Using Technology to Help Engineers Learn Mathematics

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Abstract: For a number of years, Loughborough’s Mathematics Education Centre, recently awarded national Centre for Excellence in Teaching and Learning status, has worked with others to provide innovative learning resources to enhance the teaching and learning of mathematics. This paper describes some of the uses we are making of technology to help and support our engineering undergraduates learning of mathematics; most are taught mathematics using the HELM (Helping Engineers Learn Mathematics) resources, which we briefly describe. HELM was a major curriculum development project involving a consortium of five UK universities led by Loughborough that uses interactive technology for teaching and assessment. Its success is evidenced by their considerable uptake at other institutions. E-learning is part of our overall learning and teaching strategy and a key component of our work is to develop the use of emerging modern technology in the delivery of mathematics at all levels, whilst pursuing the relevant pedagogic research. Consequently we describe some recent developments, we are using in our teaching with engineers; these include the use of Tablet PCs and electronic voting systems. We make significant use of our university’s VLE to deliver the material, and ongoing developments include the use of podcasts and Moodle quizzes for formative assessment, which we outline. Finally, we comment on some of the issues involved.

1. Introduction

Loughborough University’s Mathematics Education Centre (MEC), recently awarded national Centre for Excellence status, was established in 2002. One of its main activities is the teaching of mathematics to engineers for which it is essential to take into account their differing mathematical backgrounds; to underpin its activities, the MEC pursues relevant pedagogic research related to mathematics education in higher education. MEC staff are proactive in using technology in teaching and this paper describes some of the uses made of technology to help engineers learn mathematics.

First we outline the HELM (Helping Engineers Learn Mathematics) project; HELM [1] was a major curriculum development project involving a consortium of five UK universities led by Loughborough that uses interactive technology for teaching and assessment. It grew out of the realization that there was a need for more flexible mathematics learning resources for engineering undergraduates, due in particular to the increasing diversity of university intake standards, who were unable to cope with existing textbooks and struggling to construct accurate and meaningful lecture notes. Over a number of years, we have worked with others to provide innovative learning resources, which overcome these difficulties. Most of the staff involved with teaching mathematics to engineering students now use HELM workbooks and associated computer assisted assessment.

A key component of our work is to develop the use of emerging technology in the delivery of mathematics [2]. In this context, secondly we explore the potential offered by the tablet PC for teaching and learning and the use of narrated lectures. In the same vein, thirdly we consider the opportunity for interactivity offered by the use of electronic voting systems.

Fourthly, we make significant use of our university’s Virtual Learning Environment (VLE) to deliver the material and other recent developments have included the use of quizzes for formative assessment and podcasts, which we outline.

Finally, we comment on some of the issues involved recognising that further evidence-based research is needed to evaluate the effectiveness of these uses of technology on student learning.
2. HELM

2.1 Background
The HELM project used the expertise of the consortium and computer technology to enhance the mathematical and statistical education of engineering undergraduates in the UK through the development of a range of flexible learning resources in the form of workbooks, supplemented by web-delivered interactive lessons, together with an extensive integrated web-delivered CAA (Computer Aided Assessment) regime, to drive student learning. During the development phase 2002-05, the focus was on the consortium members developing teaching materials, all of which was then critically read by external academics, amended and then trialled at over fifty volunteer UK HEIs and FEIs (Higher and Further Education Institutions). During a follow-up project 2005-06, the focus moved to working with six different UK universities who were trialling the materials, integrating them into their own provision and performing their own evaluation. This enabled the resources to be adopted, evaluated and, where necessary, refined and thus encouraged wider transferability of the project’s deliverables across UK HEIs.

2.2 Learning Resources
The 50 HELM workbooks (see [1]) are available in both web-based and paper-based forms. These innovative workbooks can be the main text providing excellent ‘lecture notes’, supplementary text alongside the lecturer’s own notes, or just provide additional engineering related examples and exercises. They present the basic mathematical and statistical content essential for engineering undergraduates, with key points highlighted, and provide both fully worked examples and student tasks to encourage student engagement during lectures. The tasks are often broken down into stages with intermediate answers provided and incorporate space to write in the solution. Real applications make mathematics more attractive to students and so, for additional motivation, engineering examples and case studies closely related to the mathematics and statistics are presented throughout workbooks 1 to 46 and in two dedicated case study workbooks 47 and 48. Workbooks 49 and 50 comprise a student’s guide and a tutor’s guide. The total workbook pages developed numbers 2830, which indicates the unique scale of the project. HELM courseware, consisting of 80 on-line interactive lessons to aid understanding, is web-delivered and supports parts of 23 of the more basic workbooks.

An integrated web-delivered CAA regime, based on Questionmark Perception [3], with 5000 questions (see [1]) covering around 275 concepts supporting the workbooks can be used for both formative and summative assessment. This facilitates the regular testing of large numbers of students and is used to encourage students to engage more in their own learning.

2.3 Trialling
The HELM learning resources have now been extensively trialled at many UK HEIs over a number of years; the number of students involved in these trials has been at least 6000. They can be used in many different pedagogic ways and trialling has shown them to be very popular with staff and students alike. They may be used to support the teaching of engineering students for a complete mathematics or statistics course or module, or just selected parts. Alternatively, the resources can be used for independent learning. The success of HELM can be seen from some of the comments received from HEI staff, students and external assessors. One lecturer [4] reported: “We can verify that the inclusion of HELM resources has been popular with students and has made a significant contribution to pass rates, student attainment and more generally student learning skills.” Another noted from the student learning diary summaries HELM referred to as [5]: “brilliant piece of work”, “my best friend” and “exceptional”. The external evaluator commented:
“It was noticeable that mature students and those with special needs (typically dyslexics) were very appreciative of the textbooks and made extensive use of them, citing being able to work at their own pace and being confident they that they had a large set of notes as being large positives”.

The success of the HELM project is evidenced by the considerable uptake at other institutions, both colleges and universities, that have now become HELM users, a feat which past experience has shown to be notoriously difficult to achieve. At present there are 132 registered HELM users spanning 91 institutions, mostly in the UK but some overseas.

3. Teaching with Tablet PCs

3.1 Background

Many students entering university find the traditional ‘lecture-tutorial’ approach too different from what they were used to at school; like the rest of us, they find it very difficult to take notes and think at the same time. A major issue is whether we can adapt our approach to students, and our benefit so here we consider a more sympathetic approach. We believe in giving students as much help and support as we can, especially in the early stages of their academic careers. On a number of modules, we now encourage students to view (over the web) pre-prepared lectures so that they can begin the process of understanding before they get to the lecture hall. We also encourage interactivity though, with large groups, this is only possible at a superficial level.

3.2 Use of Narrated Lectures

Many students find it difficult to take notes, especially in mathematics, in which every step must be recorded and because unusual fonts and symbols are used as a matter of course. Student notes are often incomprehensible when viewed at a later date; their writing may be poor, as may be their ‘interpretation’ of notation and there may be ‘gaps’ in the logic flow. Some even invent their own notation as if they were a Gauss or a Newton! However, we do think students have a genuine cause for complaint here. A lecturer can write quite fast – after all s/he knows what’s coming next – whereas the student is continually having to look up and down and listen and try to understand at the same time! There has got to be a better way and we would suggest the use of narrated lectures to be a possibility well worth considering.

Lectures are prepared in advance: handwritten on a tablet PC, see Figure 1, directly into a PowerPoint file. There is no particular attempt to be over professional, as long as handwriting is legible. There is no attempt to put the material into printed form; that would simply be a textbook ‘on screen’ and, in mathematics, quite a time-consuming exercise. In any case, a handwritten version carries more ‘authenticity’. As much as it can do, it has the ‘feel’ of a real lecture. Because of the properties of the tablet PC, the lecturer can draw pictures, change colours for emphasis, highlight sections and so on. Experienced lecturers rarely have to cross things out or re-do pages. But if the odd mistake is made and a student spots it – excellent! It proves they’re reading it!

In a single module we prepare up to 22 lectures – each one a PowerPoint file holding as much material as required. In a normal lecture, students would begin to complain if they had more than eight, say, pages of notes per lecture so it is perhaps wise to restrict the narrated lectures in the same way. To each page we add an audio commentary, which guides the student through the work. Students can jump from page to page in any direction and can choose to access the audio commentary, or not, as they wish. We could have prepared the lectures and commentary synchronously; however doing so produces an audio file with long periods of silence (whilst the lecturer is writing) and, in our view, is less helpful than when commentary is added later. In preparation each narrated lecture takes (writing pages and adding commentary) about one hour. Of course, with a relatively stable curriculum, this overhead need not be repeated year after year.
The narrated lectures are hosted on the VLE. Students can easily access individual pages (with attendant commentary) allowing them to repeatedly go over difficult material or material of specific interest. Before each lecture students are advised to look at the corresponding lecture on the web. We argue that if they have a preview of the material, even though they may not understand it, they will find the ‘actual’ lecture more interesting and more accessible. Students need not now take notes in lectures (though most still do as a kind of support mechanism), as the material is available electronically. We strongly encourage students to attend ‘live’ lectures as extra examples are discussed, points of confusion can be clarified and it helps them gauge their understanding of the material. Further, if electronic voting systems (described below) are used, they are better able to judge their ‘position’ in the group and questions raised by other students can help to illuminate the subject material. Some students will still absent themselves from lectures arguing now that they ‘have the notes’, albeit on the web, and they are capable of sorting out their own learning. Of course we encourage students to do both, access the audio lecture first and then attend the live lecture – they must surely be in a better position than if they simply take the traditional approach and attend the lecture, find it difficult to follow (as they have to take notes and try and understand at the same time) and go away frustrated. The good student will do both (because they are sensible and listen to advice), the average/weak student has more support, because of the repetition, and the feckless student is in no worse a position than before.

The existence of a complete suite of narrated lectures is still rare in Loughborough University but has elicited very positive feedback from students studying these mathematics modules. Some of the (typical) responses from students on the narrated lectures were:
a) “The online material was really useful, I used it throughout the Semester. All lecturers should do it.”

b) “I found the material on Learn (the VLE) really useful, as both an aid to revision and throughout the semesters whenever I felt I hadn’t understood something properly. Depending on what it was I covered I skimmed or read + listened in depth.”

c) “So useful in fact I have asked **** to suggest all lecturers to record their lectures. As a dyslexic student I may also suggest it to the special needs department that their lecture scribes (note takers) use the audio lectures.”

Although there is ample evidence that students perceive the narrated lectures as a positive addition to the learning mechanisms presented in this module, these responses do not represent very strong evidence that the use of narrated lectures is effective, but are merely an indication that this approach is worth pursuing. A more detailed study over a longer period is envisaged, which would hopefully correlate the use of this extra resource with examination success.

3.3 Usage Statistics

In a module taught by one of the authors to a class of nearly 100 2nd year Mechanical Engineers in 2007-08, regular in-class summative tests were used to further encourage engagement. One of the tests was worth 10% of the overall module mark and the other two were worth 5% each; a two-hour end of Semester examination was worth 80%. Each test was advertised well in advance and covered a single topic, material taught in the previous two or three weeks.

‘Hits’ and the date of access to the learning pages were recorded throughout the entire module and are shown in Figure 2. Access rates are encouraging. During the semester there is regular, but low(ish), access which increases dramatically in the days prior to tests. Some of this access is superficial feeding, perhaps to print off a page or two or just using the facility as a quick ‘look-up’ of what they will be tested on. By recording the duration times that students spend looking at learning pages, it is clear that much deep feeding is going on as well.

![Graph showing access to narrated lectures](image_url)

**Figure 2** Access to narrated lectures shows strong correlation with exam dates
4. Electronic Voting Systems (EVS)

4.1 Introducing Interactivity

There is an argument that the more the student ‘interacts’ in a lecture, the richer will be the learning experience and consequently the easier will be the process of understanding. Whilst we feel that the overall benefits of encouraging interactivity in lectures become less significant with large classes, we have made an effort to encourage student feedback by using voting systems.

The mode of operation is as follows: several questions are prepared in advance to be delivered sometime during a class. On entering the lecture hall students collect a handset (see Figure 3), which they use to make responses. The student body is asked a question, invariably multiple choice, see Figure 4 for a typical example) presented through a data projector and respond using their handsets. Their (anonymous) responses are received via a ‘dongle’ which slots into the USB port of the lecturer’s lap-top. On expiry of the time limit, the class’s response to the question is displayed to the whole group. The worked solution of each question is then usually presented. Those who accept the challenge can make a judgement on their own performance and (crucially) on the group performance. Conscientious students can decide what their position is within the group and make a judgement, based on fact rather than supposition, whether or not their level of effort) is adequate.

\[ \text{What is } \int_{y=1}^{y=2} x^2 y \, dy ? \]

1. \(3x^2/2\)
2. \(7y/3\)
3. \(3 \cdot x^2/2 \cdot y\)
4. none of the above

![Image of a typical Voting System question](image-url)
4.2 Student Feedback
The segment of the lecture involving voting systems can be started with a warm up question (not necessarily anything to do with mathematics and having an element of humour or surprise) to get students into the right ‘mood’. Whether the mood that is induced is of any relevance to learning mathematics is difficult to ascertain, but it appears to be an aspect much appreciated by students and gets the student body on the side of the lecturer, feeling that some effort has been made on his/her part. Below we note some typical student responses following a feedback exercise:

a) “I think the voting handsets have potential for the future. They did highlight a couple of questions no one could do.”

b) “The technology was innovative! A breakthrough in teaching”

c) “The keypad (voting) system was good for learning and made it more interesting”

Anecdotal evidence suggests that this rather naïve approach to interactivity is popular with students; this is confirmed by student feedback. A more enlightened, but time-consuming, approach would be to carry out detailed analysis of student responses and amend lecture content appropriately.

5. Using Moodle to enhance the student learning experience

5.1 Background
Loughborough University recently upgraded its Virtual Learning Environment to incorporate the open source course management system known as Moodle. Some of its features suggested opportunities to develop and promulgate additional learning resources for the benefit of our Engineering students; the particular features of interest we explored were Quiz and Podcasting

5.2 Quiz
As mentioned earlier, the HELM philosophy is to encourage students to learn and practice their mathematics by the incorporation of testing, both summative and formative, at regular intervals throughout their course; Moodle’s built-in Quiz facility can be used to facilitate this. In practice, short tests of 5 questions were delivered. A 5-question test covering 5 concepts involved the construction of 5 small question banks of similar questions from each of which a single question was randomly selected; the order in which the questions were presented was varied within each new instance of a particular test. Individual questions were generally on different aspects of one topic, or occasionally, for revision, on different topics. Topics covered for our second year engineering students included Laplace transforms, Fourier series, Second order ODEs, Eigenvectors, Partial Differentiation, Probability, Continuous Distributions, and Hypothesis Testing; and for first year students, Matrices, Complex Numbers, Differentiation, Integration and First order ODE’s. The advantages to the lecturer were that the test could be created, and scheduled for delivery, directly from within Moodle without the involvement of a 3rd party, and great flexibility was provided within the test creation structure itself for the actual format that the test would take. From a mathematical perspective, a minor limitation of the Quiz structure was the requirement for a single response from the student, either numeric or, in the case of multiple choice, a letter. Sometimes the ability to enter more than one response might be useful, for example the roots of a quadratic equation. Also the question and feedback images, involving mathematical symbolism, required the prior creation of a pdf from which the images could be captured; this is initially a time consuming process, but the questions once created may be incorporated in any future test.

Typically a test was scheduled to run for a period of 5 days and Moodle automatically flagged up the test to the students on the built-in calendar of forthcoming events. An option within Moodle allowed for the number of times that the test could be taken to be varied and this was set to 3 times.
A further option allowed for a time limit to be set for the test duration (60 minutes) and another option allowed for a time delay to be set between the instances when the test was presented. This was felt to be a very useful facility, and a 1-hour delay was set up between the first and second attempts, followed by a 12-hour delay between the second and third attempts. This was intended to encourage students to look again at the material pertinent to the test, since having seen their individual question scores, they would have the opportunity to review the sections where incorrect responses had been made and not rush, still unprepared, into an immediate new attempt.

Within each presentation of the test itself, the option to allow more than one attempt at a question was made available by selecting Adaptive mode. It was also possible to set penalties for incorrect responses, and while each correct response was worth a nominal 1 mark, each incorrect response incurred a loss of 0.5 marks. Since students had the facility to submit their responses on a question-by-question basis, and immediately saw their result, they had the opportunity to review their work, before making a second attempt at that question, and thus partially redeem themselves.

The Quiz facility allowed one to specify, for a series of tests, whether the average mark, the highest mark, or the last mark of the series would be used. For half of the tests the average score option was selected and for the remainder the higher score option. Student feedback tended to favour the latter! A sample test question and feedback image are shown in Figure 5.

For conditional probabilities we have:

\[ p(A \cap B) = p(A) \times p(B|A) = p(B) \times p(A|B) \]

It follows that

\[ p(A|B) = \frac{p(A) \times p(B|A)}{p(B)} \]

But

\[ p(B) = p(B|A)p(A) + p(B|\bar{A})p(\bar{A}) \]

\[ \Rightarrow p(A|B) = \frac{p(A)p(B|A)}{p(B|A)p(A) + p(B|\bar{A})p(\bar{A})} \]

In this case:

\[
\frac{(0.4)(0.8)}{(0.4)(0.8) + (0.3)(0.6)} = 0.64
\]

Figure 5 Test question and feedback

By presenting frequent short tests, focussed on recent lectures, with the opportunity for students to correct themselves within a test and to improve their mark by attempting tests again, it was hoped that students would not feel intimidated by the test procedure, but use it as an opportunity to improve their understanding. A rigorous analysis of student responses has not yet been carried out, but the initial impression was that some students did take tests more than once even when their initial mark was high, which suggests that they may have found the test useful in developing their mastery of the material. However, it is equally clear that others only did the bare minimum.

On completion of each test, students were made aware of their final score, and at the conclusion of the testing period were able to revisit their test and see detailed feedback, in the form of specific solutions including detailed working, to the questions that they had attempted. While a total of 10 such tests were delivered over the 20-week period, students were aware that, even if they took all 10 tests, only their best 5 scores would contribute to their formal coursework assessment. This again was meant to incentivise students and to give them opportunities to demonstrate success; while it did allow many students to smooth out some weaker scores, and did show that many
students attempted all 10 tests, some stopped after 5 tests regardless of the quality of their results. A further advantage to the lecturer was that, even while the test was running, direct access to individual student results and responses was available, via a searchable interface. Thus, it was very easy to see whether a topic had been generally well understood or whether it needed to be revisited. Test marks could also be downloaded at the conclusion of a test as an Excel spreadsheet.

Student feedback was generally positive including comments such as:

a) “…Moodle tests give us a chance to test our knowledge so we can see clearly what we don’t understand.”

b) “I liked the small Moodle tests every week. Kept us learning. Helps me re-read my notes.”

5.3 Podcasting

The built-in podcasting facility within Moodle presented the opportunity of making instructional materials available to students with access to computers and mp3 players with video capability. Once the original source material had been created, Moodle enabled the lecturer to create and later update a video podcast from within the student module, while allowing the student to either directly display such video files on their computer (see Figure 6) or choose to subscribe to the podcast and view files at a later date.

![Fourier example](image)

**Figure 6** Fourier example

By subscribing, current and future video files of the selected podcast, may be automatically downloaded to the students computer via iTunes (see Figure 6), and when an appropriate player, such as an Apple video iPod is next connected to the computer, these files may be automatically transferred to the player. The advantage of this mode of delivery is that only one action is required by the recipient, namely – subscribing to the podcast – which usually just means clicking on an icon on the relevant website that delivers the podcast.

Podcasting seemed an ideal way of providing short videos lasting just 2 or 3 minutes, together with an audio commentary, which could reinforce a key concept or mathematical technique, and have the added benefit of portability and provide the opportunity for frequent review. By keeping the duration to just a few minutes, it is hoped that the student will not find it burdensome to review the material on a frequent basis. The short duration and optimisation process also ensures that the file to be downloaded is only of the order of 2-3 MB and consequently is unlikely to cause problems when downloading. It should be noted that the “videos” comprised a short series of static slides, each of which contained its own audio commentary. All of the audio associated with each
slide plays automatically, and the slide will not transition to the next until the audio has completed. A short delay (3 seconds) between transitions was incorporated so that students might have sufficient time to re-read text or make a short note. Podcasts were created to cover aspects of Fourier series, Laplace transforms and Matrices. Figure 7 shows a typical slide sequence.

Figure 7 Podcast: slide sequence

6. Final comments

This paper describes some of the uses we have made of technology to develop innovative learning resources in an attempt to enhance the teaching and learning of mathematics to engineering undergraduates. The HELM resources, in the form of workbooks supplemented by web-delivered interactive lessons, use a web-delivered CAA regime to drive student learning. They can be used in different pedagogic ways and have been successfully used at a number of universities.

Many students now find the traditional lecture-tutorial approach difficult to cope with. We have used Tablet PCs to produce narrated lectures as an indirect attraction for students to attend lectures and to more seriously engage with the mathematics. It is too early to conclude that the use of freely available narrated lectures as described here is a beneficial learning mechanism but it certainly produces a feel-good factor within the student body and that, we think, is half the battle.

Electronic voting systems have been used to encourage more student engagement during lectures. It is clear that they are popular with students and enable the lecturer to bring live user feedback into the classroom, but it is difficult to judge whether they may effect student attainment.

We are making increasing use of our university’s VLE to deliver material to students; recent developments have included the use of Moodle quizzes for formative assessment and podcasts to reinforce key concepts and techniques. Both are popular with students.

Further evidence-based research is needed to evaluate the effectiveness of all these uses of technology on student learning.

References