Case Study in Understanding Concurrencies Related to Ceva's Theorem Using the Geometer's Sketchpad

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Abstract: Suan Sunandha Rajabhat University (SSRU) has offered Bachelor of Science Program in Informatics Mathematics, Bachelor of Education Program in Mathematics Education, and Master of Science Program in Informatics Mathematics Education. The Informatics Mathematics program focuses on two categories namely mathematics and information technology , Mathematics Education focuses on mathematics education, and Informatics Mathematics Education focuses on three categories namely mathematics, informatics, and mathematics education. Case study done in this paper was a course content of elementary geometry : Concurrencies and Ceva's Theorem. The purpose of this study is to illustrate the use of the Geometer's Sketchpad (GSP) in rethinking for teaching and learning geometry for Mathematics Education students. The teaching method is cooperative learning approach through model CEDI (**C**onstructing, **E**xploring, **D**iscussion, and **I**nformal deduction). By developing dynamic activity worksheets that can be readily modified to meet the objectives of content and needs of individual students it should be possible to greatly increase experience in " thinking " at different levels such as visualisation, analysis, and informal deduction.

1. Introduction

Since the concept of concurrence in elementary geometry for secondary school are mentioned but not often established by proof such as the medians, angle bisectors and altitudes of a triangle. However, teaching those topics for undergraduate students by traditional proof, many of difficult theorems require enormous amounts of human intelligence. This case study demonstrates the importance of establishing concurrency related to Ceva's theorem with cooperative learning approach which incorporates the use of the Geometer's Sketchpad with help of learning model CEDI (**C**onstructing, **E**xploring, **D**iscussion, and **I**nformal deduction). This learning model was adopted from the first three van Hiele levels of thinking in geometry (Visual, Analysis, and Informal Deduction, respectively) [1]. Students were encouraged to move from one level to the next with in each activity. By using the Geometer's Sketchpad makes the topic of concurrence quite simple and presents a new vista in Euclidean geometry.

2. Background and Related Work

Usually, in traditional teaching about geometry, students will use paper and pencil to learn and teacher will use chalk and blackboard to teach geometry. Classical instruments allowed in geometric constructions are the compass and straightedge. However, some problems turned out to be difficult or impossible to solve by these means alone. The compass is assumed to collapse when lifted from the page, so may not be directly used to transfer distances. The traditional teaching of geometry emphasizes very much on formal definitions and proofs. In recent years, the teaching of secondary or undergraduate geometry courses emphasizes more on manipulatives. The use of manipulatives help students learn more by experiencing hands-on situations [2]. They also provide students do mathematics for themselves rather than learn to follow how someone else does it.

Using Dynamic Geometry

Today, software programs exist for various mathematics disciplines. Software developers have created dynamic programs useful in the teaching and learning of Geometry. The interactive geometry software, Geometer's Sketchpad, which is published by Key Curriculum Press [1] is suitable for implementation in a discovery-learning environment or for demonstration purposes. This program is most widely used in middle school, high school, and college geometry classrooms in Thailand but is occasionally used in other subject areas or with younger students [3].

Geoemter's Sketchpad is a dynamic geometrical software, it allows for dynamic explorations in geometry so that students can see the changes in the figures when they manipulate them. The students can instantly and dynamically draw points, angles or lines, experience the dynamic change of geometrical concepts, and construct the mathematical understanding from their exploring without wasting to redraw figures to calculate and compare them . GSP enables students to construct, transform and animate figures which lets them look at an infinite number of cases while testing and verifying conjectures [1]

Teachers need to help students develop problem-solving skills. Geometer's Sketchpad focuses on problem based learning. "The teacher's role under the guided-generation model is that of facilitator and scaffold builder, helping students to think about the nature of the problem and how to go about solving it on their own" [4]. Teachers are not the providers of knowledge, but instead guide students to discovering the concepts. When students can make sense of what they are doing they are able to go beyond rote learning and onto conceptual understanding.

Recent research has shown mixed results regarding the use of geometry software and its impact on academic achievement and student attitudes towards the subject area. Pijils [5] found that there were no significant differences in the mathematical level of the students when using technology, however, there were differences in how students approached the task. Both Grassi [6] and Enderson [7] found students demonstrated greater understanding of concepts when using GSP. The participants indicated that the software helped them make sense of the problems. In both of these studies, students focused on the use of Geometer's Sketchpad over the course of a unit of study.

This case study explored whether Geometer's Sketchpad enabled students to enhance understanding of geometry on concurrencies related to Ceva's theorem by changing the focus of their studies from factual recall to problem solving and higher-level thinking.

Thinking Levels with GSP

The work of Dutch mathematics educators, Pierre van Hiele and Diana van Hiele-Geldorf, focused on levels of thinking in geometry and the role of instruction in assisting students to move to the levels :

Level 0 : Visual …. Judges shapes by their appearance.

Level 1 : Analysis …. See figure in terms of their components and discovers properties of class of shapes.

Level 2 : Informal Deduction …. Logically interrelates previously discovered properties.

Level 3 : Deduction …. Proves theorems deductively.

Level 4 : Rigor …. Establishes theorems in different axiom systems.

Fuys *et al*. [8] were involved in a three year study during the early 1980's to determine whether the van Hiele model could be used to describe how students learn geometry. They developed three instructional modules as follows : Module 1 Basic geometric concepts, Module 2 : Angle measurement, and Module 3 Area measurement. 32 sample students (16 sixth graders and 16 ninth graders) during approximately six hours of one-to-one videotaped clinical interviews, using the three instructional modules are analyzed and discussed. The clinical study indicates that the van

Hiele model provides a reasonable structure for describing students' geometry learning. The analyses of videotaped clinical interviews provide insight and information not only on students' level of thinking but also on factors affecting students' performance on the instructional modules (e.g., language, visual perception, misconceptions, prior learing, students' thinking processes and learning styls).

Hannafin and Scott [4] investigated the impact of GSP on seventh and eighth grade low achieving students' ability to answer high-level questions. The results indicated that GSP proved to be an effective tool for improving low achieving students' performance on "hard questions", thus leading to significant gains in their ability to answer higher-level questions. This study indicated that the technology can help level the playing field for low achieving students. The rationale behind this is that high achieving students may have better school "coping" strategies than students that are lower achievers. The open learning environment developed in these studies took the high-achieving students out of their comfort zone and demanded that they learn in a different way. These higher-level students were required to take their learning to the next level. This new way of learning proved to be more effective for the low achieving students and more challenging for high level students. The activities presented to both teachers and students who used GSP in these studies either asked a question for students to solve or gave a series of questions to lead the students to gain content knowledge and conceptual understanding. The teacher had to take a backseat in these studies and allow the students to explore and rely largely on their partner as the more knowledgeable other. Teachers did not give answers, but instead gave guidance.

Cooperative Learning: Think-Pair-Share with GSP

Cooperative learning aims to create a positive environment and to develop cooperataion and understanding of others as individuals while learning together. The model may be used to supplement other forms of instruction by giving students the opportunity to teach one another, discuss in groups, or put into practice skills or information presented by the instructor [9].

There are many types of cooperative learning. The simplest to implement in this case study is discussion in Think-Pair-Share. The value of the model is improved thinking and social skills and being to work together, discuss, listen, and contribute. The thinking activity can include students writing about what they are thinking and then sharing what they have written with others. The pairing could also include larger group of three or four students, depending on the nature of the task and the number of students in the group [10].

For the purpose of this case study it was only concentrate on the first three van Hiele levels and cooperative learning (Think-Pair-Share) to enhance understanding of concurrencies related to Ceva's theorem by using Geometer's Sketchpad. The learning approach is CEDI model which has 4 steps as follows :

Constructing : Students use GSP to experiment with drawing and labeling points. They identify, name and compare geometric figures on the basis of their appearance as a whole.

Exploring : Students investigate relationships by dragging, measuring, and looking for certain properties and think idividually.

Discussion (Think-pair-share): Students share what they have found with another students.

Informal deduction: Students understand the relationship within and between figures, determine which properties are necessary and sufficient for defining or give informal deductive argument or construct the conclusion (conjectures).

3. Purpose

(1) To develop the learning materials to enhance understanding of concurrencies related to Ceva's theorem by using Geometer's Sketchpad.

(2) To evaluate the level of thinking by using self-evaluation and interviews.

Methodology

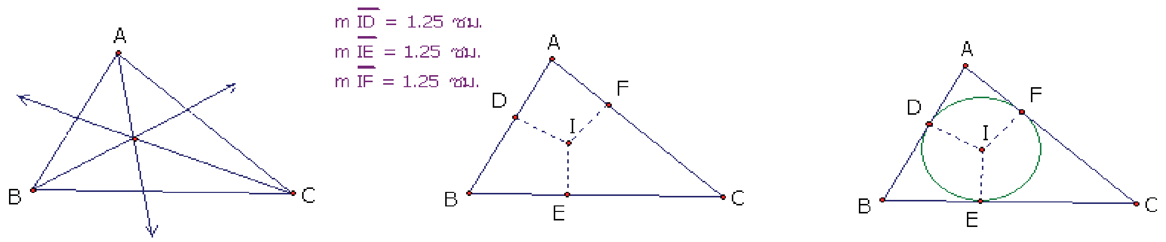
Subjects:

6 third year undergraduate students from Mathematics Education program in SSRU who had never learned concurrencies related to Ceva's theorem. Subjects were mainly at average achievement in Mathematics Teaching course.

The Design of an Experiment:

Beginning activities in each worksheet were designed to reflect the level of thinking in geometry adopted from the first three van Hiele levels with CEDI model using GSP. Worksheet 1 Warm-Up step-by-step and let the students continue the rest of worksheets. More specifically, Worksheet 2 to 5 presented the concepts of concurrencies named incenter, circumcenter, centroid, and orthocenter, respectively. In Worksheet 6 presented the Ceva's theorem and Worksheet 7 challenge to informal proof related to these congruencies.

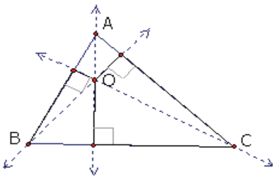
Incenter

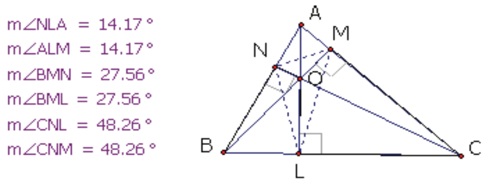


Circumcenter

Centroid

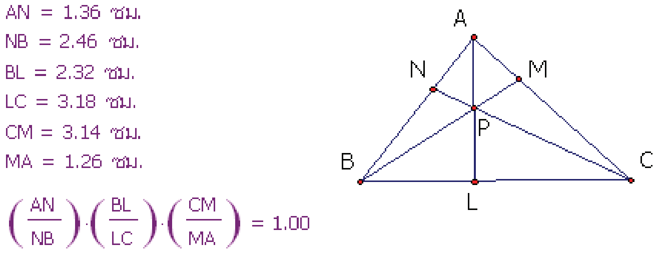
Orthocenter





Ceva's Theorem

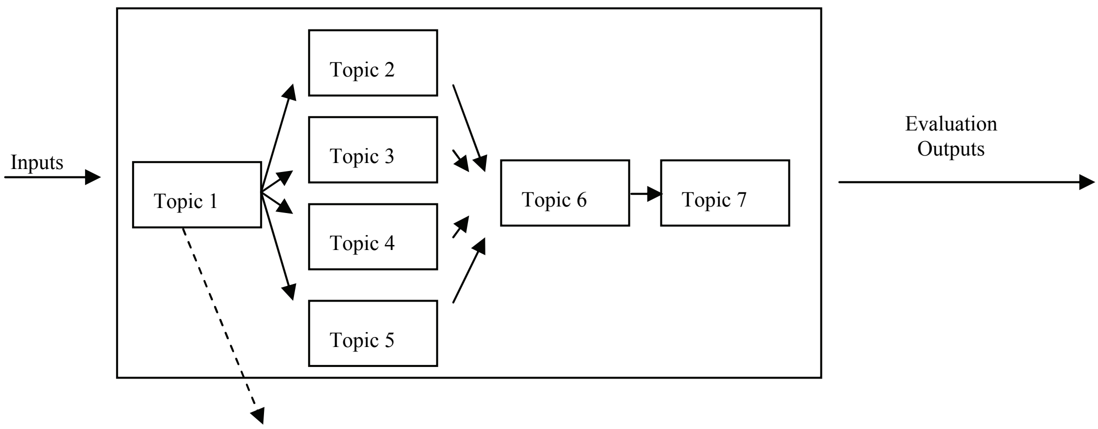
The three lines containing the vertices A, B, *and* C *of* ΔABC *and intersecting the opposite sides at points* L, M *and* N, *respectively, are concurrent if and only* $\frac{AN}{NB} \cdot \frac{BL}{LC} \cdot \frac{CM}{MA} = 1$



Initial assessment activity provided the observation on the basis of students' performances on key activities in the worksheets. The final assessment activities provided the self-evaluation and interviewer information about students' abilities to follow each step of CEDI and students' attitude toward geometry by using GSP.

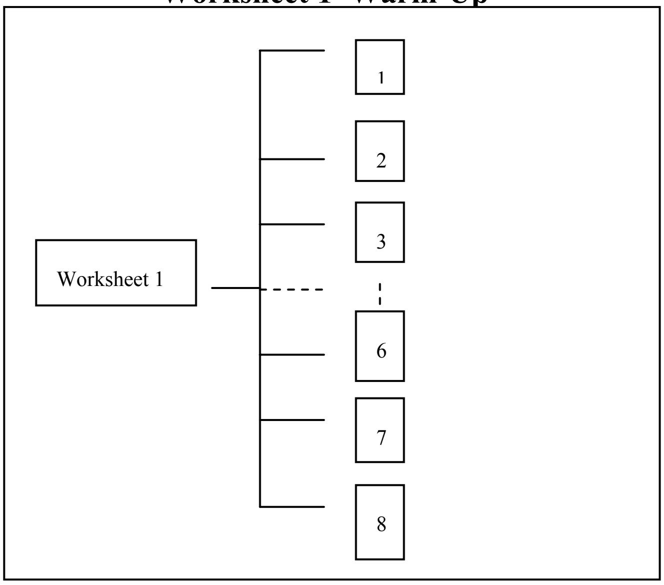
Instructional System

Stage 1 Design Topics

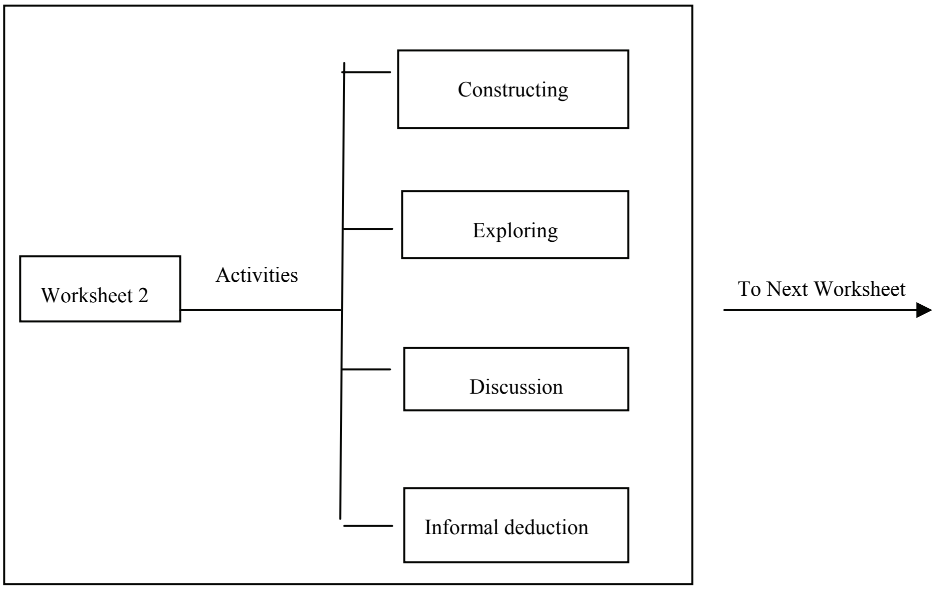


Stage 2 Design Worksheets

Worksheet 1 Warm-Up



Worksheet 2 *Incenter*



Stage 3 Evaluation

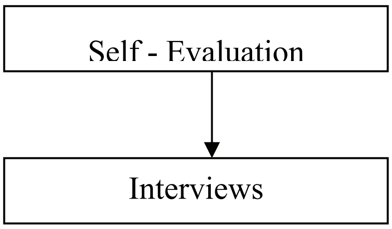


Figure 1 The design of case studies

4. Results: An Overview

A student's level of performance was determined mainly by his/her using GSP responses to questions in 7 worksheets, in particular, to questions in CEDI model (constructing, exploring, discussion, and informal deduction) in these worksheets as shown in Table 1. A student's level of thinking was determined mainly by self-evaluation his/her ability in CEDI model in these worksheets as shown in Table 2.

Table 1

Students' Level of Performance on CEDI Activities by Using GSP

	Student no.1	Student no.2	Student no.3	Student no.4	Student no.5	Student no.6
Worksheet 1 : *Warm Up*						
(1) Constructing a line segment and midpoint	*s*	*s*	*s*	*s*	*s*	*s*
(2) Constructing an angle and bisector	*s*	*p*	*s*	*g*	*s*	*p*
(3) Constructing a perpendicular line at midpoint	*s*	*p*	*s*	*s*	*s*	*s*
(4) Constructing a perpendicular line from a point to a line	*s*	*p*	*s*	*g*	*s*	*p*
(5) Finding the point of intersection of 2 lines	*s*	*p*	*s*	*p*	*s*	*p*
(6) Finding the point of intersection of 3 lines	*s*	*s*	*p*	*p*	*s*	*p*
(7) Constructing a circle given a center and a point on the circle	*s*	*s*	*s*	*p*	*s*	*p*
(8) Measuring the length of a segment.	*s*	*s*	*s*	*p*	*s*	*p*
(9) Dividing and multipling two numbers.	*s*	*s*	*s*	*p*	*s*	*p*
Worksheet 2 : *Incenter*						
Constructing(8)	*s(8)*	*s(8)*	*s(8)*	*s(7)p(1)*	*s(8)*	*s(5)p(3)*
Exploring(3)	*s(3)*	*s(2)p(1)*	*s(3)*	*s(1)p(2)*	*s(2)p(1)*	*s(1)p(2)*
Discussion(2)	*s(1)p(1)*	*s(1)p(1)*	*s(2)*	*p(2)*	*s(1)p(1)*	*p(2)*
Informal Deduction (1)	*p(1)*	*p(1)*	*s(1)*	*p(1)*	*p(1)*	*p(1)*

Key: 0* weak response; *g* responded with guidance

p responded after a prompt; *s* responded spontaneously

( ) number of item to response

	Student no.1	Student no.2	Student no.3	Student no.4	Student no.5	Student no.6
Worksheet 2 : *Incenter*						

Constructing(8) Exploring(3) Discussion(2) Informal Deduction (1)	*s(8)* *s(3)* *s(1)p(1)* *p(1)*	*s(8)* *s(2)p(1)* *s(1)p(1)* *p(1)*	*s(8)* *s(3)* *s(2)* *s(1)*	*s(7)p(1)* *s(1)p(2)* *p(2)* *p(1)*	*s(8)* *s(2)p(1)* *s(1)p(1)* *p(1)*	*s(5)p(3)* *s(1)p(2)* *p(2)* *p(1)*
Worksheet 3 : *Circumcenter* Constructing(8) Exploring(3) Discussion(2) Informal Deduction (1)	*s(8)* *s(3)* *s(1)p(1)* *p(1)*	*s(8)* *s(3)* *s(1)p(1)* *p(1)*	*s(8)* *s(3)* *s(2)* *s(1)*	*s(8)* *s(2)p(1)* *p(2)* *p(1)*	*s(8)* *s(2)p(1)* *s(1)p(1)* *p(1)*	*s(6)p(2)* *s(2)p(1)* *p(2)* *p(1)*
Worksheet 4 : *Centroid* Constructing(7) Exploring(3) Discussion(2) Informal Deduction (1)	*s(7)* *s(3)* *s(1)p(1)* *s(1)*	*s(7)* *s(3)* *s(1)p(1)* *s(1)*	*s(7)* *s(3)* *s(2)* *s(1)*	*s(7)* *s(2)p(1)* *s(1)p(1)* *s(1)*	*s(7)* *s(2)p(1)* *s(2)* *s(1)*	*s(7)* *s(2)p(1)* *s(1)p(1)* *p(1)*
Worksheet 5 : *Orthocenter* Constructing(6) Exploring(3) Discussion(2) Informal Deduction (2)	*s(6)* *s(3)* *s(2)* *s(1)p(1)*	*s(6)* *s(3)* *s(2)* *s(1)p(1)*	*s(6)* *s(3)* *s(2)* *s(1)p(1)*	*s(6)* *s(2)p(1)* *s(1)p(1)* *s(1)p(1)*	*s(6)* *s(2)p(1)* *s(2)* *s(1)p(1)*	*s(6)* *s(2)p(1)* *s(1)p(1)* *s(1)p(1)*
Worksheet 6 : *Ceva's Theorem* Constructing(4) Exploring(3) Discussion(4) Informal Deduction (4)	*s(6)* *s(3)* *s(2)* *s(1)p(1)*	*s(6)* *s(3)* *s(2)* *s(1)p(1)*	*s(6)* *s(3)* *s(2)* *s(1)p(1)*	*s(6)* *s(2)p(1)* *s(1)p(1)* *s(1)p(1)*	*s(6)* *s(2)p(1)* *s(2)* *s(1)p(1)*	*s(6)* *s(2)p(1)* *s(1)p(1)* *s(1)p(1)*
Worksheet 7 : ***Challenge Informal Proof*** ***Property 1***: All medians of a triangle areconcurrent. ***Property 2***: All altitudes of a triangle are concurrent. ***Property 3***: All bisectors of interior angles of a triangle are concurrent.	*p* *s* *p*	*s* *p* *p*	*s* *s* *s*	*s* *p* *p*	*s* *s* *p*	*p* *p* *p*

Table 2

Students' Level of Thinking on CEDI Activities by Using GSP

(Self-Evaluation)

	Student no.1	Student no.2	Student no.3	Student no.4	Student no.5	Student no.6
Visualization						
- constructing and representation.(16)	5	4	4	4	4	4
Analysis						
- exploring, problem solving and connection.(6)	4	4	4	4	5	4
Informal deduction						
- discussion, reasoning, and communication. (8)	4	4	4	4	4	4

Key: 1 poor, 2 weak, 3 average, 4 good, 5 very good

Analyses of the interviews indicated that the students in the case study fell roughly into four categories.

Category 1 : Thinking level in visualization using GSP

Students no.1, 3, and 5 have more experiences in using GSP than students no. 2, 4, and 6. Student no. 2 responded after a prompt for constructing bisector, perpendicular line, and intersection. Student no. 4 responded after a prompt for constructing intersection, circle, measuring, and computation. However, she responded with guidance for constructing bisector and perpendicular line. Student no. 6 responded after a prompt for constructing most of them, because she came to class late.

Category 2 : Thinking level in analysis

Students no.2, 3, and 5 observed that after dragging one vertex of triangle about the screen to form different types of triangles (acute, right, and obtuse) the point of concurrency located inside, outside, or anyplace of triangle. When measure the length of given line segments they can compare and compute the given ratios correctly. Students no.1, 4, and 6 had miss concepts about location of concurrent point. All students identified which triangle with concurrent point related to Ceva's theorem.

Category 3 : Thinking level in formulating informal arguments (informal deduction)

After discussion by using cooperative learning (Think-Pair-Share) student no.3, 4, and 6 formulated the definitions and properties of concurrent points sufficiently. They demonstrated how Ceva's theorem relate to the concurrency of median, altitude, and bisector. But students no.1, 2, and 5 had some problems to formulate the properties of concurrent points, they lack of precision of language and lack of experience in explaining. However, they knew the relationship between Ceva's theorem and concurrent points properties.

Category 4 : Challenging to formal proof

Since Worksheet 7 focused on challenging to formal proof, assessment of students' level of thinking across concepts of similarity of two triangles formed by medians and altitudes. Students no. 1 and 5 confused about corresponding sides. All students had done weak in concurrent of bisectors of interior angles of a triangle. Because they lack of prerequisite knowledge, " The bisector of an angle of a triangle divides the opposite side into segments that are proportional to the adjacent sides ".

5. Conclusion

Results of this study suggest that the Geometer's Sketchpad is capable of helping students to learn concurrency concepts and Ceva's theorem clearly. Students can work through CEDI model by following a simple constructing and exploring. They can share their experiences with their partners in order to construct the conjectures or draw informal conclusion . The worksheets are designed to encourage teamworking and develop students' thinking skills in mathematical process such as problem solving, reasoning, communication, connection, and representation. The Geometer's Sketchpad can be useful in designing instructional tools for learning geometry. However, the effective ways to use dynamic geometry software means of teaching geometry is essential for all students and teachers if schools are well equipped with technology and useful tools. Further study might explore levels of thinking by using GSP with CEDI model in any topics of geometry, algebra, and calculus.

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