

Deconstructing Chinese Lattices with MuPAD

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Abstract: *Chinese lattices are an interesting form of applied mathematical art. This paper is a brief attempt to analyze mathematical structure of two simple examples of Chinese lattices and show how they can be modeled using MuPAD – a Computer Algebra System developed by a group of scientists from Paderborn University and SciFace Software GmbH & Co. KG, in Germany.*

1 Introduction

The climate of South-East Asia is hot and humid. For this reason, traditional architecture in all countries in this region is very spacious and contains a lot of openings. In order to protect the inhabitants, and to stop birds and small animals entering houses, people started protecting window and door openings using wooden grids. Later, these wooden grids evolved into a very sophisticated form of applied art known as Chinese lattices. Although the name suggest China as the place of origin of Chinese lattices, we can find them in many other countries in the Far East, e.g. in Korea, Japan or Vietnam. Today, in many Far East countries Chinese lattices are used not only as protection for window openings but also as dividers of large interiors or as decorations on walls and in furniture.

Chinese lattices from China itself are stylistically slightly different to those from Japan or Korea. However, in each case, we usually deal with a creation that can be very intriguing for a mathematician or a computer scientist.

Figure 1 shows Chinese lattices in Zhouzhuang near Shanghai. The village, or rather a small city, is known as the oldest Chinese village on water. The village was restored recently and it contains a number of 800 year-old houses with very traditional Chinese lattices. We can find similar creations in the imperial palace in Nanjing, in the old town in Shanghai, Beijing and other Chinese towns, as well as in completely new apartments or villas in China.

Chinese lattices are very interesting objects from a mathematical point of view. They contain many sophisticated mathematical patterns. Analyzing these lattices with Computer Algebra Systems (CAS) often lets us recover the algorithms used to create these patterns. We can model them using any CAS where algorithmic constructions are available. In many cases, a form of turtle graphics and L-systems can be very useful.

In this paper, we will concentrate on two simple examples of Chinese lattices. If we wanted to describe the mathematical nature of some of the more complex lattices, we would need more than a few pages of a conference article.

In our investigations, we will use MuPAD, a modern Computer Algebra System from Germany. MuPAD was developed by a group of scientists from Paderborn University, in Germany. MuPAD's development is now continued by SciFace Software GmbH & Co. KG – a computer

company specializing in software for scientific research. MuPAD contains all the tools that we need to model and visualize Chinese lattices (see [3]).

The literature for Chinese lattices is very limited. There are only two books (see [1, 2]) written by Daniel Sheets Dye – former professor in West China Union University Chengtu. These two books contain thousands of drawings of Chinese lattices from various places in West China collected by the author since 1916 until his death on January 16, 1936. There is not much written information about these examples – just some basic classification and location data.

There are very few papers on Chinese lattices and only one or two simple web pages on the Internet. We, the authors, believe that it is worthwhile to investigate the mathematical nature of Chinese lattices and this paper is just a starting point for these investigations.

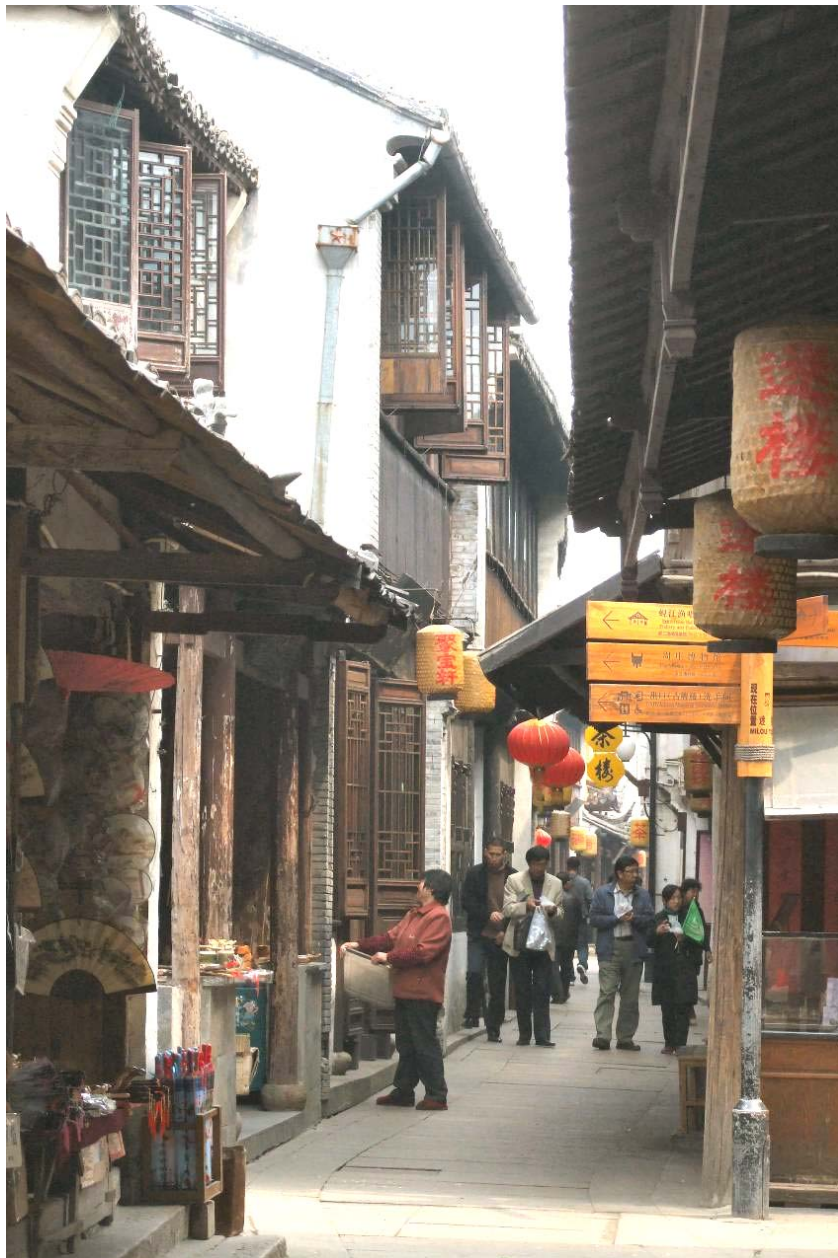


Figure 1. Street in Zhouzhuang – most of the windows are protected by wooden lattices

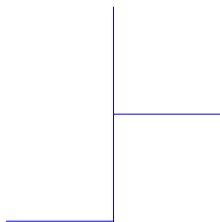
2 Line segments and turtle graphics

Most of Chinese lattice patterns can be modeled using line segments in 2D. To do so, we need a set of commands that will allow us to produce line segments and transform the obtained geometric objects. MuPAD gives us two methods of working with line segments.

The coordinate method: a method using coordinates of points in 2D or 3D and the commands `plot::Line2d([x1, y1], [x2, y2])` to create a line segment in 2D, and `plot::Line3d([x1, y1, z1], [x2, y2, z2])` to create line segment in 3D.

The turtle graphics method: a method that does not require coordinates of points, instead using a sequence of instructions describing the movements of the turtle. This may look like in the following MuPAD code:

```
Z := plot::Turtle():
Z::right(PI/2):
Z::forward(1):
Z::left(PI/2):
Z::forward(1):
Z::push():
Z::forward(1):
Z::pop():
Z::right(PI/2):
Z::forward(1):
plot(Z);
```



At the start, our turtle is located at the center of the coordinate system and is facing upwards (we call it North). In our code, the first command after initiating the turtle was to order the turtle to turn right (East), and then draw a line segment one unit long; this is then followed by further commands. Turtle graphics in MuPAD are described in detail in [3].

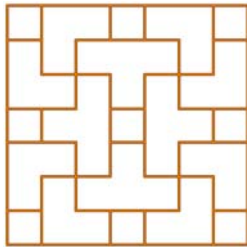
In many situations, using turtle graphics is more convenient than using the coordinate method – we can more easily follow the pattern of a lattice. In other situations, it will be faster to develop the building blocks using coordinates than with the turtle method. In general, the turtle method will require more code than the coordinate method.

3 The Zhouzhuang lattice

The very first Chinese lattice that everybody can see entering Zhouzhuang is the lattice on the balcony of the entrance gate (see fig. 2). The same or very similar pattern can be found in many other places in Zhouzhuang. For this reason, in this paper we will call it the Zhouzhuang lattice¹. The lattice looks very simple and we will use it in our first investigation.

There are five windows, all with an identical lattice. We can also see that each lattice itself is built out of repeating blocks separated by horizontal lines.

¹ It is important to note that the Zhouzhuang lattice does not exist in Daniel Sheets Dye's book, though a similar pattern is mentioned as L8a ([1], page 203).



Finally each of these blocks (see the picture next to this text) can be decomposed to four, almost identical parts: bottom-left, bottom-right, top-left and top-right. These four parts overlap in the center of the block. The central square is the common element of all four parts.

In fact, the bottom-left and top-right parts are identical, while the bottom-right and top-left parts are a mirror reflection of the bottom-left. Therefore, we only have to create one of these parts and then transform them to obtain the three other parts.

We will start our construction by creating the swastika-like pattern that forms the bottom-left part of a single block.



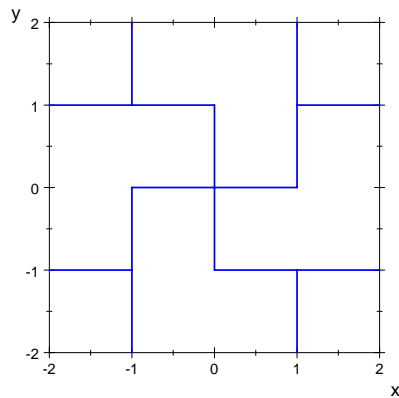
Figure 2. Lattices on the balcony of Zhouzhuang's entrance gate

The MuPAD turtle code for the bottom-left part of the block will look as follows:

```
Z := plot::Turtle():
Z::right(PI/2):
Z::forward(1):
Z::left(PI/2):
Z::forward(1):
Z::push():
Z::forward(1):
Z::pop():
Z::right(PI/2):
Z::forward(1):

Cross1 := plot::Group2d(
  Z,
  plot::Rotate2d(Z,PI/2),
  plot::Rotate2d(Z,PI),
  plot::Rotate2d(Z,3*PI/2)
):

plot(Cross1, Axes=Boxed)
```



While plotting this pattern, we deliberately forced MuPAD to plot the axes of the coordinate system. This way, we are able to check the size of the pattern and its location. Note – the swastika-like cross was obtained by taking four rotated instances of the very same zigzag that we had created while demonstrating how turtle graphics work.

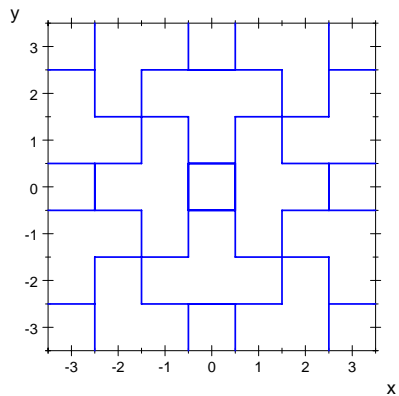
We can now produce a mirrored version of this cross. We take Cross1 and we reflect it about the line passing through the points (0,-1) and (0,1).

```
Cross2 := plot::Reflect2d([0,-1],[0,1], Cross1):
```

Having these two elements ready, we can start assembling the interior pattern of the whole block.

```
TL := plot::Translate2d([-1.5, 1.5], Cross2):
TR := plot::Translate2d([ 1.5, 1.5], Cross1):
BL := plot::Translate2d([-1.5, -1.5], Cross1):
BR := plot::Translate2d([ 1.5, -1.5], Cross2):

plot(TL, TR, BL, BR, Axes=Boxed)
```

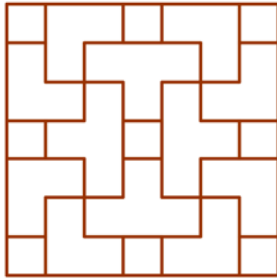


In order to complete the whole block we need a frame around of it. This can be done with a few simple orders for the turtle. We will simplify our work by writing a simple program using the for-do loop.

```
F := plot::Turtle():
for i from 1 to 4 do
  F::forward(7), F::right(PI/2)
end_for:
F := plot::Translate2d([-3.5, -3.5], F):
plot(F, Axes=Boxed)
```

By grouping the obtained parts BL, BR, TL, TR and F we produce the whole block.

```
Net1 := plot::Group2d(TL, TR, BL, BR, F, LineWidth=4*unit::pt, LineColor=RGB::Brown):
Net2 := plot::Group2d(TL, TR, BL, BR, F, LineWidth=2*unit::pt, LineColor=RGB::Gold):
plot(Net1)
```



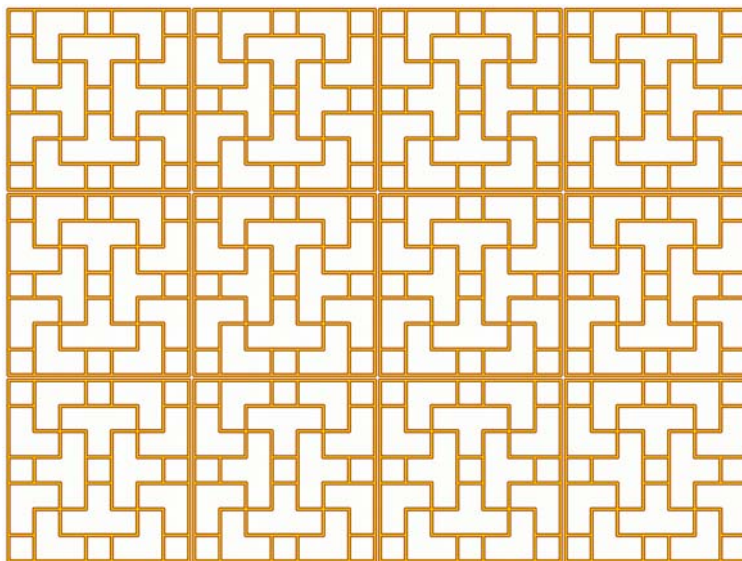
Finally, we can develop a procedure that will allow us to replicate the obtained block rightwards and upwards as many times as we wish.

```
Lattice := proc(thing, w, h, n, m, offset)
begin
  objects := []:
  for i from 1 to n do
    for j from 1 to m do
      objects := objects.[plot::Translate2d([i*(w+offset),j*(h+offset)], thing)]
    end_for
  end_for
  return(op(objects))
end_proc:
```

The procedure uses six input parameters: *thing* – this is the object that we wish to repeat; *w* and *h* are width and height of the block respectively; *n* and *m* are the number of times we wish to repeat the block to the right and up. Finally *offset* is the small gap that we have to add between two neighboring blocks to avoid overlapping them.

The code below will create a Zhouzhuang lattice with 3 rows and four columns. In order to make the final result more appealing we produce two blocks: *Net1* with 4 points line width (brown color), and *Net2* with 2 point line width (gold color); and we overlap them.

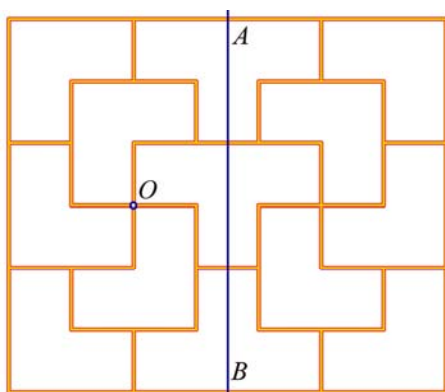
```
plot(Lattice(Net1, 7, 7, 4, 3, 0.2), Lattice(Net2, 7, 7, 4, 3, 0.2))
```



4 The lattice from a balcony in Shanghai



Figure 3. Traditional Chinese architecture in Shanghai contains numerous wooden lattices



Traditional architecture in Shanghai contains numerous wooden lattices. There is one very characteristic type of lattice that we can find in many balconies in old Shanghai (see fig. 3). The lattice², like the Zhouzhuang lattice, is constructed by repeating a single block (shown next to this text). However, the construction of the block is completely different. In the middle of the block there is a vertical symmetry line AB . The left and right parts overlap slightly in the middle of the block. Each of these parts can be split into two patterns – top and bottom. The top part can be obtained by rotating the bottom part 180 degrees about the only common point O of both parts. Therefore, we have to

construct only the top-left part of the pattern, rotate it 180 degrees to obtain the bottom-left part, and then make a reflection of the obtained left side of the block to produce the right side of the block. After combining all the parts together and adding a frame, we will produce the whole block. Then we may use the procedure `Lattice`, which we developed in the previous example, to create a whole balcony lattice.

We will start by creating the top-left part of the block pattern.

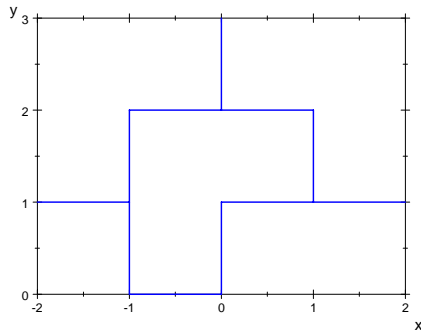
```
S := plot::Turtle(  
  Forward(1), Right(PI/2),  
  Forward(1),  
  Push(), Forward(1), Pop(),
```

² In Daniel Sheets Dye's book [1] this type of lattice was classified as S11b – swastikas of simple rectangular waves.

```

Left(PI/2), Forward(1),
Left(PI/2), Forward(1),
Push(), Right(PI/2), Forward(1), Pop(),
Forward(1),
Left(PI/2), Forward(1),
Push(), Right(PI/2), Forward(1), Pop(),
Forward(1),
Left(PI/2), Forward(1)
):
plot(S, Axes=Boxed)

```

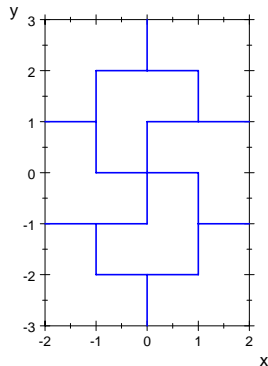


Now we can develop the whole left side of the block. We have to combine the obtained pattern with its copy rotated 180 degrees about the point (0,0).

```

LeftSide := plot::Group2d(
  S,
  plot::Rotate2d(S,PI)
):
plot(LeftSide, Axes=Boxed)

```



The right side of the pattern will be a mirror reflection of the left side.

```

RightSide := plot::Reflect2d([0, -1], [0, 1], LeftSide):

```

Finally, we will produce the whole interior as well as the frame around the block.

```

Interior := plot::Group2d(
  plot::Translate2d([-1.5, 0], LeftSide),
  plot::Translate2d([ 1.5, 0], RightSide)
):

F := plot::Turtle():
for i from 1 to 2 do
  F::forward(6), F::right(PI/2), F::forward(7), F::right(PI/2)
end_for:

F := plot::Translate2d([-3.5, -3], F):

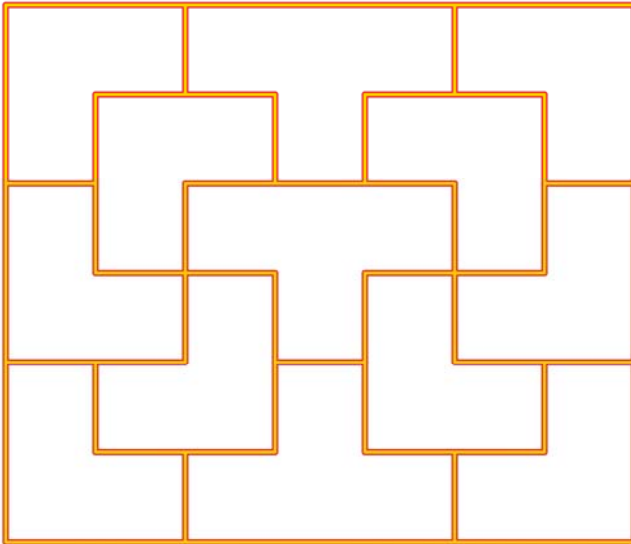
```



```

BalconyBlock := plot::Group2d(Interior, F,
  LineWidth=4*unit::pt, LineColor= RGB::BrownOadder
):
BalconyLight := plot::Group2d(Interior, F,
  LineWidth=2*unit::pt, LineColor= RGB::Gold
):
plot(BalconyBlock, BalconyLight)

```

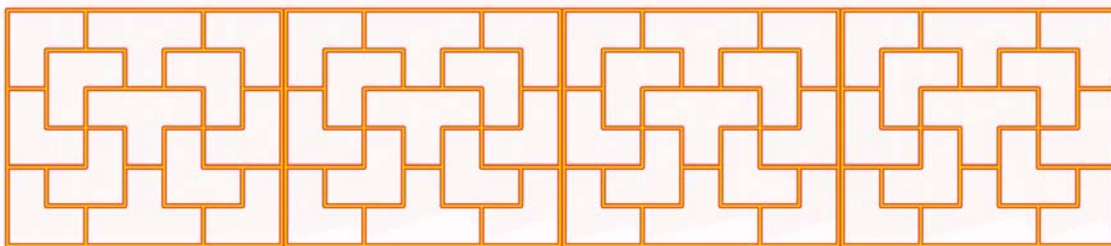


The very last task is to create the whole lattice for the balcony.

```

plot(Lattice(BalconyBlock, 7, 6, 4, 1, 0.1),Lattice(BalconyLight, 7, 6, 4, 1, 0.1))

```



5 Summary

Modeling Chinese lattices with MuPAD can be a very interesting activity. Some of the patterns are relatively easy to develop using simple elements followed by translations and rotations, finally grouping the simple patterns into more complex ones. Some of the lattices may require a completely different approach. For example, many of them will require a continuous turtle path covering the whole lattice. Such a lattice may be then obtained using one or more turtle paths combined together. Then there are lattices where an application of the L-system method seems to be appropriate. In such case, higher generations of the L-system may lead to new, very complex patterns.

In each case, the method that we use to model a lattice should be as natural as possible, and probably should mimic the way of thinking of the craftsman designing the real lattice. For example, a natural way is to create the main blocks of the interior of a lattice, then join them together and combine with a frame.

6 Credits and final comments

The idea for this article was born during the visit of one of the authors at East China Normal University in Shanghai. While walking through Zhouzhuang and later through the old town in Shanghai I realized how interesting it would be to describe mathematical patterns that I have seen everywhere around me. Therefore, I started making photographs and sketches of some of the Chinese lattices. Later, after a brief discussion with my great friend Jiyan Wang (王继延), we decided to write this paper. I believe this paper is just an introduction to a longer adventure with Chinese lattices.

I am very grateful to the East China Normal University for giving me the opportunity of visiting their old campus in Shanghai, as well as to the two wonderful girls Di Liu (柳笛), and Youwen Yuan (袁有雯) for guiding me through the mazes of Zhouzhuang and Shanghai. Learning with you the secrets of Shanghai and Zhouzhuang was a great pleasure and a memorable adventure.

Mirek Majewski (馬裕祺).

7 References

- [1] Dye D. S., Chinese Lattice Designs, Dover Publications, Inc., New York, 1974.
- [2] Dye D. S., The New Book of Chinese Lattice Designs, Dover Publications, Inc., New York, 1981.
- [3] Majewski M., Getting Started with MuPAD, Springer Verlag, Berlin-Heidelberg-New York, 2005.