Modelling with Cabri 3d to enhance a more constructivist approach to 3D geometry

Jean-Jacques Dahan
jjdahan@wanadoo.fr
IREM of Toulouse France

Our purpose in this paper is to show that a training session to Cabri 3D based on a specific modelling task, with specific planned stages can be didactically successful both in the short and in the long term. Our choices are justified by the results obtained by Kate Mackrell and Janet Ainley and Dave Pratt about "designing tasks". The concept of geometric working space (GWS) of Alain Kuzniak will help us to justify the didactical importance of these choices. We will be able to observe particularly if the environment chosen by the teacher or the trainer can be adopted or approached by the learner during such a training. We will also justify the importance of the choice of a task of modelling a phenomenon coming from real life.

1. Cabri 3d workshop (ATM Keele 2008)

1.1. Aims

I have led at ATM 2008 a training session about Cabri 3D. Ten teachers have attended this session and most of them were Cabri 3D beginners. The aim was for them, basically, to discover the principal features of Cabri 3D. My aim was to involve them quickly in a process of problem solving which could be motivating even at the very first beginning of the training. The task I have designed is "modelling the basic step of Cha Cha dance". I will describe below how the task was designed in order to obtain the Brousseau's devolution (involvement) during the training and a big chance that the work will be continued after the training.

1.2. What happens

Stage 1 (introduction)

→ The attendees were asked to look at me dancing the basic step of Cha Cha with one of the women who has accepted to help me. They had to observe carefully the motion of my feet.

→ After this short presentation with music, the task of the workshop was announced to everybody : we will use Cabri 3D to model what they have observed.

→ A Cabri 3D file was displayed showing with an animation what could be the result of such a work if led successfully. A little character "Filou" (created in 1998 in 2D) is dancing the basic step of Cha Cha with music (figure 1). The geometric constructions are visible in this file.

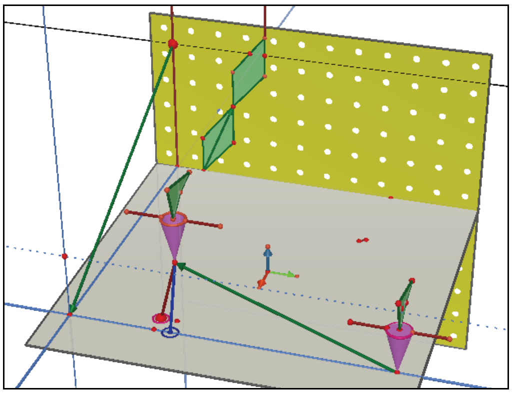

Figure1

Stage 2 (step by step teaching)

The participants are taught step by step how to begin the construction of the motion of the two feet of the dancer. They are involved in a process of "instrumentation" in the meaning of Rabardel as we are building an instrument with the artifact "Cabri" and some its schemes of using. They can see the interest of the math tools of Cabri to model a phenomenon of real life. During this stage, the participants (following step by step the instructions of the trainer) model on their screen the motion of two points (of the horizontal plane) representing the left and the right feet of the dancer : this motion is modelling Cha and Cha, that is to say : at first, the right foot R slides rightwards from A to B as the left one L stays in A (Cha 1 in figure 3) and secondly, the left foot L slides rightward from A to join the right foot R which is now fix in B (Cha 2 in figure 3). This construction uses the parallelogram $A_1B_1A_2B_1'$ (figure 4). This motion is commanded by the two intersection points of an horizontal line of the front plane with the sides of this parallelogram previously constructed in this plane. A translation allows to link these two points to the right and left feet, poits ***1*** and ***1'*** (figure 4). The participants understand very quickly the technique used to generate the Cha-Cha first step. At this moment, they have learnt enough from the trainer about the Cabri tools and the techniques of modelling the steps of such a dance.

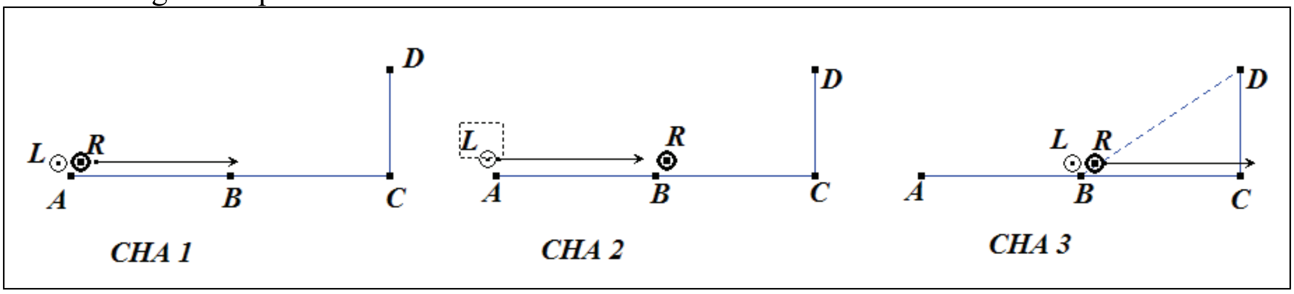

Figure 3

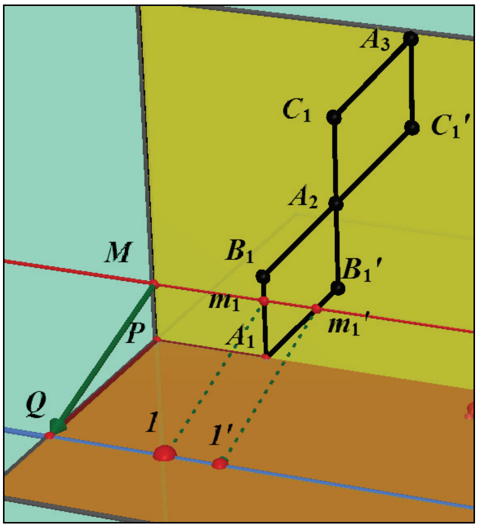

Figure 4

Stage 3 (modelling on their own in a process of imitation)

The trainer stops to give the constructions to the participants who are given the task of continuing the modelling (on their own). Some of them tried to find out the good constructions in a process of thinking, some other like Sue Johnston Wilder (the president of ATM) tried to observe herself while dancing really Cha Cha. Very quickly most of them succeed to model the next step (Cha 3 in figure

3) by constructing another parallelogram symmetric of the initial one with respect to its upper vertex

Stage 4 (modelling on their own in a process of creation)
A real problem appears when the participants have to model the motion of the left foot forward to reach a position leaving the right foot behind it (L slides from B to D along [BD] as R stays in C: Cha 3 in figure 3). This is a real problem because it cannot be solved in using the techniques used before; it needs to use some tricks that must be created with some tools of Cabri such as rotation for example. Some of the participants try some nice ideas showing that they can see in 3D and take initiatives involving the concept which is one of the purpose of the activity (Hoyles and Noss in 1987 have called this *"using before knowing"*).

Stage 5 (preparing the after session work)
At the end of the session, most of the participants have solved the proposed task in programming the animation of Cha, Cha, Cha and the step forward of the left foot. During the last ten minutes of the workshop, I have proposed to everybody to observe again a demonstration of Cha Cha dance in order to model the motion of the legs: the principal problem is that the weight of the body is always supported by the moving leg. I displayed the file I have realised to show the result of such a modelling. The after session work was accepted : to continue the work we began in order to obtain a Cabri model of Filou dancing the basic step of Cha Cha.

1.3. What is expected after this training

To finish the modelling in a long and collaborative process in order to obtain if possible the visualisation of a character dancing completely the basic sequence of Cha Cha steps.
To be able to model like Kate Mackrell did with her Claude's files:
At a learner level : construction of a house with opening doors and windows
At an intermediate level: construction of a carousel, of a swinging Claude (figure 5).
At an expert level : construction of the Möbius strip with Claude sliding on it (figure 5).

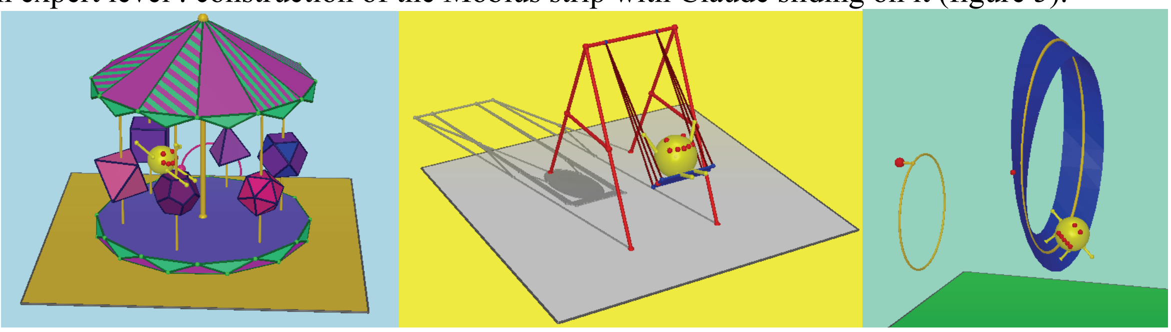

Figure 5

1.4. Some short didactical remarks

As pointed by Ainley and Pratt, such an activity can help the participant to understand the **utility** of the tools used all along this work and help the trainer to reach its **purpose** which is here the concept.
The step by step stage 2 is focused basically on the "*construction*" tools of the software and not on the "*creation*" tools. It allows the realisation of robust figures in the meaning of Laborde and gives to the attendees a lots of useful techniques of modelling.
The problem of the *play paradox* (pointed by Noss and Hoyles) is solved essentially in preparing the stage 3 by the stage 2 which is carefully designed to give the attendees the impression that the

tools to solve the problem are available as well as the techniques. They have the impression to be in a process of imitation as really very quickly, they are in a process of creation.

2. THEORETICAL FRAMEWORK

2.1. Modelling

2.1.1. Noël Mouloud's options : (in Encyclopedia Universalis), the modelling process can be
→ Either the representation of a law of the science with an object of the real world that can be more easily understood (for example "visualisation of the set of graphs of the solutions of differential equations" Dahan Time 2006, Dresde); we can call this process M_1.
→ Or the translation of a concrete or empiricist phenomenon into an abstract representation because a direct study of this phenomenon could only lead to non accurate relationships (example of the modelling of the lampshade of the Convention Centre of Hong Kong, ATCM 2008); we can call this process M_2.
2.1.2. The special case of microworlds such as Cabri (Balacheff 1999)
Working under Cabri means that we use the M_2 modelling; we work and operate on the theoretical objects thanks to the power of direct manipulation and the ergonomy of this DGS that shortens the distance between the real object and its theoretical representation (or model).
Lakatos describes mechanisms of modelling very close from those of physics. Balacheff has proven that the mechanism of proof cannot be reduced to demonstration but is an intense back and forth between theory and experiment

2.2. Geometric paradigms (Houdement and Kuzniak) and their extensions in a DGS environment (Dahan)

2.2.1. The notion of paradigm according to Kuhn
He defined it on two aspects.
1. In its most global use, the term paradigm stands for the entire constellation of beliefs, values, techniques, and so on shared by the members of a given community.
2. On the other, it denotes one sort of element in that constellation, the concrete puzzle-solutions which, employed as models or examples, can replace explicit rules as basis for the solution of the remaining puzzle of normal science.

2.2.2. The geometric paradigms G1 and G2 according to Houdement and Kuzniak
Geometry I (Natural Geometry). The source of validation is the sensitive. It is intimately related to reality. Intuition is often assimilated to immediate perception, experiment and deduction act on material objects by means of the perception and the instruments. The backward and forward motion between the model and the reality is permanent and allowed to prove the assertions. For example, dynamic proofs are accepted in this Geometry.
Geometry II (Natural Axiomatic Geometry). The source of validation bases itself on the hypothetical deductive laws in an axiomatic system. A system of axioms is necessary but the axioms are as close as possible to the intuition of the space around us. The axiom system can be uncompleted, but the demonstrations inside the system are necessary requested for progress and for reaching certainty.

2.2.3. The geometric paradigms G1 informatique and G2 informatique according to Dahan
Geometry I Informatique: this paradigm extends G1 during a work of geometry problem solving within a DGS environment because the source of validation even still sensitive is enriched by the accuracy of the drawing and by the robustness of figures during the process of dragging which opens a widest window for conjectures (the properties used to build the figure are kept all the time).

Geometry 2 Informatique: the principal characteristic is that the deductive activity, similar to the demonstration activity is assisted by the software. Indeed, certainty is not reached but the plausibility of a result obtained under this paradigm is very big. We have a debate with Alain Kuzniak about this paradigm, because he argues that G2 Informatique cannot be qualified as G2 because, we don't use real formal demonstration. My point of view is that this paradigm is closer to G2 than to G1. The concept of geometric working space can probably help us to agree about a concept including this paradigms.

2.3. Geometric working space (Kuzniak)

2.3.1. Definition (Houdement et Kuzniak 2006)

The Geometrical Working Space(GWS) is the place organized to ensure the geometrical work. It makes networking the three following components:

- objects located in a real and local space as material support,
- artifacts as drawings tools and computers put in the service of the geometrician
- a theoretical system of reference possibly organized in a theoretical model.

It can be compared in one way to the concept of *webbing* The notion of *webbing* offers a way of thinking about the child's developing knowledge as dynamically constituted from interaction with both internal and external resources (Noss and Hoyles)

2.3.2. The three basic GWS

The reference GWS

When people agree with a given paradigm, they can formulate problems and organize their solutions within a frame which favors some tools or some ways of thinking determining a GWS named GWS of reference. To know the nature of this GWS, it is necessary to describe how the geometrical work is done and to focus on the rules of discourse, treatment and communication.

The suitable GWS

When the general paradigm is accepted and the reference GWS is built, it remains to teach geometry to students and for that it's necessary to organize a suitable GWS to convey the kind of geometry expected by the educational institution. The geometrical working space turns to be suitable only if it allows the user to link and master the three components defining the working space.

The personal GWS

When the problem is proposed to somebody (a pupil, a student or a teacher) even to an expert, this person handles the problem with its personal GWS. This GWS will not be as rich and powerful than the one of an expert.

Focusing on the personal GWS, needs to introduce a cognitive dimension into the GWS approach and especially the one of Duval (1995) who points out three kinds of cognitive processes (figure 6):

- visualization process with regard to space representation and the material support,
- construction process depending on the used tools (rules, compass) and on the configuration,
- reasoning in relation to a discursive process.

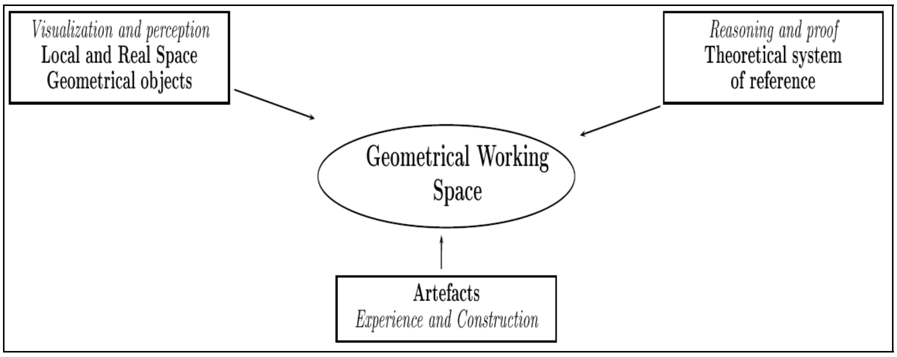

Figure 6

3. OUR HYPOTHESIS

3.1. The prototypical ATM training (modelling reality in five stages with a specific use of transformations) generates an instrumental genesis (Rabardel)

3.2. This instrumental genesis is composed by a sequence Reference-Suitable-Personal of GWS.

3.2. This type of training allows us to minimize the distance between the Reference and the Personal of GWS.

4. DIDACTICAL ANALYSIS OF THE ATM TRAINING UNDER OUR THEORETICAL FRAMEWORK

4.1. The stages of such a training

Stage 1: this workshop begins in the real world and gives the learner the opportunity to realise what are the stages of the process of modelling: to observe, to simplify, to abstract and to construct in the software the result of this abstraction which will be the expected model.

This example has been chosen in the real life in relation with street mathematics because as Schliemann said, "for meaningful mathematical learning to take place in the classroom, reflection upon mathematical relations must be embedded in meaningful socially relevant situations".

it is one reason that can motivate the engagement of the learner at all level (Brousseau's devolution). This choice is here especially important because we know that teachers who are trained (instrumented) with particular activities, most of the time, propose the same activities or similar and adapted activities to their students.

This stage gives some informations about the reference GWS in which the trainer wants to attract the learner. The artifact is given (here Cabri) but not the schemes of using which will be presented in stage 2; the objects of the microworld Cabri are also given as the objects on which the work will be done (really the objects are figures of Cabri: a figure is considered here as a set of properties). The choice of Cabri can be justify by the fact that "*Microworld settings provide the possibility of pupils working with concepts that they do not yet understand. Understanding emerges through activity*" (Noss and Hoyles). We think that this can be extended to adult learners.

Stage 2: in this stage, learners are involved in a process of construction under the paradigm G1 Informatique. The tools used in the software are essentially the "*Construction*" tools in order to build robust figures and also to avoid the poverty of the approach of such a problem with "*creation*" tools as pointed by Mackrell, Ainley and Pratt. The reference GWS includes knowledge in Euclidian geometry and some techniques specifically related to DGS (for example modelling the motion of a couple of points); for the learner it is mostly the presentation of the suitable GWS, the one he could be able to work on his own. The experiments done under Cabri are only validatives experiments (in the meaning of Dahan). They allow in a perceptive way to confirm that the Cabri model is a good model because an animation confirms that the points built on the screen moves like the feet of the real dancer. In this stage the learner is entirely responsible of the "*instrumental genesis*": the learner discovers Cabri as an instrument and not like a tool: the tools are introduced with their "***utility***" to solve problems, with their schemes of using. For Rabardel an instrument is a couple built with a tool and some of its schemes of using. That is also another reason to motivate more the learner in a deep engagement. If this stage seems to be a "behaviourist stage", it is only an impression because the way this stage was designed is a way that will allow the trainer to be involve in a more constructivist approach in the next stage.

Stage 3: like all learners, the participants, at this moment try to use their immediate knowledge, that is to say what they have learnt in the previous stage: it was my will when I have designed stage 2. It happened because all the works observed were led under the same paradigm, G1 informatique; the same techniques are reinvested to model a motion similar to the previous one. So we can say that the GWS in which the participants worked was the reference one: they have used the same reasoning. Really, they have used the same constructions, the same techniques and the same reasoning because all these settings were adapted to their knowledge, skills and ability. So we can say that they are working under a suitable GWS.

Stage 4: we know also that all learners, in front of a real problem try to use their deep knowledge when the use of the immediate one is not successful. It is what we have observed when some participants have imagined (figure 7)

1/ to rotate the upper part of parallelogram $A_2C_1A_3C_1'$ and the intersection points m_4 and m_4' around line (C_1C_1') (90°)

2/ To translate what they have obtained onto the horizontal plane in order to build a fix point (4') modelling the right foot and an animated point (4) modelling the left foot sliding forward.

At this point the participants were not familiar with rotations in 3D; nevertheless, they have understood the "***utility***" of this tools, making a lot of mistakes in the choices of their parameters. This way of working obliged them to return to their knowledge but also to work on all the aspects of the concepts of rotation and translation. Here they were "*using before knowing*". The validations were perceptive: the model was definitely the good model when we can validate perceptively on the screen that points m_4 and m_4' are moving on segments included in the horizontal plane.

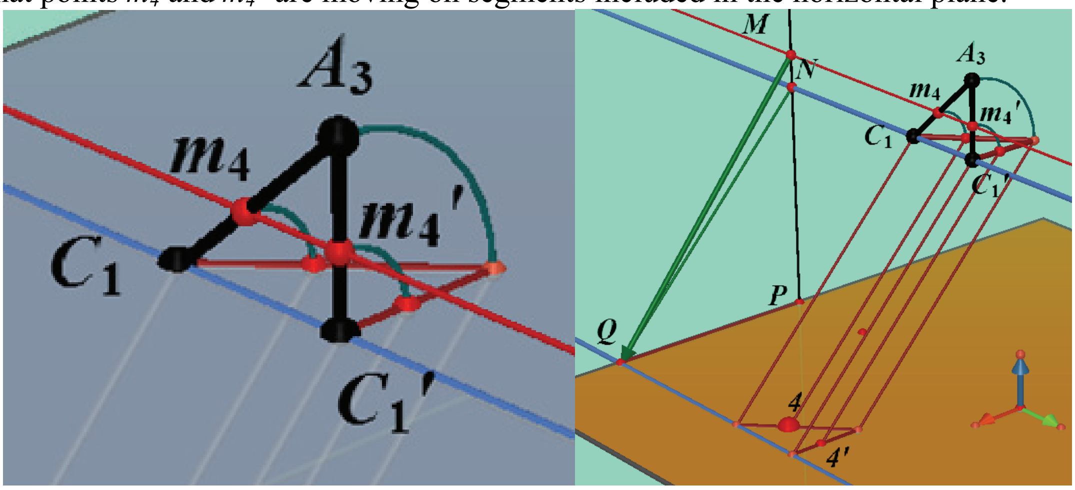

Figure 7

At this stage the GWS turns onto a personal one, richer than the one used during the previous stage. It shows that, even if the participant has not the expertise of the trainer, he is able after the preparation of stages 2 and 3 to enrich his power of experimentation to manipulate new objects leading to modelling.

Stage 5: we have not observed what happened after this session because, at the moment this paper is written, I have not received any feedback from the participants. We can at least expect that the learner will work in the GWS he has reached in stage 4 and why not improve it.

4. Prototypical examples leading to the ATM Model training

4.1. The prototypical example of Filou dancing in 2D

This example was proposed in 1998 (and published in a book about geometry with the TI-92) and the training took place in a middle school with students of 11 to 14 year old (figure 8). The children were taught how to construct a little character called Filou with an animated belly button, a Filou dancing (stage 2). They were especially shown how to redefine the belly button as a point of a guide-segment. As they wanted to model a Filou playing yoyo, I showed them only that the circle-eye could be dragged along the polygon-face (in dragging its centre). It was sufficient for them to enter in stage 4 to create a circle with its centre on a segment and to redefine one vertex of this segment on a segment-finger of Filou. After that, they animated the centre of the yoyo along the initial segment (they indeed have hidden this segment before the animation).

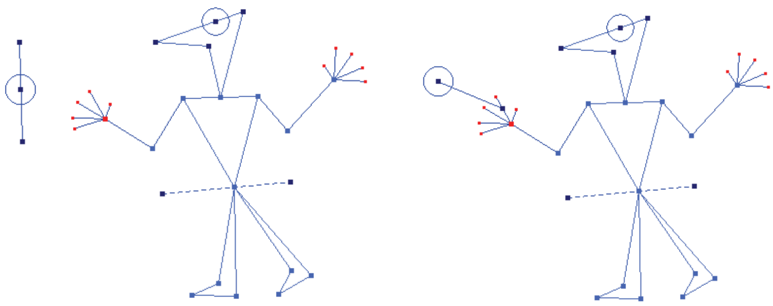

Figure 8

4.2. The prototypical example of the swinging Claude in 3D

This example has been used during workshops at ATM before 2008, during trainings for French highschool teachers and also for the initiation of students of french highshools level at Cabri 3D (figure 5). These trainings have been handled approximatively like the one we have described in paragraph 1. They have allowed the participants (all beginners in Cabri 3D) to reach stages 3 and 4 within a "personal" GWS looking like the "reference" one introduced in stage 1. We have noticed that the personal GWS obtained with such a training was richer than the one obtained after a training led in showing first the features of the software and letting secondly the participant free in front of a problem of modelling to investigate without any preparation (especially our stage 2)

5. Conclusion

All the trainings about Cabri 3D for beginners (in Cabri 3D) led before the Keele ATM training were the roots of the one we have described and analyzed in this paper. We have shown that the trainer in choosing this type of training can insert the participant inside a "*reference*" GWS to lead him progressively into a "*personal*" and rich GWS. During that sort of training, he can also lead a very efficient instrumental genesis with the sequence 1-2-3 of the stages The choice of the problem is also very relevant : coming from real life according to the model of "Realistic Mathematics Education", it helps to use concepts before knowing them really. It helps to justify the utility of the tools when associated to the techniques presented or discovered and reach at the end the purpose of the trainer : to know more mathematics in embedding the tools of Cabri 3D with some ways of using them for solving problems. This type of training helps definitely the trainer to transmit an instrument in the meaning of Rabardel.

Bibliography

[1] AINLEY J., PRATT D.,1997, The construction of meanings for geometric construction: two contrasting cases, International Journal of Computers for Mathematical Learning, 1(3), 293–322.

[2] AINLEY J., PRATT D. and HANSEN A., 2006, Connecting Engagement and Focus in Pedagogic Task Design, *British Educational Research Journal,* 32(1), 23-38.

[3] BALACHEFF N., 1982, Preuve et démonstration en Mathématiques au Collège, *Recherches en didactique des mathématiques*, 3, (3), 261-303.

[4] BROUSSEAU G., 1998, *Théorie des situations didactiques,* La Pensée Sauvage Éditions, Grenoble.

[5] DAHAN J.J., 1998, *Introduction à la géométrie avec la TI-92*, Ellipses Publishing.

[6] DAHAN J.J., 2005, La démarche de découverte expérimentalement médiée par Cabri-géomètre en mathématiques. Un essai de formalisation à partir de l'analyse de démarches de résolutions de problèmes de boîtes noires, thèse de doctorat, Université Joseph Fourier, Grenoble. http://www-iam.imag.fr/ThesesIAM/ThesejjDahan.pdf

[7] DAHAN J.J., 2006, Visualizing Solutions of Differential Equations of the Second Order with Cabri 2 Plus *in Proceeding*s DES-TIME 2006, Int. Symp. on Technology and its Integration into Mathematic Education, July 20-23 2006; Dresden, Germany (CD-ROM bk Teachware Series).

[8] DAHAN J.J., 2007, Two explorations with Cabri 3D leading to two theorems : The maximum of the volume of the convex envelope of a net of a cube. Quasi-tessellations of a cylinder with isosceles triangles (linked to the Schwarz paradox) *in Proceeding*s ATCM 2007 Taipei, Taïwan.

[9] DAHAN J.J., 2008, Les paramètres didactiques cruciaux pour comprendre l'intégration de l'expérimental dans la pratique et l'enseignement de la géométrie. Exemplification grâce à Cabri 2 Plus et Cabri 3D *in proceedings of 5th International Colloquium on the Didactics of Mathematics,* University of Crete, Rethymnon.

[10] FREUDENTHAL H., 1968, Why to teach mathematics so as to be useful, Educational Studies in Mathematics, 1, 3–8.

[11] HOUDEMENT, C., KUZNIAK, A., 2006, "Paradigmes géométriques et enseignement de la Géométrie", Annales de didactique et de sciences cognitives pp. 175-193.

[12] KUZNIAK A., 2008, Diversity of geometrical problems and trouble in the teaching and learning of geometry *in proceedings of 5th International Colloquium on the Didactics of Mathematics,* University of Crete, Rethymnon.

[13] KUHN, T.S., 1962-1987, Structure of Scientific Revolutions, Cambridge: Cambridge University Press.

[14] LABORDE C., 1993, Do The Pupils Learn And What Do They Learn In A Computer Based_Environment? The Case of Cabri-Géomètre, in Jaworski B. (Ed), *Proceedings of the International Conference on Technology in Mathematics Teaching 93*, 39-52, Birmingham, UK.

[15] LABORDE C., 1995, Designing Tasks for Learning Geometry in a Computer-based Environment, in Burton, L. & Jaworski, B. (Eds), *Technology in Mathematics Teaching - a Bridge Between Teaching and Learning*, 35-68, Chartwell-Bratt.

[16] MACKRELL K., 2008, Cabri 3D : An Environment for Creative Mathematical Design, P. Liljedahl (Ed), *Canadian Mathematics Education Study Group Proceedings 2007 Annual Meeting*. Frederickton: University of Frederickton.
[17] NOSS R. and HOYLES C., 1992, Looking Back and Looking Forward, in NOSS R. and HOYLES C.(Eds), *Learning Mathematics and Logo,* MIT.
[18] NOSS R. and HOYLES C., 1993, Out of the Cul-De-Sac, *Proceedings of the fifteenth conference of the International Group for the Psychology of Mathematics Education North American Chapter*, 1, p83-90, California.
[19] NOSS R., HOYLES C., 1996, *Windows on Mathematical Meanings*, Mathematic Education Library, Kluwer Academic Publishers, Dordrecht, Pays Bas.
[20] NUNES T., SCHLIEMANN A. D., CARRAHER D. W., 1993, Street mathematics and school mathematics. (Cambridge, Cambridge University Press).
[21] RABARDEL P., 1997, *Les hommes et les technologies*, Armand Colin Collection U, Paris. Encyclopédia Universalis 1997.