagram</u>

This method is often used in a workshop for graphing calculator. At the first, we experiment and collect data, then draw a scatter diagram and find the relation between these data. In this problem posing we expected students to pose a problem with the relation between the length of ED (x) and the area of triangle DEF (y). The Fig.6 is an example of this situation shown by TI-Nspire CAS (see [c]). We move the point E in the figure on the screen and we can collect data x and y. Then, these data are input into a spreadsheet and the scatter diagram is drawn. After that, we can also draw a regression curve using a software function. In this environment, students found there was a posed problem about the relation between the elements in the figure.

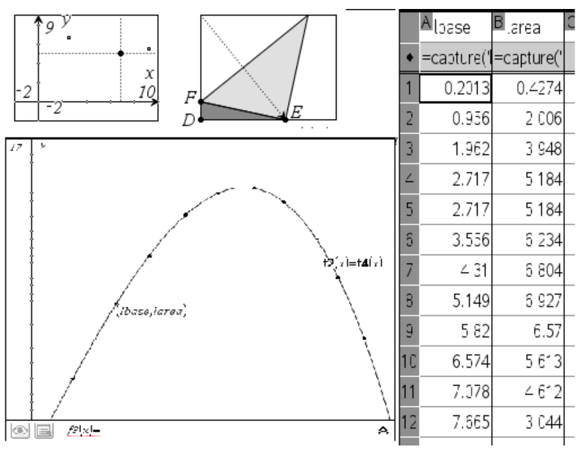

Fig.6

To approach for mathematical function

In this task, we can define various paired valuables (x) and functions (y). For example,

1) when x=DE, y=(area of triangle DEF),
2) when x=(angle of EFD), y=(area of triangle DEF) ,
3) when x=AG, y=(area of triangle GEF) and
4) When point E move on CD constantly, x=time, y=(position of G).

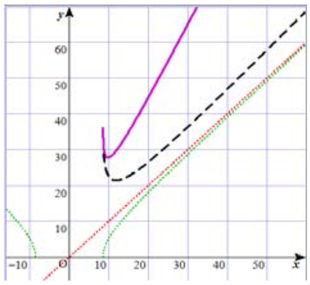

Fig.7

Although it was difficult for students to define these functions, by manipulating some points on the figure they could find there were many paired variable to have some relation in this problem. After defining an algebraic expression, students could simulate the behavior of the function. For example, the graph of the third function above was drawn by Grapes (see [b]) like the top curve in the Fig.7. So the students could see the minimum point for this function and posed a problem: what is it? Then students could construct a new problem continuously for other paired variables.

To expand to a calculus problem

If we can express algebraically the function of an area, we can pose a mini-max problem. For example in the 2nd function above, the max area of triangle DEF was found by differential calculus using TI-Nspire CAS (see Fig.8). It‘s a complicated calculation, but we could calculate it using CAS.

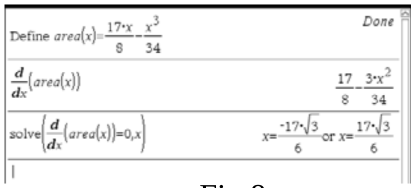

Fig.8

When the value was the max, students found AF:FD=2:1. They returned to the original problem and found an interesting geometrical relation. Moreover, their interest expanded to connect this problem with trigonometry.

To explore a conic curve problem

When we make the fold line FG continuously, we can find the rule for those lines and draw their locus. We showed students the Fig.9 drawn by GeoGebra, and it was a parabola. Students were surprised at drawing the locus. In the next step, they posed a new problem in which they expressed algebraically the function and found the focus and directrix for the parabola. But they had no confidence of this result. Then, they wanted to make sure this result. The locus of these fold lines was drawn by GeoGebra(see Fig.10). They input the command “Parabola [A, d]”. This parabola overlapped with previous one, then they could confirm their result using the software that the locus is the parabola with focus point A and directrix x-axis,.

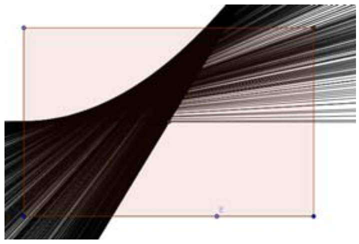

Fig.9

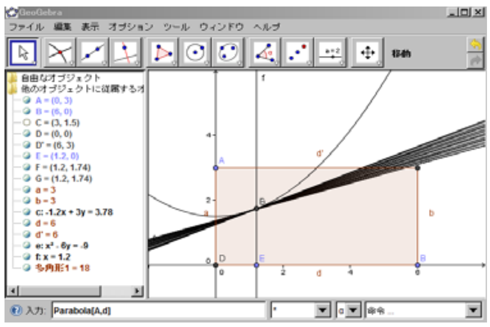

Fig.10

6. Discussions

6.1 Use of aids and tools

In [1], they decided to make the use of eight characteristic mathematical competencies necessary to “mathematise”. One of them is “*Use of aids and tools*”.

1) This involves knowing about, and being able to make use of, various aids and tools (including information technology tools) that may assist mathematical activity
2) Knowing about the limitations of such aids and tools.
3) This involves knowing about and being able to use familiar aids and tools in contexts, situations

and ways close to those in which their use was introduced and practiced.

The report describes the use of aids and tools as a positive step. And in this research especially IT tools made a lot of aids for students to expand their viewpoints.

6.2 Mathematics+Man+Computer solves the problem

In [7], Shimizu said "In European and American mathematical education, teachers and government officials think not only of mathematical performance with paper and pencil but also with computers or other technology. They bring mathematical competency into view and then require further effort by requiring students to solve problems using technology". In Japanese education, we have not set the ability of using technology as our goal, but the point is that it's necessary to cultivate this ability.

6.3 The importance of conjecture

In [8], Watanabe said "It's not interesting to solve a problem if we already know the answer. When we wonder where the answer is, we want to construct a new solution using all our knowledge. Then, technology has the possibility to lead us to the solution." But usually in most math classes, students solve a problem in which answers are known Therefore, when we gave students a posing problem, at the beginning, all students in the class said, "I didn't understand how to explore the problem or how to pose a new problem. I had no method". After we showed various approaches to pose a problem by using technology, students changed to try to conjecture new own ideas by using dynamical movements in the problem. It was a strong motivation for students to mathematise a situation.

6.4 Starting from mathematisation and successfully progressing

In [1], the five steps of the mathematisation process are explained and these steps are shown in the Fig.11 and listed below:

1. Starting with a problem situated in reality
2. Organizing it according to mathematical concepts and identifying the relevant mathematics
3. Gradually trimming away the reality through processes such as making assumptions, generalizing and formalizing, which promote the mathematical features of the situation and which transform the real-world problem into a mathematical problem that faithfully represents the situation
4. Solving the mathematical problem
5. Making sense of the mathematical solution in terms of the real situation, including identifying the limitations of the solution.

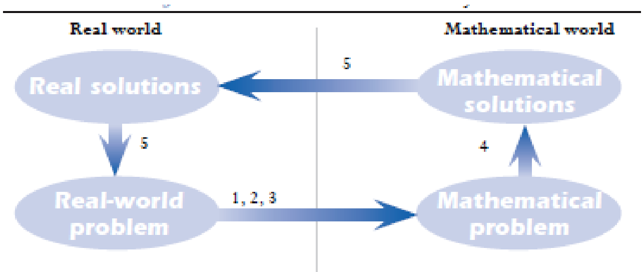

Fig.11 The mathematisation cycle (from Reference [1])

This reminded us the importance of the role of these five steps and gave us a warning about performing only the fourth step to answer a question. So, we gave students the task of posing problem and solving own posed problem. It gave students a chance to mathematise a situation and to change their viewpoints in a math problem.

7. Conclusions

We summarize the results of the activities in this research as follows:

1) Through posing problems and solving friends' posed problems, students' attitude to approach mathematics changed from passive to active and students changed to discuss actively during their activities even in the paper and pencil environment.

2) Students understood the problems more deeply by reconstructing the expressions in the problem by using technology. This step is very important because students summarize and confirm

the problem's conditions.
3) By using a technology students got various intuitions, and their explorations were widespread.
4) A leaning environment with technology encouraged students to find two or more methods to pose his/her own problem and to confirm the forecasted solution through their experiments.
5) Students could develop a dynamic viewpoint with which they could move the figure under a given condition.
6) Students could form an aspect by which they expressed a problem symbolically or made generalizations since they could change the value or move the point in a figure.

Software for Education

- [a] Cabri Geometry II Plus. Dynamic Geometry Software, Product of Cabri log. http://www.cabri.com/
- [b] Grapes. Graph Presentation and Experiment System, Developed by Tomoda Katsuhisa, It's free, and download by this URL : http://www.osaka-kyoiku.ac.jp/~tomodak/grapes/
- [c] TI-Nspire CAS - Teacher Edition. Product of Texas Instruments, It can simulate the Graphing calculator TI-Nspire on PC. It includes dynamic geometry software, graphing calculator, spreadsheet, and CAS. http://education.ti.com/
- [d] GeoGebra. Developed by Markus Hohenwarter, It's a multi-platform dynamic mathematics software for all levels of education that joins arithmetic, geometry, algebra and calculus. Recently includes spreadsheets. Free and download from following website. http://www.geogebra.org/

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