

Graphing Calculator Assessment Research Directions

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Abstract: Teachers play a key role in the school assessment process through their contribution to the teaching and learning of classroom mathematics and through their choice of assessment goals and instruments. Research points to the influence that knowledge and beliefs of teachers play in the quality of classroom use of graphing calculator. It has been argued that there are five broad categories of teacher behaviour when it comes to the integration of ICTs within their classroom. Teachers tend to view ICT as either: a demon; a servant; an idol; a partner; or, a liberator. These categories are discussed in relation to research findings and their implications for assessment with graphing calculators. This is a theoretical paper which attempts to address the question: In what ways does the graphing calculator expand the range of performances accessible through assessment? The paper concludes with a number of brief directions for designing mathematical questions involving the use of graphing calculators.

1. Background

To address the issue of assessment involving the graphing calculator, it is necessary to briefly consider the wider background issues relating to assessment in mathematics. While there is a considerable body of scholarship and resources focussed on the construction and delivery of tests and examinations, there is also an increasing body of scholarship and resources focussed on 'alternative assessment' (see [7]). These alternative assessment methods generally cover any technique that doesn't involve tests or examinations. They are a reaction to an increasing concern by teachers, educational leaders and researchers that tests do not accurately reflect the full range of student learning outcomes and that tests suffered in comparison with other methods (see [8]). This movement away from tests has created greater pressure upon mathematics teachers and systems to integrate assessment into the classroom teaching and learning cycle. As further evidence of this building pressure, the recent National Council of Teachers of Mathematics Second Handbook on Research (see[18]) devoted a chapter that aimed to ignore the formal methods of tests and examinations in order to "redress the balance by focussing on the role that assessment can play in supporting learning, rather than just measuring it - sometimes described as a distinction between assessment *for* learning and assessment *of* learning" ([30], p. 1053).

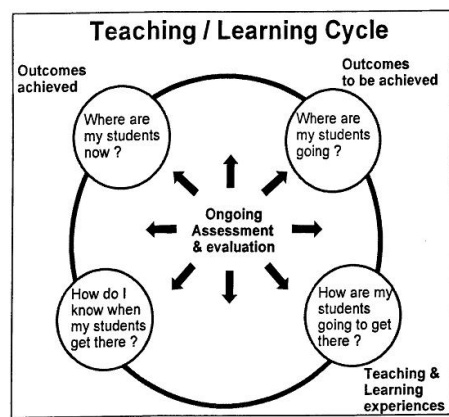


Figure 1 The teaching and learning cycle (BOS, 2003)

As early as 1993, the USA's Mathematical Sciences Education Board (MESB) produced a guide for mathematics assessment. In a recent statement, [24] changed the word 'content' to 'mathematics' in the first principle and used the three principles to present a conceptual guide for use with a range of technology in the assessment of mathematics. Their discussion focussed upon examples involving graphing calculators including those with symbolic manipulation capability such as computer algebra systems (CAS) available on graphing calculators.

"Three fundamental educational principles form the foundation of all assessment that supports effective education:

THE CONTENT PRINCIPLE

Assessment should reflect the mathematics that is most important to learn.

THE LEARNING PRINCIPLE

Assessment should enhance mathematics learning and support good instructional practice.

THE EQUITY PRINCIPLE

Assessment should support every student's opportunity to learn important mathematics"

([24], p.1)

[24] used the content principle to raise questions about what knowledge and skills are most valuable. "Technology, including dynamic geometry, spreadsheets, and calculators, enables students to explore mathematical ideas for themselves, formulate and test hypotheses ... so even examinations now need to look beyond assessing a narrow bandwidth of mathematical activity" (p. 16).

So what constitutes important mathematics activity that lies beyond the narrow bandwidth? [32] make a distinction between two different types of mathematics activity: technical and conceptual, "The technical dimension of mathematical activity is about taking mathematical actions on mathematical objects or on representations of those objects. Procedures can then be built out of sequences of mathematical objects... Conceptual mathematical activity involves understanding, communicating, and using mathematical connections, structures, and relationships (p. 1120).

Elaborating upon the learning principle [24] highlight the power of assessment to encourage good teaching and learning. "Mathematics is not button pushing ... Hence, assessment items must emphasise the formulating and interpreting and the understanding of generalizations that machines do not do" (p. 16). Good assessment will achieve higher-order thinking benefits.

[24] used the equity principle to draw attention to "High performance in school mathematics, even without technology, is often associated with social advantage [25], so it is important that technology use in class or in assessment does not operate to worsen the situation" (p. 16).

This concludes a very brief discussion of the background issues which readers can read in greater depth elsewhere (for example, [14] and [18]). The remaining sections will adopt the three principles and the wider view of assessment as one of: "functioning as a bridge between teaching and learning, helping teachers collect evidence about student achievement in order to adjust instruction to better meet student learning needs in real time" ([30], 1054). The discussion of assessment involving graphing calculators will include tests and examinations and will also seek a wider range of methods and student learning outcomes. It will also address the question: In what ways does the graphing calculator expand the range of performances accessible through assessment? Thus the next sections will focus on the important role the views of the teacher plays in the integration of technology into the classroom and the implications these views have for assessment using graphing calculators.

2. Teacher integration of technology

The process of transforming information into knowledge is a demanding and purposeful process that requires both instruction and a community of learners. In this transformation process the role of the teacher is crucial. There has been an explosive growth in the availability of technology for mathematics classrooms during the last quarter of a century, and this growth has been accompanied by an enthusiasm for the potential of new technologies in teaching and learning of mathematics [32].

Teachers have adopted different approaches in response to their experience and beliefs about teaching and learning and the use of technology. A teacher's beliefs about the teaching and learning of mathematics have a strong influence upon his or her integration of ICT [1]. [28] and [29] conducted an examination of the research literature and proposed five broad categories or perspectives of teacher response to the integration of technology within their classroom and how teachers tend to view technology. The categories briefly summarised below are not meant to be used for labelling teachers but as means for understanding why there are such a diversity of approaches in the classroom, in assessment involving technology, and in the use of graphing calculators.

ICT as demon

In this category [29] claimed that evidence for this approach is observable in the teachers who actively oppose and subvert any attempt to integrate technology into the curriculum. These were regarded as the resistance fighters who are either afraid or unwilling to learn and conducted an active or passive campaign of resistance. When directed by authority, they did the minimum effort required and this often resulted in inappropriate use.

ICT as servant

Teachers in this category [29] states, adopt a conservative position where technology is used "yet the pedagogy remains much the same as in the past" ([11], p. 26). Thus technology is a tool for enhancing students' learning outcomes but within the existing curriculum and using existing learning processes and teaching strategies [23].

ICT as idol

This approach includes teachers who are seduced by the 'dazzle effect' of these 'techno toys' and who fail to consider the teaching and learning implications beyond a surface level. This approach promotes technology as a tool for use across the curriculum where the emphasis is upon the development of technology related skills, knowledge, processes and attitudes ([23], p. 3). It is more focused upon teaching about technology rather than teaching mathematics with technology.

ICT as partner

Here "students are actively engaged in gathering data, aggregating their data with those gathered by other students, and making meaning of their results" ([11], p. 26). Teachers have tried to change the classroom orientation from teaching *about* computers to teaching *with* computers [23]. Here technology is integral to the pedagogy which seeks to change not only *how* students learn but *what* they learn.

ICT as liberator

The final category, [29] claims is a radical approach where integration is a component of the reforms that will alter the organisation and structure of schooling itself. Here schools, systems and teachers are working to seek new and innovative ways that use technology to genuinely improve student learning outcomes.

If the previous five categories (groups of teachers) approximately capture the reactions of the majority of teachers towards technology then the following discussion makes use of research findings to speculate on teachers' views towards assessment with graphing calculators. Do the different approaches to integration manifest themselves in different assessment approaches?

3. A teacher's view of graphing calculator use and the possible assessment implications

As [29] proposed his five categories of teacher's views towards the integration of technology into the classroom, these categories will now be adapted as an organising device to examine research involving mathematical assessment including graphing calculators.

Graphing calculator as demon

There is evidence to suggest that teachers in this category make use of every difficulty to resist the inclusion of the graphing calculator into mathematics assessment. As a consequence, teachers either ban the devices from examinations or attempt to write graphing calculator-free questions. Another tactic is to place so many conditions on which model can be used that "due to the limitations of devices that are allowed in an examination, it is difficult to design appropriate questions to measure understanding" ([31], p.60).

The availability of CAS on graphing calculators lead to early research on assessment that concentrated upon (a) the effects on the solutions of past examination questions when CAS is included, with algebra and calculus procedures being most effected, (b) changes (or lack of change) in what is tested and the style of questions when CAS has been introduced, (c) setting CAS neutral questions (no advantage to CAS users), which was deemed possible, and (d) setting CAS-active questions for users of different CAS brands and models. For example, [4] investigated function and calculus questions in the year 2000 for Danish upper secondary public examinations, the International Baccalaureate (IB), and the Victorian Certificate of Education (VCE) examinations, for which graphics calculators were allowed. They found that questions often afforded no advantage to calculator use or they excluded their use.

Graphing calculator as servant

While teachers at this stage cling to current practice, there is evidence that the presence of technology gradually influences these teachers to change. A longitudinal case study inquiry by [15] studied instructional practices of a teacher using CAS. In the early teaching using CAS, the teacher continued emphasising by-hand skills and only employed CAS for demonstrations and solving problems that required difficult by-hand symbolic manipulation. Yet over time she increased her use of CAS as a pedagogical tool for students to learn mathematics via activities requiring exploration and investigation. This shift in teaching style was also evident in changes to worksheets. This teacher case study illustrates the challenges faced by teachers in introducing software into a classroom. It also alludes to the possibility of a developmental process that teachers may go through. As they become more comfortable and knowledgeable about the software, they are then more comfortable with extending their range of teaching strategies [29].

As further evidence of the power of graphing calculators to change current practice, [21] studied (the two years before 1996-1997 and the two years after 1998-1999) a government mandated implementation of graphing calculators into the Years 11 and 12 Tertiary Entrance Examination (TEE) program in Western Australia. They reported that the percentage of algorithmic, procedurally-orientated questions in the calculus examinations dropped across the four years, from 70% in 1996 and 80% in 1997, to 67% in 1998 and 65% in 1999. Where graphs were required, students could generate them on the graphing calculator and circumvent algebraic working. There

was a trend of increased numbers of application questions for which calculation could be done on the graphing calculator. In a later Western Australian study of Year 12 Applicable Mathematics, three years after mandated use of graphing calculators, [19] collected assessment data from eight schools for one year. They found that only 17% of test/examination marks, on average, were related to items for which calculator use was essential or advantageous.

[20] examined student understanding of function graphs when using graphing calculators. The results indicated that students had a weak understanding of the effects of scale-changes on a graph, of numerical accuracy, and accepted graphs as they saw them. The researchers concluded that misinterpretation of calculator graphs and the effects of scale on graphs needed to be explicitly addressed if students were to develop appropriate ideas and strategies for working with the technology. They also reported a tendency of students to accept the graphic image uncritically, without attempting to relate it to other symbolic or numerical information.

Graphing calculator as idol

In the beginning when the use of graphing calculators in the mathematics classroom was an option, many teachers were faced with the issue of whether the time spent on tool familiarity was worth the gains in student understanding and learning. Those teachers caught up in the excitement of using a new tool were sometimes distracted away from the mathematics. However while there exist early assessment items where correct button sequences were sought as a measure of student technical mathematics knowledge they should not be discounted. The entering of correct expressions requires a conceptual appreciation for the structure of the expressions being entered. The use of CAS requires technical knowledge of the notations and conventions of CAS [32].

Even this stage can provide opportunities for new ways of learning, teaching and assessing mathematics. A recognition of differences in students' embrace of graphing calculators can be found in the work of [32] (p. 1179) who use the terms 'instrumental genesis' to describe how technology does not have the same automatic power for all users and how intelligent use requires both conceptual and technical knowledge. Drijvers (2003, as cited in [32], p. 1179) studied 9th and 10th graders use of CAS with graphing calculators to develop higher order understanding of the concept of parameter. Drijvers analysis of the obstacles revealed both technical and conceptual difficulties. Thus knowledge of these obstacles has the potential to provide further teaching and learning opportunities and directions as well as assessment items for these technical and conceptual areas.

Graphing calculator as partner

The availability of CAS calculators has generated considerable research and debate about mathematics content and teaching practices at secondary level and is relevant to include here however briefly due to the overlap with graphing calculators with CAS. [24] provide an overview of the place of CAS in the Victorian State mathematics examination system in Australia and some other systems. Apart from emphasising the place of CAS in the three principles discussed earlier, they required "examinations to be rigorously scrutinized by people knowledgeable about all the technologies in use" (p. 15). This has implications for teachers' integration of technology in their classrooms.

[26] used a pre-post-test methodology with a module of work implemented over four lessons, addressing the solution of linear equations using CAS. It was implemented in a New Zealand Form 4 secondary class of 13-14 year old students. In the pre-test, students scored well on standard questions which could be solved symbolically and there was no significant difference in favour of CAS in performance on those in the post-test. There was a significant difference on questions that were more conceptually demanding and involved relating data between two or more

representations. They concluded that one value of graphing and CAS calculators is that they can be used to make explicit links between different representations.

The early belief that good teaching of functions and calculus preceded by simultaneously developing the three representations (numeric, graphic and symbolic) has lost its favour. Research cautions that the adoption of multiple representations may need to be introduced over time rather than from the start, with graphical and symbolic representation taking precedence over numeric representations [16]. Research indicated that regular access to the technology can have a positive influence on linking different representations of functions. The principal findings of [6] were that teachers who believed in multi-representational approaches used such approaches when teaching and that effective technology use was buttressed by well-developed mathematical knowledge.

The limitations of the graphing calculator can be used as an opportunity for learning, teaching and assessing mathematics. [10] uses the term 'mathematical fidelity' to highlight that a tool such as a graphing calculator doesn't always represent mathematics as it is understood by the mathematics community (eg., the graph of the sine function with increasingly small periods). He noted three areas in which a lack of mathematical fidelity can occur: discrepancies between tool and mathematical syntax conventions; under-specification in mathematical structure; and, limitations in representing continuous phenomena and discrete structures and infinite precision numerical computations. The limitations can become assessment opportunities for students to explore and apply their mathematics.

Graphing calculators often include dynamic geometry which "helps students in defining and identifying geometrical properties and the dependencies between them, but not in proving them" ([13], p. 114). [26] conducted a study of a Year 8 secondary school classroom where students worked in pairs on an exploratory task using *Cabri Geometry* to generate and then prove conjectures about quadrilateral properties. She reported that the dynamic environment supported students' argumentation and helped to connect conjecturing with proving. The dynamic feedback from the software (e.g., "dragging" figures to look for invariant properties) and teacher support helped students to construct a written proof. Teacher support is important as [22] found that elementary students produced diagrams more often than constructions when given minimal guidance in their introduction to a dynamic geometry. These diagrams did not lead to insights about the powerful mathematical ideas embedded in the Cabri program. For [13] the challenge is the "search for learning contexts which help students switch naturally between deductive and inductive concerns - contexts in which it makes sense to formulate statements and definitions through agreed procedures of deduction without severing any connection from empirical justification" (p. 103).

Graphing calculator as liberator

In a description of experimental mathematics, Borwein (2005, [32], p. 1170) proposed the following for computer use which could act as a guide to the use of graphing calculators assessment items. Assessment items using graphing calculators could be used in (a) gaining insight and intuition, (b) discovering new patterns and relationships, (c) graphing to expose mathematical principles, (d) testing and especially falsifying conjectures, (e) exploring a possible result to see whether it merits formal proof, (f) suggesting approaches for formal proof, (g) replacing lengthy hand derivations with tool computations, and (h) confirming analytically derived results.

The graphing calculator allows students to quickly generate graphs from algebraic and numerical data. Assessment questions can shift the emphasis to the teaching of higher-order thinking skills such as interpretation rather than the procedural skills of drawing graphs. "Previously, by the time students had drawn a graph, the class was nearly over - now they can talk about what it means" ([31], p. 60).

Another example of the liberation effect of graphing calculators for mathematics assessment relies upon graphing calculators ability to support students' engagement in authentic mathematical activity by using the classroom as a knowledge-building community. [9] discusses shared-passions in which students are involved in a collective effort of understanding with an emphasis on diversity of expertise, shared objectives, learning how to learn and sharing what is learned. Learning communities are groups of people who investigate issues and share what they learn with others in the community, thus advancing both their individual knowledge and the community's knowledge.

The motivational aspects of student use of graphing calculators and the social context can be rich in assessment opportunities. [2] examined the factors which influenced a student's choice or reorganisation of mathematical procedures. Surprisingly, these factors were found to be social rather than mathematical in origin and were influenced by the goals that students establish, the contexts in which they interpret them, and has very little to do with the understanding of mathematical concepts. If student behaviour is embedded within social interaction, then what are the implications for graphing calculators? Graphing calculators were designed as personal tools, yet research has reported that students tended to use them as a shared device. Graphing calculators played an important role in group activities as a kind of conversation piece for sharing mathematical ideas and making thought processes publicly available in the classroom. The graphing calculators facilitated social interaction in the classroom because it acted as a common point of reference for students as they discussed their ideas and results [5]. Thus the effectiveness of graphics calculators may reside more in this social role than in their ability to reveal mathematical concepts. Alternative assessment approaches can harness this social context.

4. In what ways does the graphing calculator expand the range of performances accessible through assessment?

Using the graphing calculator means some mathematics assessment becomes more important because graphing calculator use requires it; some mathematics assessment becomes less important because graphing calculator use replaces it; and some mathematics assessment becomes possible because graphing calculator allows it. The lists that follow are not meant to be comprehensive but should provide guidance to teachers who regard the graphing calculator as a partner or liberator. It will attempt to answer in a partial way the question: In what ways does the graphing calculator expand the range of performances accessible through assessment?

Mathematics assessment becomes more important because graphing calculator use requires it

Mathematics assessment using graphing calculators should achieve higher-order thinking thus assessment items must emphasise the formulating and interpreting and the understanding of generalizations [24]. There should be a greater number of assessment questions that were more conceptually demanding and involved relating data between two or more representation.

Assessment items should make use of the fact that graphing calculators allow the use of real world, real time data through the use of the various probes where data can be captured outside the classroom to be later transferred and analysed [11]. Greater assessment of students' understanding of function graphs when using graphing calculators which includes an understanding of the effects of scale-changes on a graph, of numerical accuracy, critical examination of graphical images, and the skills needed to relate graphs to other symbolic or numerical information [20] Items should have a greater emphasis on higher-order thinking skills such as interpretation rather than the procedural skills of drawing graphs [31].

Increase the number of application questions [21]. The dynamic geometry environment provides opportunities for assessing students' argumentation and their ability to connect conjecturing with proving [27] Dynamic geometry can provide learning contexts which help assess

students' ability to switch naturally between deductive and inductive concerns. Assessment contexts where students formulate statements and definitions through agreed procedures of deduction from empirical justification [13].

Mathematics assessment becomes less important because graphing calculator use replaces it.

The use of graphing calculators in mathematics assessment requires a reduction in the percentage of algorithmic, procedurally-orientated questions in calculus items. On items requiring graphs, students could generate them on the graphing calculator and circumvent algebraic working [21].

While the graphing calculator can be used to reduce the tedium of mundane computations this is not a reason for the complete elimination of computations from assessment. Correct button sequences are a measure of student technical mathematics knowledge and the entering of correct expressions requires a conceptual appreciation for the structure of the expressions being entered [32].

Mathematics assessment becomes possible because graphing calculator allows it.

In this section opportunities for assessing mathematics learning that arise due to the strengths and weaknesses of the graphing calculator are canvassed.

Enyedy and Suthers (2000 cited in [32], p. 1192) defined representational fluency as "being able to interpret and construct various disciplinary representations, and to be able to move between representations appropriately. This includes knowing what particular representations are able to illustrate or explain, and to be able to use representations as justifications for other claims. This also includes an ability to link multiple representations in meaningful ways". Thus assessment opportunities arise when the students' draw meaning across representations and when they generalise across different representations. Representational fluency across multiple representations affords the opportunity to focus on central mathematical objects and processes [32].

Assessment of mathematics can make greater use of items that involve examining 'mathematical fidelity'. Items could investigate discrepancies between tool and mathematical syntax conventions; under-specification in mathematical structure; and, limitations in representing continuous phenomena and discrete structures and infinite precision numerical computations [10].

Graphing calculators expand the possibilities for assessment items that can be designed to measure the extent to which students are able to (a) gain insight and intuition, (b) discover new patterns and relationships, (c) graph to expose mathematical principles, (d) test and especially falsify conjectures, (e) explore a possible result to see whether it merits formal proof, (f) suggest approaches for formal proof, (g) replace lengthy hand derivations with tool computations, and (h) confirm analytically derived results (Borwein (2005, cited in [32]).

Finally the graphing calculators expand the range of opportunities for assessing students performance in social interaction and collaborative working situations [5] Items can be developed to assess the degree of students' engagement in authentic mathematical activity and their contribution to the knowledge-building community [9].

5. Conclusion

This paper has sought to inform the current debate on the assessment of mathematics in an environment where students have access to graphing calculators. It has tried to situate the debate within the wider one which focuses upon a greater range of strategies and outcomes than can be delivered by a formal test and examination context.

The paper was guided by the principles of mathematics, learning and equity. Mathematics assessment in a graphing calculator environment creates new challenges where creativity is required to design items that focus upon valuable mathematical knowledge, encourage good learning, and are equitable. The directions provided by the research studies outlined in this paper have the potential to transform assessment while transforming teaching and learning.

6. References

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