

# Perspectives on Instrumental Orchestration and Assessment - from Challenges to Opportunities

*Lenni Haapasalo*

[lenni.haapasalo@joensuu.fi](mailto:lenni.haapasalo@joensuu.fi)

University of Joensuu, Finland

**Abstract.** *When developing instructional praxis of mathematics, there are following tensions to be considered: (1) Objectivism vs. Radical Constructivism, (2) Developmental Approach emphasizing procedural knowledge vs. Educational Approach stressing conceptual knowledge, (3) Gagne's Systematization emphasizing guided learning vs. Minimalist Instruction emphasizing student's volition to learn, (4) Instrumentation where technology is shaping the actions of doing mathematics vs. Instrumentalization where technology is shaping also the mathematical objects, (5) Learning by Instructional Materials vs. Learning by Design, (6) Teaching mathematical contents vs. Emphasizing sustainable heuristics from the history of mathematics, and (7) Looking internal problems of mathematics education vs. Applying business principles to shift the bad reputation of mathematics. This article represents responds to these dilemmas and discusses pedagogical implementations with critical issues.*

## Introduction

To design research-based frameworks for instructional praxis we need to combine knowledge of mathematics and its history, philosophy, psychology, sociology, physiology and ICT, for example. Technology has caused a holistic change in our "mental art", i.e. in the way we think, plan, work and evaluate in a modern society. This implies that educators are forced to shift their views of mathematics and learning of it. Even though the development of solid praxis theories might be tough, viable and sustainable products are very often beautiful, simple and easy to use. This has been the starting point of my *MODEM* –project, having been a basis for me as teacher educator about two decades. The idea to use a quasi-systematic model for the planning of learning environments but on the other hand to allow free architecture of learning in the authentic learning situations sounds contradictory. However, this is the way our everyday life runs. We pick our food quite randomly from buffet, for example, even though the kitchen personnel have used all its expertise to plan and construct things for us very systematically. To emphasize challenges arising from this meta-text, I will start with socio-constructivist issues. Even though moving from old-fashioned pacing within Gagne's ([7]) approach to Minimalist Instruction appears to be necessary in authentic learning situations, quasi-systematic models are useful for the planning of learning environments and assessing of students' cognitive level. On the basis of the development of scientific and individual knowledge and the history of heuristics, I will discuss how mathematical knowledge might come into student's heads, into life and into action. Zimmermann's [21]) model is meant to be an invitation to develop new kind of methodology, especially because the use of modern technology appears on different levels of instrumentation and instrumentalization.

Technological development together with a changed conception of knowledge and learning could lead to a paradigm shift: learning of mathematics is more distributive (i.e. independent of time, place and formal modes), socio-constructivist (learning community centred) and technologically enhanced ([12], p. 167). This potential is used not only via networks or

computers but also on calculators and communicators, which students use in informal way on their free time. When using a tool within more or less spontaneous procedural<sup>1</sup> actions, the tool, especially at the beginning, puts certain limitations on what can be investigated and how. Adapting the term of Trouche ([20]), I mean by *instrumentation* the process when the tool shapes the actions of the users. On the other hand, users often find their own schemas and schemes to utilize the tool. In this process of *instrumentalization* not only the usage of the tool, but also the objects to be investigated are shaped by the users. Today, any sophisticated user skips unnecessary manipulation of  $1/\sqrt{2}$  to  $\sqrt{2}/2$ , for example, by using the calculator keys SQR and 1/X. This instrumental genesis has an impact on how mathematical representations appear for a modern citizen.

The instrumentation of a sophisticated impersonal device to instrumentalized personal device is often a long process of instrumental genesis. This forces educators to re-consider the interaction between process and object features of mathematical knowledge. At the same time, to make problem-solving devices to pedagogical tools, intensive co-operation is needed among hardware/software designers and researchers of mathematical learning processes. I will interpret Trouche's term *instrumental orchestration* in wide sense to discuss some of the most important challenges (Ch1-Ch7) and opportunities when developing research-based models to scaffold student's mathematical development.

## Solid theories for collaborative social constructions (Ch1)

*Mathematics education research should produce sustainable and viable frameworks for instructional praxis so that teachers would be able to scaffold students' collaborative constructions towards viable knowledge.*

This demand can be simplified by quoting von Glasersfeld ([8], p. xiv): "Teacher's efforts have the purpose of helping students to cope in the world of their experience". The slogan of constructivism in our scientific community appeared about twenty years ago (ICME 6). Later on, this discussion extended to include collaborative social group processes. Still, there are quite few empirically tested models how teachers can plan and realize learning environments within that paradigm. Even though there are numerous studies on *collaboration* and *group development*<sup>2</sup>, the impact of knowledge structure, pedagogical philosophy and support for reflective communication has been neglected.

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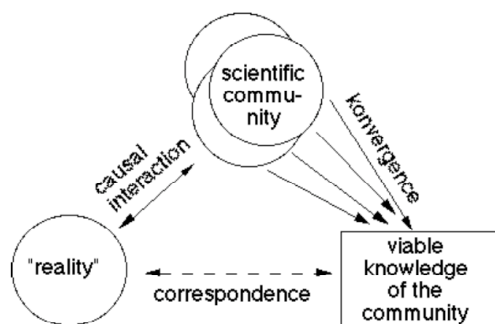
<sup>1</sup> I adopt the following characterizations of [11]:

- *Procedural knowledge* denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.
- *Conceptual knowledge* denotes knowledge of particular networks and a skilful "drive" along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.

Based on the logical relation between these knowledge types, two pedagogical approaches are defined: *developmental* and *educational*. The first one is based upon *genetic view* (i.e. procedural knowledge is necessary for conceptual one) or *simultaneous activation view* again. The logical basis of the second one is *dynamic interaction view* (i.e. conceptual knowledge is necessary for procedural one), or the *simultaneous activation view* (i.e. procedural knowledge is necessary and sufficient for conceptual knowledge). The latter means that the learner has opportunities to activate conceptual and procedural features of the current object simultaneously by making mental or concrete manipulations of the representatives of each type of knowledge, for example.

<sup>2</sup> When typing-in 'Tuckman group process', for example, Google produced more than 72000 hits with interesting links, whilst for "collaboration" there were almost 100 million hits (retrieved 10<sup>th</sup> of Feb 2008).

As regards an appropriate *socio-constructivist* framework, the famous *pragmatic theory of truth* emphasized by the famous philosopher Charles Peirce would give a solid basis, making the debate between radical and weak constructivism sound unnecessary and even naïve. When an investigation space has been designed (an open problem is given), namely, the teams work in causal interaction with this problem under collaboration. After testing the viability of radical ideas among the teams and between the teams, only those ideas finally remain which are viable for the whole social group consisting of those teams (see Figure 1). The objectivity of knowledge is related to what the teacher and students see necessary “to be able to cope” in the sense of von Glasersfeld. Just to recall the fact that among top meteorologists there are strong disagreements about the reasons beyond global warming.



**Figure 1.** Viable knowledge as a result of radical social constructions ([6]).

## Solid theories to link conceptual and procedural knowledge (Ch 2)

- (1) *Do students have to understand being able to do, or vice versa?*
- (2) *Should mathematical objects be emphasized as objects or as processes?*

These two dilemmas have been neglected in our scientific community (cf. [17]), even though they might appear throughout our life. Literature analysis reveals the dominance of procedural knowledge over conceptual one in the development of scientific and individual knowledge ([11], [21]). We know from the basics from cognitive psychology that *our world is a world of meanings, not a world of stimuli*. The ‘investigation space’ (‘Entdeckungsraum’, labeled “reality” in Figure 1) should be psychologically meaningful for the students. Today, an interactive manipulation at computer screen can be for children more “real” than what is conventionally called “real world”. This implies the need to apply the so-called *developmental approach* in the instructional design: students should have opportunities to go for their more or less spontaneous procedural knowledge. On the other hand, perhaps the most important educational goal in a modern society is – especially if we trust on mathematics’ power to trigger general educational goals – to scaffold citizens’ abilities to identify and construct links within complicated multi-causal and multi-disciplined knowledge networks. This means investing on conceptual knowledge, even in such a way, that students also learn appropriate procedural skills. This so-called *educational approach* causes the following conflict: *Does a student have to understand being able to do, or vice versa* ([9]). This dilemma is forced by the fact that for too many students, one of the basic difficulties for the learning of mathematics is that very often entities appear as objects as well as processes. A solid balance between the two approaches can hardly be reached if teachers follow textbooks written by their colleagues without any theoretical framework of knowing and learning.

### **Dilemma between systematization and minimalism (Ch 3)**

*To allow students construct solid mathematical knowledge without continuous guidance by the teacher.*

To emphasize the genesis of heuristic processes and students' ability to develop intuition and mathematical ideas within constructivist or minimalist approach a systematic planning of the learning environments (including instrumental orchestration) is needed. In learning situations, however, students must have freedom to choose the problems that they want to solve within continuous self-evaluation instead of relying on guidance by the teacher. Students frequently neglect teacher's tutoring or they feel they do not have time to learn how to use technical tools. Teachers similarly feel they do not have time to teach how these tools should be used. This problem becomes even more severe when the versatility of advanced technology cannot be accessed without first reading heavy manuals. The term *Minimalist Instruction*, introduced by Carroll ([1]), is crucial not only for teachers but also for those who write manuals and software tutorials. Modifying Lambrecht ([19]), I pick up the following characteristics:

- Only a core knowledge domain may be pre-specified, not specific content and outcomes;
- Learning is modelled and coached for students with unscripted teacher responses;
- Learning goals are determined from authentic tasks stressing doing and exploring;
- Errors are not avoided but used for instruction;
- Learners construct multiple perspectives or solutions through discussion and collaboration;
- Learning focuses on the process of knowledge construction and awareness of that process;
- Criterion for success is the transfer of learning and a change in students' action potential;
- The assessment is ongoing and based on learner needs.

### **Relating instructional design and assessment to instrumental genesis (Ch 4)**

*Teaching and assessment should reflect the development of instrumentation and instrumentalization in modern society.*

Educators on all levels must be aware of the instrumental genesis, which has already changed - and it will change even more radically - our views on interpreting, making and applying of mathematics. "Changing mathematical representation" by using technology is not only a powerful thinking tool to enhance problem solving processes but it also promotes links between procedural and conceptual knowledge. The fact that several kinds of instrumentations and instrumentalizations very often happen on students' free time in natural way, implies that educators should shift the focus from well-prepared classroom lessons on minimalism. Instead of acting like a pace car in a race, institutions should be types of pit stops to scaffold students' "race" outside the classroom. By looking the relationship between technology and mathematics education from five perspectives, I have suggested that 'instead of speaking about *'implementing modern technology into classroom'* it might be more appropriate to speak about *'adapting mathematics teaching to the needs of information technology in modern society'* (see [10]). This means emphasizing more the making of informal than formal mathematics within the framework of the eight main activities and motives, which have proved to be sustainable in the history of human thinking processes and making of mathematics (cf. Ch 6 below).

As regards the assessment, I would like to emphasize that we should give students a variety of ways to show their *action potential*. This demand concerns also when assessment is used to measure student's cognitive performance, as by matriculation examination, for example. I feel uncomfortable with the fact that at the same time when advanced technology is becoming everybody's tool, administrative regulations of the most of countries factually prevent its usage in all educational ways, especially in assessment. Overcome the gap between traditional methodologies targeting "perfect manual performance of algebraic manipulations and geometric constructions" and new ones accenting "understanding of mathematical concepts and their role in solving real-life problems" is difficult but challenging. Modern approaches accent technology not only because it reduces the amount of tedious exercises. ICT, if applied properly, demonstrates the role of mathematics in our everyday life better than many traditional ones do.

## **Learning by Design (Ch 5)**

*Those who learn more from the instructional materials are their developers, not users.*

Even though this challenge could be seen as an implication from previous ones, I wanted to emphasize this view of Jonassen ([16]) by calling it Challenge – not at least because I feel this expression "cool" and I strongly believe on it. Students should be seen as designers of their own lessons, whether ICT-based or not, rather than just as knowledge users.

Kolodner *et al.* ([18]) attach among others the following components to the conception of Learning by Design:

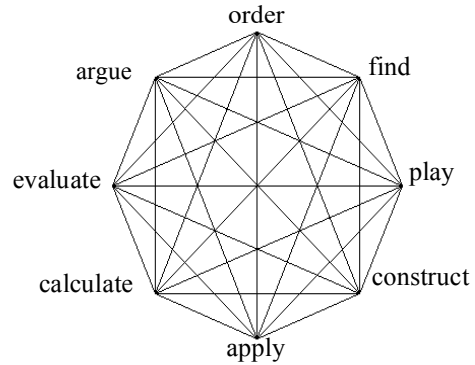
- Authentic, engaging design-and-build activities that enliven students' interest,
- Case-based reasoning and use of analogical reasoning with concrete cases to inform design decisions,
- Multiple contexts for design activities,
- A balance of constrained, scaffolded challenges with more open-ended design tasks,
- Rich varied feedback for designers, through real-world testing, peer and expert analysis of conceptual designs, comparisons to expert model-case solutions and comparisons between peer designs, and extant expert design guidelines,
- Well-orchestrated approaches to generating classroom discussions and collaboration.
- Experimental and exploratory laboratory work that supports the design challenge and engages students in investigation,
- Support for process thinking and self-monitoring and a strong reflection,
- Activities to introduce students to the subject specific skills and complex cognitive and social skills critical to doing, learning, and applying.

Design has four dimensions: the purpose, structure, typical instance and justification. According to this conception, theory is practice-oriented and seeks to explain the latter. Knowledge always contains the tools by which it is developed, evaluated and justified. Clearly learning by design has connections with mathematical modelling processes and more generally with problem-based approach especially when the problems are fairly large, open and adapting themselves to the redefinition dating from the time of the process.

## Revitalizing sustainable heuristics (Ch 6)

*Teaching and assessment should reflect tools, which have been proved to be sustainable in the history of human thinking processes and for the generation of new mathematics.*

Zimmermann's ([21]) study of the history of mathematics reveals eight main activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years (Figure 2). It might be appropriate to use this framework not only by instructional design but to develop an instrument to measure student profiles. Getting desired shifts in those profiles might provide new kinds of challenges for mathematics education and teachers' pedagogical studies. Quantifying each activity to denote the distance of the activity from the centre of the octagon tells us how strong a student thinks this activity is represented by each of the following three profiles, for example:



**Figure 2.** Network of thinking tools and activities which proved to be successful in producing new mathematics ([21], p. 42)

- *Math-profile:* How strong each activity appears when using the term ‘mathematics’,
- *Identity-profile:* How good the student thinks he or she is performing each activity,
- *Techno-profile:* How strongly a computer can help to perform each activity.

## Applying business principles to shift the bad reputation of mathematics (Ch 7)

*Instead of talking about internal problems of mathematics, educators should discuss the issue as a managerial problem.*

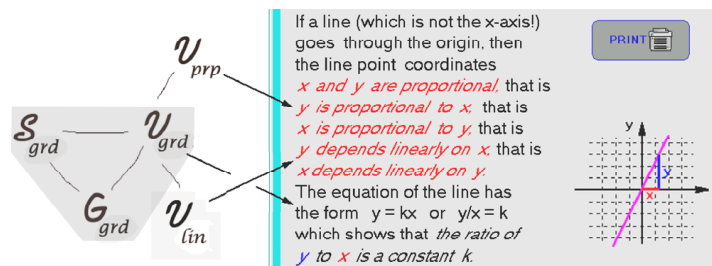
I share the metaphor of Hvorecky ([15]) that “if would be an enterprise named ‘Teaching Mathematics’, it would have probably already crashed and ‘Mathematics’ would disappear from most schools like it was the case with the Latin language”. Technology probably accelerates this process. From a managerial point of view, Hvorecky considers teaching Mathematics as a (virtual) company and suggests the following measures to be involved in educational research: (1) Each “market segment” has its own expectations. Thus, we should set up relevant priorities for different groups of pupils/students; (2) As “the customer is always right”, we should make mathematics more “edible and digestible” for each segment i.e. closer to their environment and cultural values ([15]).

When people criticize mathematics, they may mean the science itself or its teaching in schools. There are numerous researches pointing out the public image of mathematics. Again, when testing how eager the discussion is, I typed “public image of mathematics” in Google, getting more than two million hits. Educators should try to pick up “from this mess” elements, which could lead to successful marketing ideas and new curriculum design.

## Opportunities to respond the Challenges

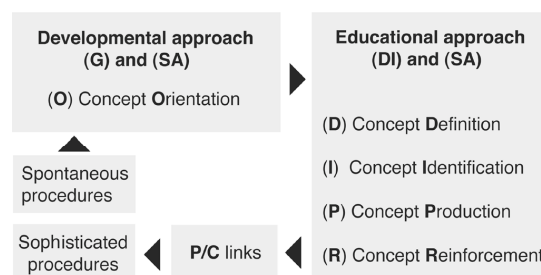
### Social constructions utilizing quasi-systematic framework

I would like to represent the framework gained from the *MODEM*-project<sup>3</sup> because there is a ATCM08 workshop to discuss the utilization of this theory. I will consider the large conceptual field<sup>4</sup>  $C$  [*Proportionality* ( $C_{prp}$ ), *Gradient of a Straight Line through origin* ( $C_{grd}$ ), *“Depends Linearly on”* ( $C_{lin}$ )]. By the abbreviations I would like to stress that those three concepts differ actually just by Verbal ( $V$ ) descriptions, the Symbolic ( $S$ ) and Graphical ( $G$ ) expressions being exactly the same ones. Figure 3 illustrates an appropriate pedagogical meta-strategy when trying to scaffold students in constructing of conceptual knowledge  $C$ : Dynamical links between representation forms of gradient ( $V_{grd}$ ,  $G_{grd}$ ,  $S_{grd}$ ) are constructed, at first. After that, the student can easily link two new verbal expressions  $V_{prp}$  and  $V_{lin}$  to his or her dynamical cognitive network by just “rename command”.



**Figure 3.** Extending the conceptual knowledge  $C_{grd}$  to  $C$  within educational approach.

This meta-strategy seems to be fine but there is a problem: Figure 3 represents abstract conceptual knowledge, which might not be psychologically meaningful for the student and therefore cannot be a natural starting point for the investigations. It must be seen as an end point of the learning process, whereas an appropriate pedagogical idea at starting point would be to go for spontaneous procedural knowledge. Figure 4 illustrates a sophisticated interplay between conceptual and procedural knowledge within the *MODEM* –framework. When planning a constructivist approach to the mathematical concepts under consideration, the focus is on the left-hand side of the figure. On the other hand, when offering students opportunities to construct links between representation forms of the concept, the focus is on the right-hand box, which describes the stages of mathematical concept building.



**Figure 4.** Sophisticated interplay between developmental and educational approach.

<sup>3</sup> See [9], and <http://www.joensuu.fi/lenni/modemempe.html>. The CAL–software illustrating Figure 4 is freely downloadable at <http://www.joensuu.fi/lenni/programs.html>

<sup>4</sup> Isolated and split contents without appropriate links to this conceptual field occupy a lot of room in school curriculum and textbooks, learning results being still very poor (see [9]).

*Orientation (O)* forms the first phase of the systematic concept building. It basically utilizes a developmental approach: the interpretations of the situation can be based on mental models of the pupils, coming, more or less, from their naïve procedural ideas. These act like a wake-up voltage in an electric circuit that triggers another, much more powerful current to be amplified again. The procedural and conceptual knowledge types start to support each other, offering a nice opportunity to use the principle of simultaneous activation, for example. This principle, being at the intersection of the logical definitions of the two approaches, links the developmental approach and educational approach in a most natural way.

The role of the Concept *Definition (D)* is to offer students opportunities to make their own investigations, to express results especially in verbal forms in each case, and to argue about these results within the collaborative teams and between the teams. As a result of social construction, a definition for the concept is born, meaning that students try to fix the relevant determiners of the concept in verbal, symbolic and graphic forms. Especially in the phases of orientation and definition, creative thinking and productive work is needed. This shows how the response to Challenge 1 (cf. Figure 1) is embedded within the knowledge construction, including interesting pedagogical variables.

The next phases of concept building utilize the principle of dynamic interaction. The idea is to give students opportunities to construct concept attributes and procedural knowledge based on them. In the phase of *Identification (I)* we have to give students opportunities to train themselves in identifying concept attributes in verbal (*V*), symbolic (*S*) and graphic (*G*) forms. For this we need six kinds of tasks (*I*): *IVV*, *IVG*, *IVS*, *IGG*, *ISS* and *ISG*. During the learning process, the teacher must be ready, if necessary, to begin with tasks that require distinguishing between only two elements before going on to the identification of several elements.

In the phase of *Production (P)* we have to give pupils the possibility to produce from a given presentation of the concept another representation in a different form. The development of production (*P*) requires nine combinations: *PGV*, *PGS*, *PGG*, *PSG*, *PSV*, *PSS*, *PVS*, *PVV* and *PVG*. The tasks of identification and production must be achievable without any complicated processing of information on the student's part.

In the phase of *Reinforcement*, the goal is to train and utilize concept attributes and to develop procedural knowledge to be used in problem solving and applications.

*MODEM*- framework, being from the first beginning planned to promote conceptual knowledge, is turned out to be suitable to design conceptual tasks for as well learning as assessment. Several experimental studies revealed, among others, the following issues:

- For most students, concept building was a long process in which the five phases could be utilized systematically and successfully;
- Identification phase played a central role in concept building, and seemed to create a pleasant learning environment for the formation of concept attributes;
- Concept understanding was most reliably measured by the production tasks, which were usually difficult for most students, especially the *PVV* type.
- The framework probably promoted also conceptual-procedural links, because a significant positive correlation was found between students' scores on those task types.
- Students scores in most verbal problems concerning fractions or decimals were significantly higher than their scores in these same tasks, which were rewritten directly in



the symbolic form at the end of the test (it was not allowed to return to the previous tasks, namely). Thus, *MODEM* studies not only anticipated but partly explained the interesting and contradictory Finnish findings in TIMSS and PISA studies (i.e. bad scores in symbolic/mechanical tasks and high scores in context-oriented tasks).

- The framework offers promising opportunities to remove the gap between school and university, which is a serious concern among ICMI ([14]). As regards constructivist demands, Ehmke *et al.* ([2]) found that missing of the creative orientation and definition phases caused difficulties to university students when trying to utilize dynamical applets, which were aimed to whitewash their naïve and stereotypic conceptions gained in school.

### **Promoting instrumentation and instrumentalization on student' free time**

By applying minimalism, Eronen & Haapasalo ([3]) gave at the starting point of the *ClassPad Project* students at 8<sup>th</sup> class opportunity to study voluntarily 9<sup>th</sup> class mathematics with ClassPad calculator<sup>5</sup> during their summer holiday. This tool allows manipulations between the algebraic and geometric windows with drag-and-drop manoeuvre or Geometry Link operation. The tool was shortly represented to students just few days before their summer holiday in case they would like to take it for a playing tool. The only duty was to write a portfolio if they worked with the tool. The examples in [10, p. 4], taken from the portfolio of an average student, show that she was able to utilize modern technology in sophisticated way on her free time. She moved from instrumentation to instrumentalization without any tutoring from teacher's side. Furthermore, our findings reveal that doing mathematics with ClassPad, even during a short period of time outside the classroom, enlarged school student's mathematical and identity profiles in more creative direction.

### **Combining systematic planning and minimalism**

Encouraged by the findings above we wanted to realize the mathematics curriculum at 9<sup>th</sup> grade just by using ClassPad. We planned the learning material the conceptual field *C* according to the framework represented in Figures 3-4. However, to follow the principle of Minimalism, different task types were organized to form a "problem buffet". Students had freedom to "jump the gun" by choosing from this buffet any problem that they wanted. To "go for lines", one student team, for example, initially selected a quite complicated problem on optimizing mobile phone costs, which was planned to be a reinforcement task. After realizing that the (partly linear) cost models appeared too difficult for them, they then chose a new, much easier, problem set. This happened to consist of Identification Tasks – the lowest level of understanding the links between representations (cf. Figure 4). This example shows that a sophisticated interplay between a systematic and minimalist approach can be achieved even by simple pedagogical solutions. Note this important feature of Minimalism: Tutors did not want to regulate students' work by recommending them an easier sub-problem, for example. Instead of that it was students' internal motivation that regulated their task choice. Figure 5 represents the path of another student team, which selected different kinds of problem types from the buffet. Recalling the abbreviations in Figure 4 we notice that this team did not utilize the *MODEM* framework in any optimal way. The team went directly to *PSG* tasks and also selected from that list those eleven tasks (#1 - # 11) more or less randomly (as it was the case within the Orientation Tasks). Students evidently liked the amazing drag-and-drop function, which automatically performed the *PSG* action<sup>6</sup>.

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<sup>5</sup> <http://www.classpad.org>

<sup>6</sup> *PSG*, for example, denotes *P*roduction from *S*ymbolic to *V*erbal form, and *IVS* Identification between *V*erbal and *S*ymbolic form, respectively.

→	<i>PSG</i> (#1)	→	<i>PSG</i> (#5)	→	<i>PSG</i> (#3)	→	<i>PSG</i> (#7)	→	<i>O</i> (#3)
→	<i>O</i> (#1)	→	<i>O</i> (#2)	→	<i>PSG</i> (#2)	→	<i>IVG</i> (all)	→	<i>PSG</i> (#6)
→	<i>IVS</i> (all)	→	<i>ISS</i> (all)	→	<i>R</i> (#1)	→	<i>R</i> (#1)	→	<i>R</i> (#1)
→	<i>PSG</i> (#4)	→	<i>PSG</i> (#9)	→	<i>PSG</i> (#8)	→	<i>PSG</i> (#11)	→	<i>PSG</i> (#10)
→	<i>PVV</i> (all)	→	<i>P</i> GV(all)	→	<i>PSG</i> (#12)	→	<i>P</i> GG(all)	→	<i>PVV</i> (all)

**Figure 5.** An example of a “classpath” when selecting tasks.

The learning of mathematics at 9<sup>th</sup> grade solely with ClassPad calculator without any textbooks within interaction between minimalism and quasi-systematization was successful concerning students’ cognitive development. Students scored in all test items significantly better after the working period than in the pre-test. They also showed remarkable procedural skills not only connected to the conceptual field *C* but to other function types, as well. A postponed test, after 2 months, revealed that this scoring level remained consistent, and for many students it even improved. Students liked the feeling that they had reached action potential, which was described to be one of the main aspects in assessment within minimalism. They also liked the learning without any pre-set goals or tutoring from teacher’s side. A closer analysis of video recordings during the lessons reveals amazing things about student communication. Even though the percentage of informal communication outside mathematics seems to dominate student discussions, that communication actually empowered real learning (see [3] and [22]). On the other hand, teachers and students like to go for symbols and graphs, neglecting the importance of verbal representations (cf. Fig. 5).

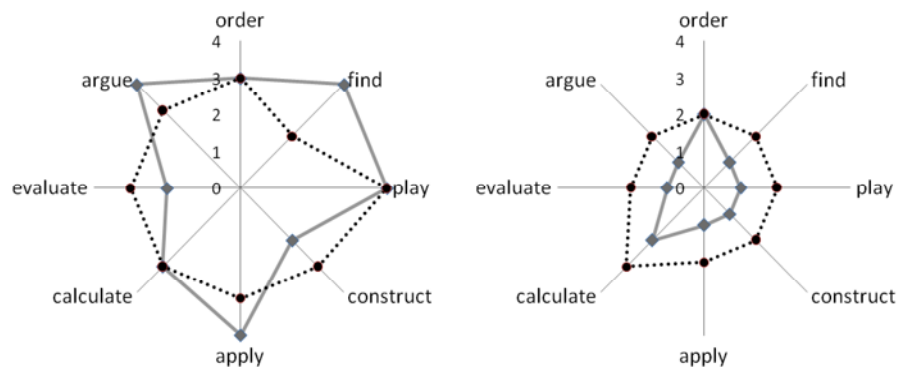
### **Learning by Design**

Eskelinen & Haapasalo ([5] and [6]) applied the Learning by Design principle within collaborative socio-constructivist approach for two student groups in teacher education. The objective of the student elementary teachers’ design process was a hypermedia, whereas the student mathematics teachers planned and realised socioconstructivist microteaching lessons within their peer group. Apart from other studies of design processes the study stresses the impact of knowledge structure, pedagogical philosophy and support for reflective communication. The results revealed that working within socio-constructivist collaborative design processes (cf. Figure 1), even during a short period of time, changed student’s conceptions of teaching and learning from an objectivist-behaviourist viewpoint to a constructivist view, and decreased students’ interest in having support for computer routines. The group dynamics varied in the design process as anticipated in the light of earlier studies: in the planning phase the amount of stress and aggression increased, whereas in the phase of implementation the self confidence increased, which was especially the case within the educational approach. Design of technology-based learning environments within an adequate constructivist theory linked to the knowledge structure might be a proper framework to respond to the main challenge of teacher education: to get students understand which are the basic components of modern constructivist theories on teaching and maintaining the learning through cognitive conflicts. The developmental approach based on spontaneous procedural knowledge seems to be appropriate in relation to both cognitive and affective variables. In order to apply the educational approach so as to stress the importance of conceptual knowledge, an educator needs to be sensitive to the cognitive and emotional variables present in the learning process.

## Critical issues

As regards constructivism, sometimes I feel that our community is contaminated with so-called “romantic constructivism” without any particular educational goals: students should be allowed to construct what they will or can. Hence, many researchers see that a systematic instructional design contradicts the starting point of constructivism. On the other hand, quite poor metacognitive abilities among teachers and students can prevent utilizing a quasi-systematic model, even though task types would be tailored to learning with very simple actions (see [9] and [10]). Hence, the own design of both investigation tasks and tasks to link mathematical representations might be even more difficult to teachers.

Eronen and Haapasalo ([4]) studied the shift in mathematical profiles among student teachers of elementary education and students of mathematics during their first study year in the university. The result transpired that these profiles were very degenerated among both sample groups. Mathematical studies in the Department of Mathematics during the first study year seemed rather to degenerate than extend the profiles among students. Furthermore, our survey (<http://www.joensuu.fi/lenni/survey2008/TestEng.html>) reveals that teacher students are equipped with very strong objectivist view of knowing and behaviourist way of teaching when entering teacher education programme. However, our studies ([5] and [6]) show that those views can be shifted towards constructivism by realizing the above-mentioned Challenges in the pedagogical studies. I would like to represent a finding, which my student teachers made when observing two pupils during their ClassPad work at 9<sup>th</sup> grade and when measuring their profiles as a part of their own pedagogical studies. A procedure-oriented (who wanted to understand later what she was doing) peer-teacher was teaching her procedure-bounded (who was not interested to understand what she was doing) classmate. The outcome of the analysis is that this kind of peer-teaching period quasi-enriched the profile of the peer-teacher but degenerated her mathematical self-confidence (Figure 6). Interestingly the profiles of those two pupils seem to run in opposite directions. The most interesting – even alarming – finding is that at the same time when the classmate seems (perhaps wrongly) to think she could find, apply and argue better than at the beginning, her peer-teacher’s own self-confidence in making mathematics seems to collapse. This kind of finding might be extremely interesting not only when thinking about the fostering of problem-solving abilities but also when considering the mental problems among teachers<sup>7</sup>.



**Figure 6.** Identity profiles of the peer-teacher (on the right) and her classmate (on the left) at the beginning (dashed) and at the end of the working period ([13], p. 71).

<sup>7</sup> When typing “teacher’s mental problems”, Google suggested more than 3 million sites 10<sup>th</sup> of January 2007.

## Coda

To design and analyze learning environments – I would like to speak about investigation spaces – several aspects should be considered at the same time. The research tradition in mathematics education community has, unfortunately, concentrated on focusing on one or few components at a time. My examples hopefully show that when looking an appropriate respond for each of the above-mentioned Challenges, we almost immediately notice that those Challenges are linked together with a variety of interesting pedagogical problems. Like in scientific research through the history: perhaps the most remarkable finding comes out as a by-product when you are searching for something else.

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