Pedagogical Use of a CAS

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Abstract: *A CAS can be used either as a functional tool or a pedagogical tool. A trial was conducted using Casio Classpad calculators with a class of year 10 extension students. They studied the same content as 8 parallel classes in the same public school for one semester. The pedagogical use of the CAS was emphasised in class work. During examinations the experimental group were not permitted to use their CAS calculators, although they could use then at any time in class and during a take-home assignment.*

A control group, who were matched on the criteria of working mathematically and thinking mathematically using their grade 9 results was used to evaluate the effectiveness of the trial. At the conclusion of the trial, student scores on knowledge and procedure the experimental group were slightly slower on average than those of the control group, but with a smaller spread. The scores on modelling and problem solving were almost identical for the experimental and control groups.

Introduction

What is a CAS?

A CAS is a Computer Algebra System. It is a sophisticated algebraic / symbolic manipulator, or more colloquially an intelligent assistant.

Although they have been available for several decades, their cost has been prohibitive for school use until recently. Mainframe CAS such as *Mathematica*™, *Derive*™, *MatLab*™ and *Maple*™ have been available since the 1970s, and for PCs since the late 1980s. Single-user licences are still prohibitive for most school situations (typically of the order of $A 1000). Hand held devices such as the Texas Instruments TI-89 & TI-92, Casio FX-2 and HP 40G have been available since the late 1990s. Over time, these applications have become much more user-friendly with a significant reduction in the need to learn program specific syntax rules. These calculators were essentially a graphing calculator with a modified CAS (such as Derive) built in as an application. The latest versions of these devices are similarly priced to graphing calculators.

Current hand-held devices such as the TI-nspire™ or Casio Classpad™ also have sophisticated geometry applications (such as Cabri) that can be interfaced with the CAS, and spread sheeting applications, but these are not the focus of this paper. The larger screens on these devices (compared to the TI-89 and FX-2) makes the use of split screens and simultaneous multiple representations much easier.

CAS use in Australia

Some Queensland schools, such as the Brisbane School of Distance Education have been using CAS calculators extensively in Mathematics B since the late 1990s. (Driver, 2003) Some other schools have been using them to a greater or lesser extent, both as hand-held and as computer based systems for similar or shorter periods.

Victorian schools have been experimenting with a CAS enabled syllabus for several years and as a result of their trials (which have been supported by extensive research and professional development) the Victorian studies authority has mandated the use of CAS in their Mathematics B equivalent courses for 2009. Western Australia has recently decided to also mandate the use of CAS calculators from 2010.

In other states, graphing calculators only are allowed. In New South Wales, they can only be used in middle school mathematics.

Although the use of graphing calculators was initially popular with senior secondary school teachers and is increasingly being used by middle school teachers, they seem to have been largely ignored by tertiary academics. CAS, however, has been used by tertiary academics for both research and teaching before being taken up by senior secondary teachers. CAS use also seems to be being introduced at increasingly earlier stages of education.

The use of CAS at the secondary level has been strongly supported by the manufacturers of hand-held technology and also by some tertiary mathematics educators and teacher educators.

What can a CAS do?

A CAS can do almost all of the algebra, calculus and trigonometry in the current Mathematics syllabuses in Australia. (Throughout this paper, illustrations are all done using a Casio Classpad. Although the syntax and menu structure may differ, similar procedures are possible on other platforms.) It "knows", for example, the:

formula for the solution of a quadratic equation

```
solve(x^2-2x-1=0,x)
```

$$\{x=-\sqrt{2}+1, x=\sqrt{2}+1\}$$

```
solve(ax^2+bx+c=0,x)
```

$$\left\{x=\frac{-\left(b-\sqrt{b^2-4\cdot a\cdot c}\right)}{2\cdot a}, x=-\right.$$

Figure 1 The quadratic formula

laws of indices

```
x^3×x^5
```

$$x^8$$

```
(x×y)^3
```

$$x^3\cdot y^3$$

```
x^a/x^b
```

$$x^{a-b}$$

Figure 2 The index laws

trigonometric identities

```
tExpand(sin(2x))
```

$$2\cdot\cos(x)\cdot\sin(x)$$

```
tCollect(tan(x))
```

$$\frac{\sin(x)}{\cos(x)}$$

Figure 3 Trigonometric identities

exact values of trigonometric ratios and their inverses

```
sin(75)
```

$$\frac{\sqrt{2}\cdot\left(\sqrt{3}+1\right)}{4}$$

```
sin⁻¹(.5)
```

$$\frac{\pi}{6}$$

Figure 4 Exact trigonometric values

rules for differentiation

```
diff(e^(-x)×x,x)
```

$$-(x-1)\cdot e^{-x}$$

```
diff(ln(sin(x)),x)
```

$$\frac{\cos(x)}{\sin(x)}$$

Figure 5 Derivative functions

indefinite integrals

```
∫(x×(x^2+1)^3,x)
```

$$\frac{(x^2+1)^4}{8}$$

```
∫(1/√(x^2-4),x)
```

$$\ln\left(x+\sqrt{x^2-4}\right)$$

Figure 6 Indefinite integrals

How can students use a CAS?

A CAS can be used to **do** Maths and to **learn** Maths. Bernhard Kutzler (2003), one of the pioneers in the use of CAS makes the following analogy between transportation and mathematics when using a calculator or computer as an automatization and compensation tool (i.e. as a functional tool):

Moving and Transportation (Physical)	Mathematics (Intellectual)
Walking	Mental calculation
Riding a bicycle	Paper-and-pencil calculation
Driving a car	Calculator or computer calculation (automation)
Using a wheelchair	Calculator or computer calculation (compensation)

Table 1 Kutzler's automatization model

Kutzler then goes on to describe the use of a calculator or computer as a tool, for trivialization, experimentation, visualization and concentration (i.e. as a pedagogical tool).

His “model” of the steps of mathematical discoveries:

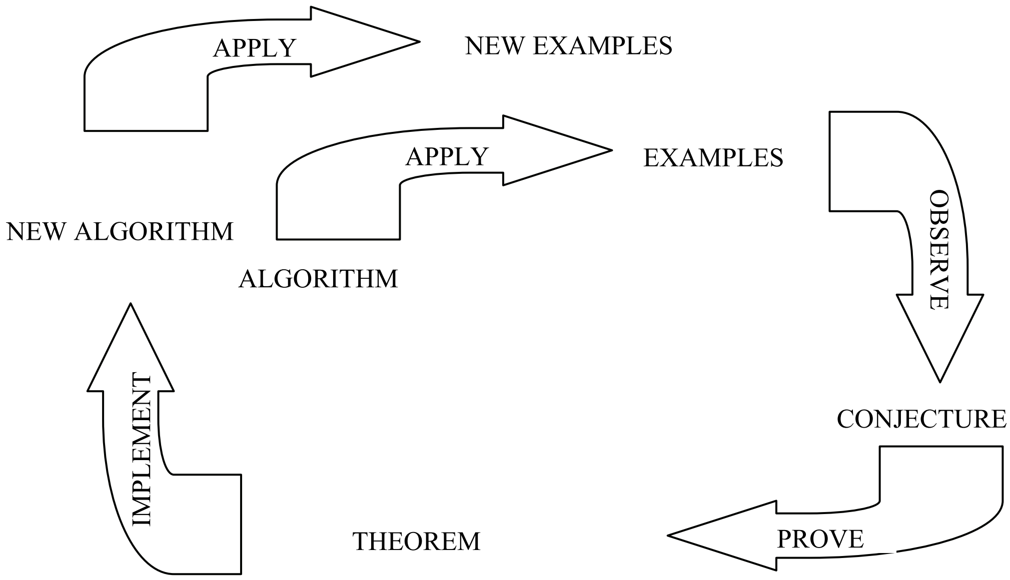

Figure 7 Kutzler’s pedagogical model

References to Heugl and Piaget highlight the constructivist model of learning that underlies Kutzler’s views on the use of a CAS as a pedagogical tool.

CAS as a functional tool when problem solving.

As per Etlinger’s reference to the four-function calculator in primary education a CAS can be used to “do the messy algebra”.

By allowing a student to focus on the selection of a problem solving strategy or appropriate procedure rather than the application of the strategy or procedure, and student can develop their higher-order thinking skills. For example, the student can:

solve a complex linear equation “step-by-step” by simply selecting an appropriate next step to find a series of equivalent equations and thus systematically reduce the equation to its solution.

e.g. solve the equation $2(x+3)+4=5(x-1)-7$

```
2(x+3)+4=5(x-1)-7
          2·(x+3)+4=5·(x-1)-7
expand(ans)
                2·x+10=5·x-12
ans-5x
                  -3·x+10=-12
ans-10
                     -3·x=-22
ans/-3
                          x=22/3
```

Figure 8 Step-by-step solution of a complex linear equation

optimise a complex situation - by allowing the calculator to: substitute a constraint into a function with two or more independent variables; differentiate the resulting single-variable function; find the zeros of the derivative; and substitute an exact solution back into the function - the student is released to focus on the goal and how to achieve it.

e.g. find the dimensions of a rectangular prism in which the length of the base is twice its width and the volume is 1000 cm^3.

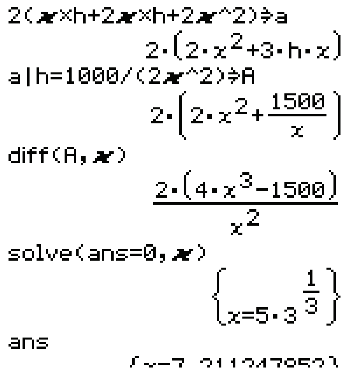

Figure 9 Solution of an Optimisation Problem

i.e the dimensions of the rectangular prism are approximately 7.2 × 14.4 × 9.6 cm.

Note that although the CAS can “do” the mathematics involved in this problem, students will need to take greater care than they normally would, with their communication and justification of their solution.

CAS as a pedagogical tool in constructivist teaching

The use of a CAS as both a functional tool and as a pedagogical tool is consistent with the Australian Association of Mathematics Teacher’s Statement on the use of calculators and computers for mathematics in Australian schools that, in part, recommends that:

- priority be given to the use of calculators and computers as natural media for mathematics learning within a technologically-rich learning; and
- teachers at all levels be actively involved in exploring ways to take full advantage of the potential of technology for mathematics learning within the total curriculum.

(AAMT, 1996)

Kath Heid proposed the following as ways in which a CAS can function is a cognitive technology to assist in the teaching and learning of mathematics:

- students can use CAS for the repeated execution of routine symbolic procedures in rapid succession without the diminished accuracy and increased fatigue usually associated with the repetitive execution of by-hand routines;
- students can assign rote symbolic tasks to the CAS so that they can concentrate on making “executive decisions;
- students can use the CAS to apply routine symbolic algorithms to complicated algebraic expressions, without the confusion students sometimes experience when trying to apply a routine procedure to a complicated expression.

I would propose that a CAS can also assist by facilitating students dealing with real world data rather than the “nice” or sterilised data and functions which we tend to use in high school mathematics.

It is not my intention to elaborate on the work done by Kaye Stacey (Stacey, 2007), except to indicate that I believe that this is the most powerful reason for the introduction of a CAS into either a middle school or senior school mathematics classroom. Some of the hands-on activities will illustrate some of the ideas summarised in the following graphic.

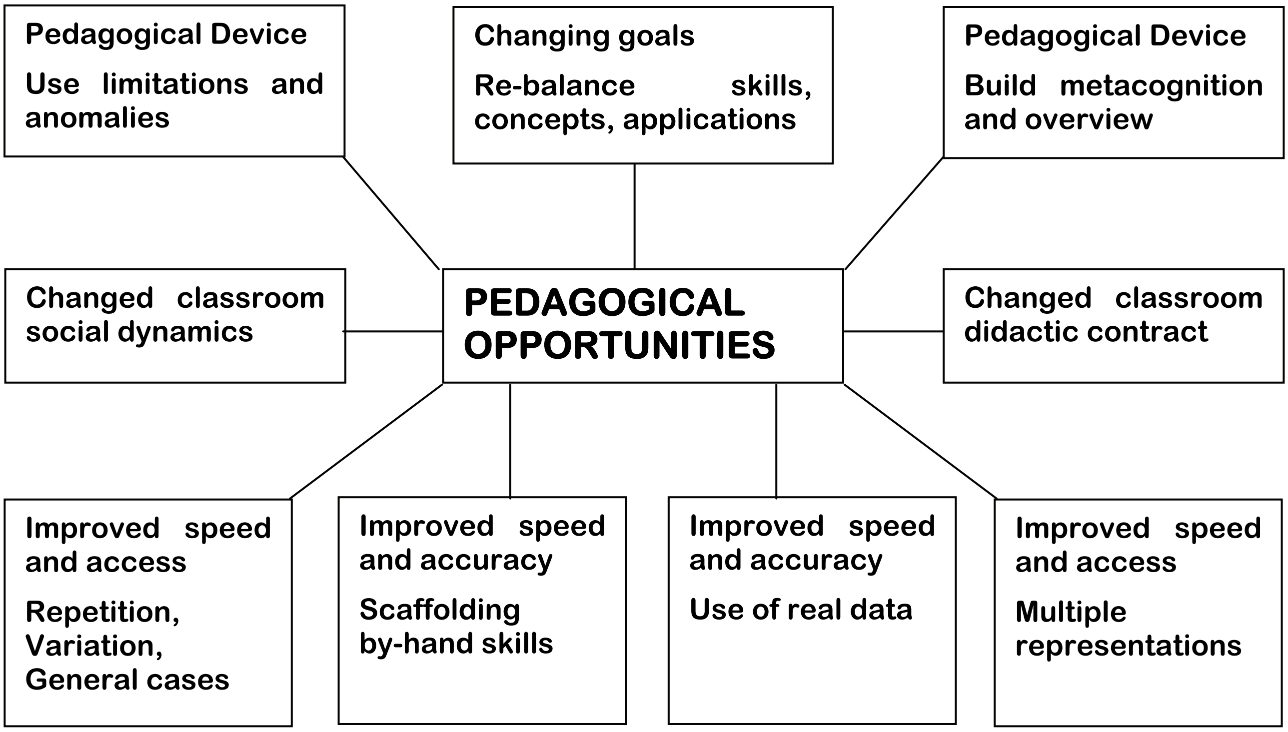

Figure 10 Stacey's Pedagogical opportunities

Teaching approaches in a CAS classroom

Stacey has outlined five teacher scenarios, in which teachers value the introduction of a CAS into their classroom for different reasons depending on whether they use the CAS primarily for functional or pedagogical purposes or both.

Initially my own use of a CAS (see Driver, 2003) would have been as a progressive teacher in Stacey's terms, but has moved since the mid 2000s to a radical viewpoint.

CAS and algebraic manipulation skills

Numerous studies have reported either no loss or an improvement in students' pen-on-paper algebraic manipulation skills following the introduction of a CAS into their classroom. In a review of 15 studies of the impact of the use of a CAS on computational or procedural skills, Heid, Blume, Hollebrands and Piez concluded that in all but one of the studies, "students whose courses included a CAS did just as well on test items that required computational and procedural skills as those students whose instruction did not include the technology" (Heid, et.al., quoted in Böhm, Forbes, Herweyers, Hugelshofer and Schomacker, 2004.)

At this stage, it is proposed that at BSHS (where I currently teach), to circumvent any loss of algebraic manipulative skills and to reinforce this to students, we will modify our assessment procedure and have separate calculator allowed and calculator not-allowed examinations in years 10, 11 and 12 rather than separate Knowledge and Procedures and Modelling and Problem Solving examinations.

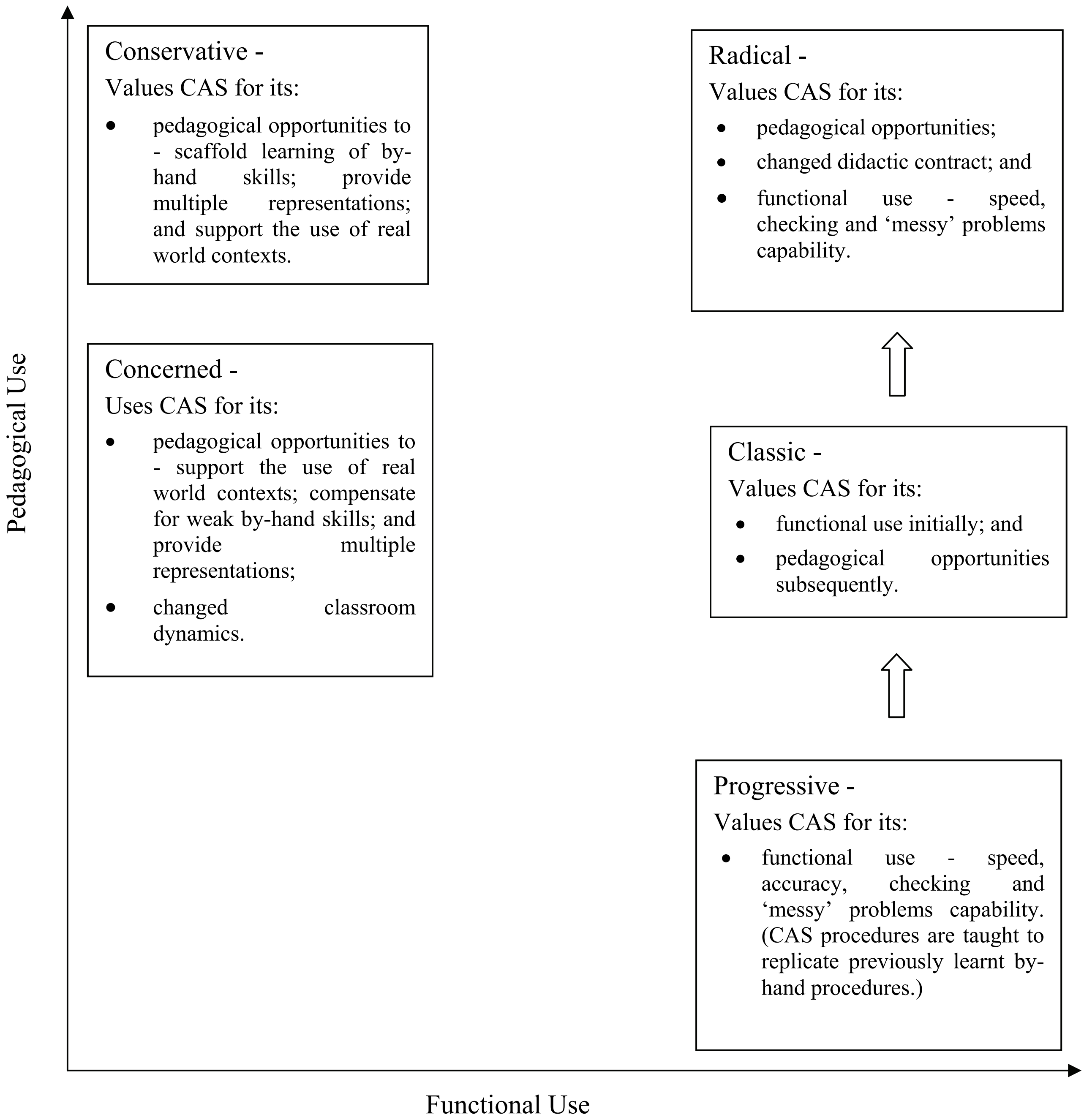

Figure 11 Two-dimensional interpretation of Stacey's Pedagogical opportunities

What knew skills do students need to effectively use a CAS?

Effective use of a CAS will necessitate an increased ability to quickly recognise algebraically equivalent expressions. Pierce and Stacey (2001) describe algebraic sense (which can be compared to number sense) as algebraic expectation and the ability to link representations.

When should students use a CAS?

I believe that students should have ready access to a CAS: when developing new concepts (primarily as a pedagogical tool) and when doing class work or for assignments (as a functional tool) and on calculator allowed tests. i.e. it should be used for teaching / learning and assessment.

Syllabus implications

The Queensland Mathematics 1 / Mathematics B syllabi show a clear trend in the use of technology. From using scientific calculators, we have progressed from the recommendation to use graphing calculator technology (1992), to the mandated use of graphing calculator technology (2002) and now the consideration of "the use of … hand-held (calculator) technologies … with and without algebraic manipulation" (2008). Will the next step be the mandating of algebraic manipulation technology in line with Victoria and Western Australia? With the push for a national curriculum, someone is going to have to give. I cannot image the authorities in Victoria and Western Australia retreating from their use of "algebraic manipulation technology" i.e. CAS usage.

Thomas, Monaghan & Pierce (2004) argue that the introduction of a CAS has the potential to influence the curriculum by

- providing a catalyst for a change in the emphasis given to aspects of the algebra curriculum;
- enabling the building up of conceptual understanding and increasing the range of available representations;
- emphasising generalisation; and
- supporting mathematical modelling.

By removing the need for nice functions that are often contrived and artificial (due to the need for ease of pep-on-paper techniques) more realistic, real-world data and models can be used.

The BSHS Trial

The standard year-10 first semester program in MAB101 is composed entirely of algebra, coordinate geometry and trigonometry. There were nine classes following this program - approximately 2/3 of the age cohort. The author's class were issued with Casio Classpad calculators at the commencement of the semester. Students had the option of taking them home, and 22 of the 28 students elected to do this. The remaining 6 students had access to the calculators only in class time. The course content was not modified. Where possible, however, the calculators were used as a pedagogical tool.

CAS activities which focused on its use as a pedagogical tool included: basic algebra; index laws; transposing formulae; solving equations step-by-step; factorizing trinomials; completing the square; and quadratic graphs. Most of the activities consisted of guided investigations which encouraged students to observe patterns, make conjectures and construct their own meaning to the underlying mathematics.

The Classpads were not permitted during assessment, which consisted of 2 one-hour Knowledge & Procedures examinations, 2 one-hour Modelling & Problem Solving examinations and a take-home assignment on transformations of quadratic functions.

Evaluation

The students thoroughly enjoyed the calculators and were the envy of other year 10, 11 and 12 mathematics students. The motivational effects of the calculator were not directly assessable. Students in the experimental group (Classpad users) were matched with students form other year 10 classes (scientific calculator users) on the basis of their semester 2 of year 9 results in both Working Mathematically (WM) and Thinking Mathematically (TM). Due to the large cohort size, it was possible to match most of the students in the experimental group with a control group student whose scores on both criteria were within 2% points of the experimental group students' scores. Six of the 28 students in the experimental group were not at BSHS inn 2007 and so have been excluded from the evaluation, as it was not possible to identify a matched-pair student. It should be noted that both the experimental and control groups are almost entirely from the upper two-thirds of a selective cohort. (Typically in Queensland, about 30% of year 11 students study Mathematics B. At BSHS, approximately 70% of students study this course and only 30% of students undertake the less demanding Mathematics A course.

As shown in Table 2, preliminary results indicate that there was no significant positive or negative effect on either of these measures of student performance. Scores on Knowledge & Procedures (K&P) and Modelling & Problem Solving (M&PS) were similar for the experimental group and the matched-pair control group. The duration of the trial may have reduced the impact of the use of the CAS calculator.

Results

	Experimental Group (Classpad)				Control Group (Scientific Calculators)			
	Year 9		Year 10		Year 9		Year 10	
	WM	TM	K&P	M&PS	WM	TM	K&P	M&PS
	85	69	78	57	85	65	79	41
	75	58	59	11	74	57	71	33
	55	31	53	41	55	35	65	27
	74	39	77	44	75	39	79	58
	77	69	77	43	77	65	71	57
	80	65	62	40	81	64	81	45
	72	52	68	33	71	52	74	26
	94	88	89	64	94	89	96	61
	86	82	90	57	87	80	84	67
	62	44	51	27	64	44	53	33
	82	77	75	43	82	73	79	35
	49	0	53	26	37	11	38	5
	76	68	82	61	75	65	86	64
	90	69	87	53	92	73	94	57
	83	87	68	33	82	85	83	43
	98	98	89	79	99	98	91	73
	91	90	83	61	92	86	82	50
	93	98	91	67	94	98	97	74
	84	63	94	86	86	63	79	35
	82	70	82	65	81	74	92	80
	73	46	89	79	79	46	89	87
mean	79.1	64.9	76.0	51.0	79.1	64.9	79.2	50.0
stdev.	12.4	23.9	13.8	19.4	14.2	21.9	14.2	20.6
r			0.792	0.461			0.887	0.577
r^2			0.627	0.212			0.786	0.332

Table 2 Pre and Post Trial results for Knowledge & Procedures (Working Mathematically) and Modelling & Problem Solving (Thinking Mathematically) for the Experimental and Control Groups

Although the correlations between years 9 and 10 performance on both criteria were similar for the experimental and control groups, the trend lines in Figures suggest that the use of the Classpad may have had a “levelling effect” on Modelling & Problem Solving, although this may be an artefact, due to the poor performance on Thinking Mathematically of one student in the experimental group. The tendency for the correlations between Working Mathematically & Knowledge & Procedures and between Thinking Mathematically & Modelling & Problem Solving to be smaller for the experimental group than for the control group, may indicate that although the Classpad was not permitted in examinations, it has had the effect of overcoming some of the weaknesses of lower achieving students in algebra. Further research is need to determine whether or not this trend is significant.

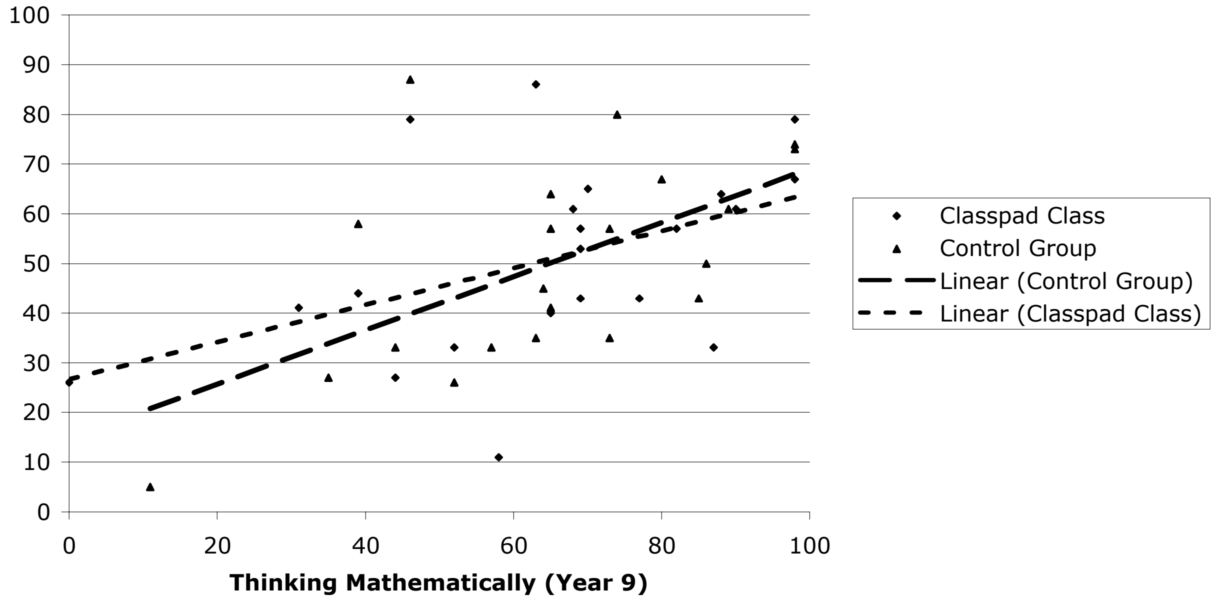

Figure 12 Working Mathematically (Pre-trial) and Knowledge & Procedures (Post-trial) Results for the Experimental and Control Groups

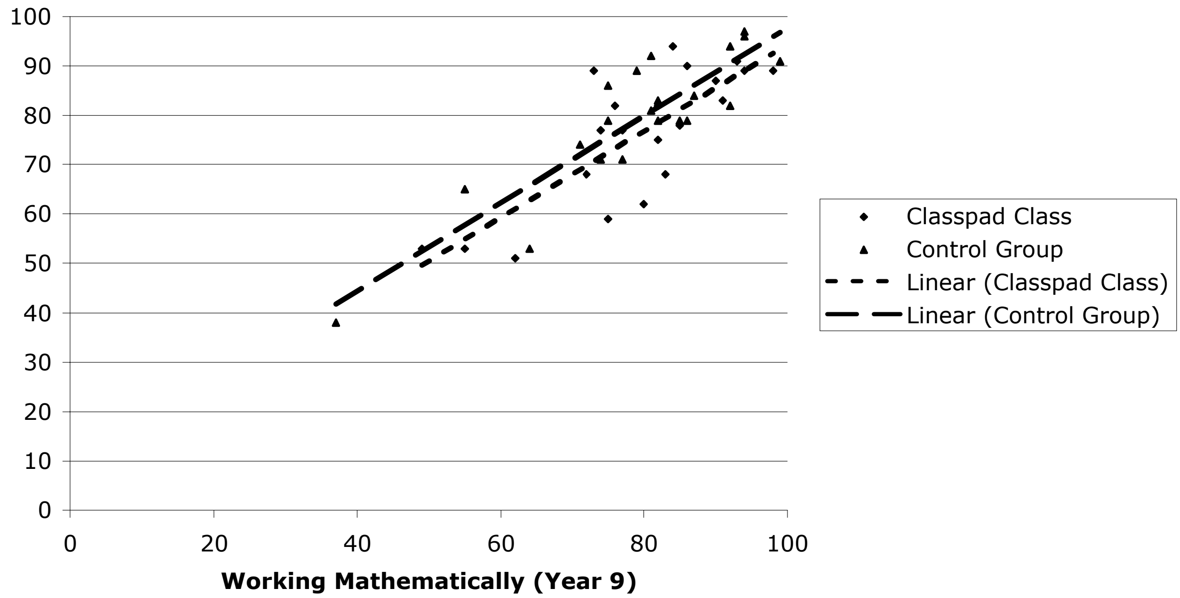

Figure 13 Thinking Mathematically (Pre-trial) and Modelling & Problem Solving (Post-trial) Results for the Experimental and Control Groups

Conclusion

Although the course work studied by the students in this trial was almost entirely algebraic, there does not appear to have been any adverse impact on pen-on-paper algebra skills during assessment in the experimental group as a result of using a CAS for class work.
If, as was hoped, the use of a constructivist approach to teaching in conjunction with the use of a CAS as a pedagogical tool, has a long term impact on algebra skills, then this will become evident in the future.

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