Learning Triangle Properties Through Geometrical Sketchpad Activities

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Abstract: *Progression in Learning Is Usually From the Concrete to the Abstract. Young people can learn most readily about things that are tangible and directly accessible to their senses visual, auditory, tactile, and kinaesthetic. With experience, they grow in their ability to understand abstract concepts, manipulate symbols, reason logically, and generalize. These skills develop slowly, however, and the dependence of most people on concrete examples of new ideas persists throughout life. Concrete experiences are most effective in learning when they occur in the context of some relevant conceptual structure. In many mathematics textbooks, the properties of triangles, and their medians, angle bisectors, altitudes, and other special lines in triangles, are listed as theorems or statements. This paper provides an opportunity for interactive, discovery activities for students to learn properties of triangles using Sketchpad. It gives them opportunities for hands-on experimentation with medians, angle bisectors. They discover the properties on their own, and help each other and discuss their discoveries through Sketchpad activities which assist them in these explorations. I find that when students have this kind of experience, they understand the properties rather than just memorising them, and not only help them to remember the properties, but they have a kind of "ownership" of the information, and their understanding is an active process.*

Triangle Properties

In many textbooks, the properties of triangles, and medians, angle bisectors, altitudes, and other special lines of triangles, are listed as theorems are just statements may be with some diagrams. But this topic provides a wonderful opportunity for interactive, discovery activities for students. In my classes, I will group the students in groups of 3 and allocate a computer per group installed with GSP 4.0 for this topic. They will use GSP to construct medians, altitudes, bisectors and other special triangles before they understand the concepts of properties of triangles. After doing the activities I will ask them to read the theorems or statements in the textbook. This gives them opportunities for hands-on experimentation with medians, angle bisectors and altitudes.

On doing these activities students will discover the properties on their own, and also help each other and discuss their discoveries with their members in their group to engage them actively through out the lesson. I hope these GSP Lab activities will assist them to discover and explore the properties of triangles. I find that when students have this kind of experience, they will understand the properties rather than just memorising the statements, and not only does this help them to remember the properties, but they have a kind of "ownership" of the information, and their understanding is an active process.

Here is an example of a problem that I might give my students:

Is the following statement always true, sometimes true, or never true? Explain your answer, and give examples in words and sketches. (*Use GSP to explain your answer*)
(i) <u>A median is perpendicular to a side of a triangle.</u>

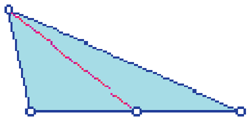
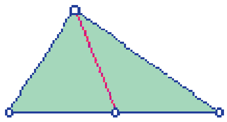
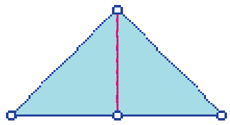
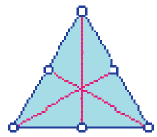
   

Figure 1 Figure 2 Figure 3 Figure 4

In Figure 1, Median is not perpendicular to the base, because it is a scalene triangle.
In Figure 2, Median is not perpendicular to the base actually hypotenuse of a right triangle.
In Figure 3, Median is perpendicular to the base, because it is an isosceles triangle.
In Figure 4, all the three Medians are perpendicular to the opposite side, because it is an equilateral triangle. In this activity, students draw all types of triangles and verify the angle between the median and the base using 'measure' tool in GSP. Students are learning these properties by experimenting with GSP software and discuss with their members in their group and give their reasons for the statement '<u>A median is perpendicular to a side of a triangle</u>' given above.

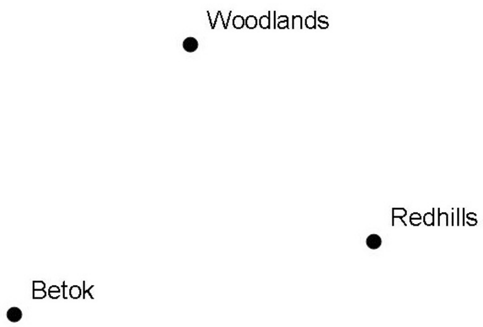
Experiential education is based on the idea that active involvement enhances students' learning. Applying this idea to mathematics is difficult, in part, because mathematics is so "abstract." One practical route for bringing experience to bear on students' mathematical understanding, however, is the use of hands-on activities. Teachers in the primary grades have generally accepted the importance of hands-on activities. Moreover, recent studies of students' learning of mathematical concepts and processes have created new interest in the use of hands-on activities across all levels.

Both Pestalozzi, in the 19th century, and Montessori, in the early 20th century, advocated the active involvement of students in the learning process. In every decade since 1940, the National Council of Teachers of Mathematics (NCTM) has encouraged the use of hands-on activities at all grade levels.

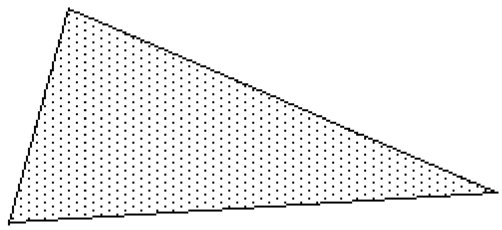
Many students have told me that when they are trying to remember a property - for example, whether or not a median of a triangle is perpendicular to a side, and if so, in what kind of triangle - they picture themselves "dragging" a vertex of a triangle and visualise the median, as they did during the hands-on GSP activity. Or other students will do a quick series of sketches of triangles, re-creating the GSP activity, and answer the question correctly from their "doodles" which allows them to revisit the original activity. This process is far superior to reading a theorem or statement in a textbook and trying to just memorise it!

A Sample Activity to Find the Center of a Triangle*:* An Exploration – Group activity

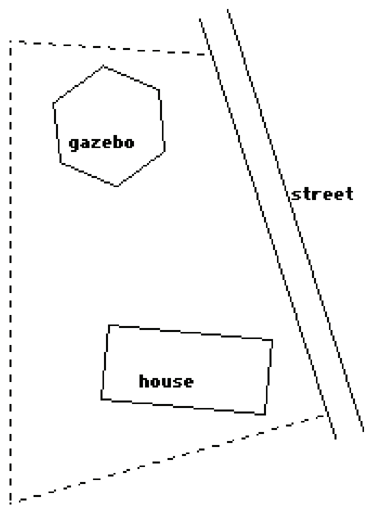
1. What is the center of a triangle? As a teacher, discuss some possible answers to this question with students allow them to discuss in groups and ask them to present their answers, before you go any further explanation. Discussion need not necessarily lead to any "right" answers to this question. You may give an answer or answers, pose more questions related to the problem, sketch ideas, or whatever you feel is appropriate.
2. Why would you want to find the center of a triangle? Consider the following situations.
    a. You are a city planner. The three towns Woodlands, Betok and Redhills have pooled their funds and want to build a recreation center. The 3 towns are sketched below. Where would you put the recreation center so as to be fair to all 3 towns? (See the Figure below). Sketch and explain your solution.



b. You are a sculptor and have just completed a large metal artifact. You want to hang this artwork in the school hall so that it will be suspended with the triangular surface parallel to the ground. From what point should it hang? Sketch and explain your solution.



c. You are an architect. You are designing a swimming pool and surrounding lawn. The client wants the lawn to be circular. The property on which it will be built is near a street, and has an existing house and a view house (gazebo). (The property line is shown as a dotted line) Where would you place the center of the lawn, to make the lawn and swimming pool as large as possible? Sketch and explain your solution. (Use the Figure below for your construction):

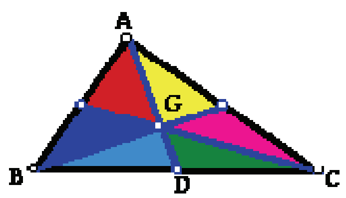


What is the center of a triangle? Let's explore this further with the help of GSP.

3. Open the Geometer's SketchPad (GSP) to a New Sketch. Construct a triangle ABC. Make the segments thick. Construct the midpoint of side AB and then a line through the midpoint perpendicular to the side AB. Make the line red and thin. Construct a point P anywhere on the perpendicular bisector.
   What is true about this point? ________________________
   Measure the distance from this point to the two endpoints of the line (A and B).
   What is true about these distances, PA and PB? ____
   Drag point P. Is this still true? ____ Delete point P.
   Construct the perpendicular bisectors of the remaining sides.
   Make these lines red and thin.
   Do all three perpendicular bisectors meet at a point? ______
   What is true about this point? ______________________________
   Is this the center of a triangle? Why or why not? Discuss within your group.
   Drag any vertex of the triangle and make some conjectures about where this point falls in different triangles.
   Sketch an example of each of the following types of triangles: scalene, isosceles, equilateral, acute, right, and obtuse triangles.
   Sketch the perpendicular bisectors for each triangle, and the point where they meet. Does this point seem like it could be called the center for any of these triangles? Discuss this, briefly with in your group and share your findings with the whole class.
4. Undo all the way back to your triangle, or save your sketch and open a New Sketch. *(**Note**: To Undo, use either Command Z back to where you want to go, or Option Z back to the very start, then Command R to redo the triangle construction)*
   Now construct an angle bisector. Construct a point P on the angle bisector.
   What do you think might be true about this point? (discuss)
   Is it equidistant from the vertices of the triangle as the perpendicular bisector was? (discuss)
   Is this point equally distant from anything? (discuss)
   When you have answered this question, delete point P and continue construct the remaining two angle bisectors.
   Do they all meet at a point? ___________
   What is true about this point? _______________________

   Could this point be considered the center of a triangle? Why or why not?
   Drag any vertex of the triangle and make some conjectures about where this point falls in different triangles.
   Does it matter whether the triangle is scalene, isosceles, equilateral, acute, right, or obtuse? Include at least one sketch in your discussion (use GSP) for help.

5. Undo the angle bisector construction, or save your sketch and open a New Sketch. Construct the medians of your triangle. Construct the point of intersection. Six triangles are formed. Construct the polygon interiors for each of these triangles and make the interiors different shades and/or different colors.
   Explore the properties of these triangles.
   What can you discover about these 6 triangles?



Are they congruent? Similar? Anything?
Drag any vertex of the triangle, and measure anything you want.
(Hint: You can measure perimeter or area by constructing the Polygon Interior, then use the Measure menu.)
Could this point be considered the center of a triangle? Why or why not?
Drag any vertex of the triangle and make some conjectures about where this point falls in different triangles. Be sure to consider scalene, isosceles, equilateral, acute, right, and obtuse triangles. Is this the center for any of these triangles? Include sketches in your discussion

6. Undo until you have a blank screen or save your sketch and open a New Sketch. Construct three non-collinear points. Select the points 2 at a time and use the Construct menu to construct lines until you have a triangle. Construct the triangle's interior and make it a light shade of any color. Construct the 3 altitudes.
   Do they meet at a point? _____
   Could this point be considered the center of a triangle? Why or why not?
   Drag any vertex of the triangle and make some conjectures about where this point falls in different triangles. Sketch an example of each of the following types of triangles: scalene, isosceles, equilateral, acute, right, and obtuse triangles. Sketch the altitudes for each triangle, and the point where they meet.
   Does this point seem like it could be called the center for any of these triangles? Discuss this, briefly with your group members and share with the whole class.
7. Take another look at the questions and your answers of question number 2 a, b and c. Discuss what you have discovered, in relation to the three questions asked in the question number 2. Look for connections between the geometric properties you discovered in the questions 3 to 6 and the "real-life" applications in the question number 2. Explain, for each of the situations in the question number 2, how the city planner, sculptor, or architect would find the center needed.

In the question number 3, students actively concentrated in construction of perpendicular bisectors of all the three sides and found the lines are concurrent and that point is equidistant from all the three vertices. In the question number 4, they concentrated on angle bisectors and they found the point of concurrency is equidistant from all the three sides not the vertices. In the question number 5, they involved in construction of three medians and the point where all the three medians meet. They found the point of intersection divides the median in the ratio 2:1 from the vertex and also the six triangles are of equal area. With the simple help of physics application they will find the centre of gravity is acting through this point. Lastly, in the question number 6, they constructed the three altitudes from the vertices to the opposite sides. They found these three altitudes are concurrent, but there is no other significance of this point. Finally in the question number 7, they will link to all the properties to the real life situation problems.

What is the final answer, which one is the centre? Each point of concurrency, namely Cir-cum-center (Question 3), In-center (Question 4), Centroid (Question 5), Ortho-center (Question 6) all centers of a triangle in one way. But they have some more properties. This is the extension of this activity. How these points lie in a scalene, right, isosceles and equilateral triangles.

Conclusion:

Actually, it is a journey of learning properties of a triangle. But students discover the properties on their own and help each other and discuss their discoveries with the help of GSP and shared their results with the whole class. This develops the communication skill of students using the language of mathematics which is one of the important areas under secondary mathematics framework of Ministry of Education Singapore under the heading processes of the well known pentagon (CPDD – MOE Singapore 2007). The focus of the whole paper is where are the 4 centers of a triangle, which I put it as an extension of the activity what I described. A triangle center (sometimes simply called a center) is a point whose tri-linear coordinates are defined in terms of the side lengths and angles of a triangle and for which a triangle center function can be defined. The four ancient centers are the triangle centroid, in-center, cir-cum-center, and ortho-center. Clearly, this paper concludes "Progression in Learning is Usually From the Concrete to the Abstract".

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