# MuPAD for the Classroom—a Discusion on Using Computer Algebra Systems in Teaching Mathematics

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**Abstract**

It is well known that Computer Algebra Systems (CAS), for instance Mathematica, Maple, MuPAD or Derive, provide computational power to solve many mathematical problems faster and more efficiently than using the old paper-and-pencil methods. The availability of such software and its ability to deal with most undergraduate mathematics problems cannot be ignored. A mathematics educator cannot stay neutral in this issue. Indeed, the emergence of such software has divided mathematics educators into two almost separate groups — those who believe that we should use such software in teaching mathematics as much as possible, and those who think that we should completely avoid technology in mathematics classes.

In this paper we will analyze various points of view on using CAS in teaching mathematics. We will show how some of the major concerns can be overcome. Finally, we will discuss the *MuPAD for the classroom* project, where the main objective is to develop a comprehensive set of materials to help educators incorporate MuPAD efficiently into undergraduate mathematics teaching.

Most of the issues discussed in this paper are related to teaching high school mathematics. In a few places we will also consider the university point of view. However, it is important to note that in many aspects, the CAS situation at the university is quite different than in high school.

## 1 Introduction

This paper starts with some issues related to MuPAD and moves towards more general analysis of using Computer Algebra Systems (CAS) in teaching mathematics. Later we will return from those general issues back to MuPAD. Therefore, all MuPAD related issues will be discussed in a wider context of teaching mathematics with CAS. Let us start by introducing MuPAD.

MuPAD is one of a few CAS used in teaching mathematics, as well as in scientific computing. Originally MuPAD, full name of this CAS is MuPAD Pro, was developed at the Paderborn University by the MuPAD Research Group under supervision of Professor Benno Fuchssteiner. In 1997 SciFace Software company was created in order to handle further development and distribution of MuPAD. For the last ten years, one of the main objectives of MuPAD development has been to create a didactical tool that could be used in teaching mathematics — starting from high school mathematics courses all the way to advanced university level courses.

The author of this paper has been involved in numerous projects related to MuPAD — suggesting and developing of some of its features, writing books and papers on using MuPAD, organizing MuPAD conferences and workshops for mathematics teachers. His main MuPAD activity in Europe is a series of *mathPAD Conferences* — six annual conferences targeting mathematics teachers in Central and Eastern Europe. He organized MuPAD workshops in Poland, Germany, Singapore, Malaysia, Thailand, Macao, and Taiwan. He is also the editor of an online magazine for the MuPAD community (www.mathpad.org).

Having been involved in MuPAD activities for so many years, a natural question is to ask *how do mathematics teachers use MuPAD in their classroom*? The findings are quite surprising. In Germany, the country where MuPAD was developed, MuPAD is used in teaching mathematics in many minor ways. However none of them is monitored by the Ministry of Education or any other educational authority. There is no common policy on using technology in the classroom and in many of federal states, using CAS in mathematics teaching and examinations is still forbidden.

Therefore, another natural question is to ask *are there countries or places where using CAS for teaching undergraduate mathematics is taken more seriously*? It was surprising for the author to find that implementing CAS in teaching mathematics in many countries is far from what we would expect, and in many developed countries is still completely ignored.

Before going into further discussion let us remind ourselves what CAS can do for mathematics educators.

## 2 CAS and Mathematics Education

Major CAS, e.g. Maple, Mathematica and MuPAD, are command-line tools for solving mathematics problems. Therefore, in order to get a response on a mathematical question, we have to type in a command, and press the [Enter] key to produce a result or a graph. Let us start with a simple middle school example.

**Example 1** *Converting a recurring decimal into a fraction*

Suppose

$$k = 0.076923076923076923076923076923076923076923076923... \qquad (1)$$

We can easily observe that the recurring cycle is 6 digits long. By multiplying the above equation by $10^6$ we get

$$1000000k = 76923.076923076923076923076923076923076923076923... \quad (2)$$

Now, by subtracting (1) from (2) we produce the equation

$$999999k = 76923$$

therefore $k = \frac{76923}{999999}$

Here comes the use of CAS:

```
simplify(76923/999999)
```

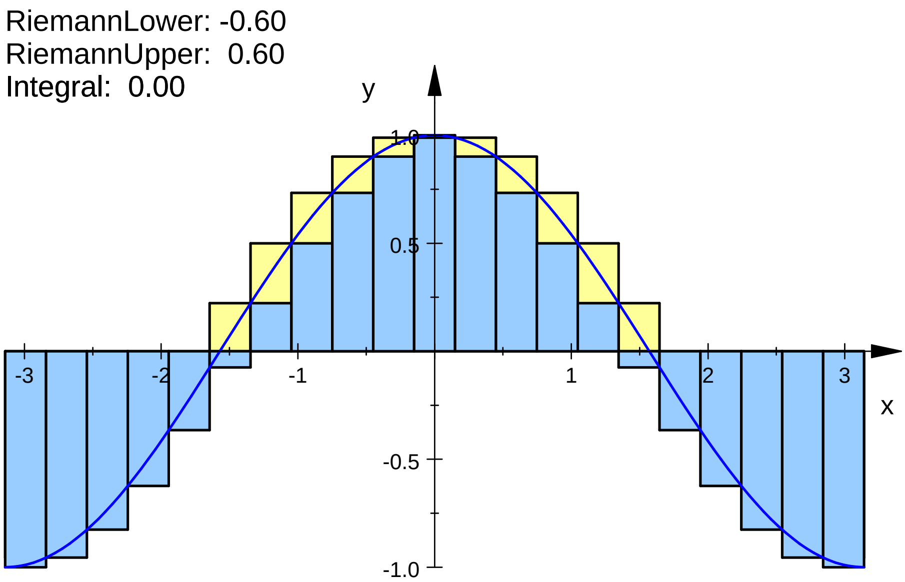
$\frac{1}{13}$

Note, we used MuPAD here in a situation where students, in this case middle school students, had to perform a most time-consuming operation of simplifying a complex fraction. There are many more situations in mathematics where CAS can free the students from spending time on tedious calculations.

Here is a simple example showing the MuPAD code to produce a graph of the function $y = \sin x$ and graph representing its Riemann lower and upper sums.

**Example 2** *Riemann lower and upper sums for the function $y = \sin x$ using 21 equal size intervals.*

```
f := plot::Function2d(cos(x), x = -PI..PI):
g := plot::Integral(f, IntMethod = RiemannLower, 21,
                       ShowInfo = [IntMethod, "", Integral]):
h := plot::Integral(f, IntMethod = RiemannUpper, 21,
                       Color = RGB::LightYellow):
plot(h, g, f)
```



**Example 3** *Consider the quadratic polynomial* $f(x) = ax^2 + bx + c$. *A graph of such a polynomial has an extreme values at* $\left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)$. *Determine the locus of the extreme points of quadratic polynomials where* $a = 1$ *and* $c = -4$.

This is a perfect example to demonstrate many applications of CAS. Let us start with visualizing the family of curves. In MuPAD we start by loading the easy plot package (more about eplot in the last section of this paper).

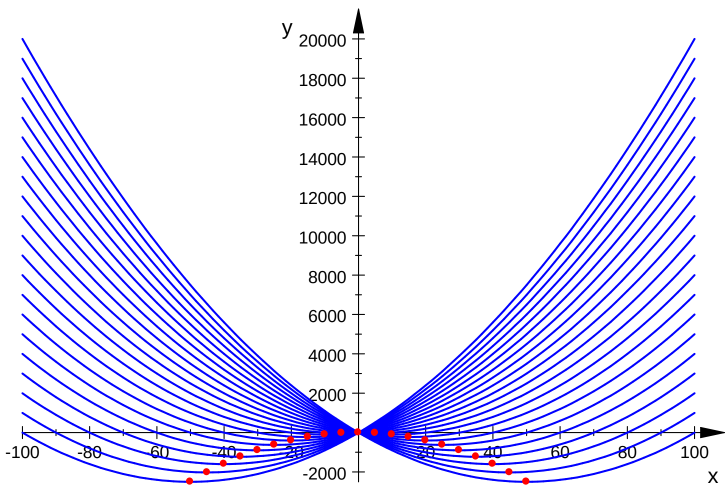
```
reset():
package("eplot"):
```

Then we define the family of polynomials and their vertices,

```
f := x -> x^2+b*x-4:
curves := {f(x) $ b=-100..100 step 10}:
points := {[-b/2, (-16-b^2)/4] $ b=-100..100 step 10}:
```

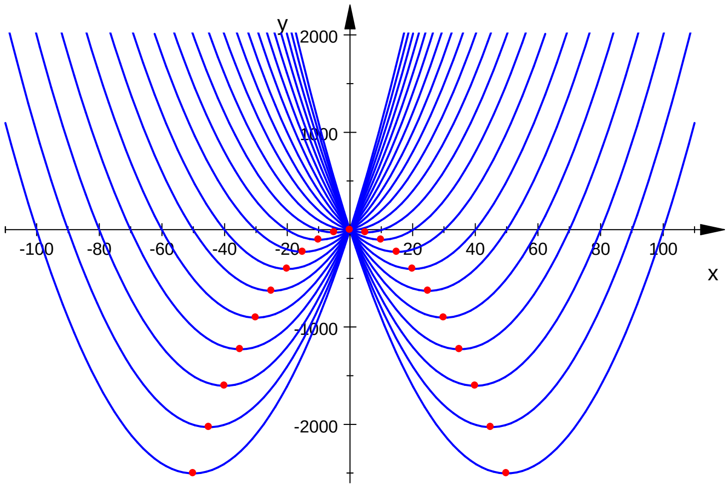
Now we plot all of them together,

```
plot(curves, points, x=-100..100)
```




Before we proceed with our investigations, we should limit the graph to the area where the vertices are shown by defining a more suitable viewing box for the plot.

```
plot(curves, points, x=-110..110, ViewingBox=[-110..110,-2600..2000])
```




At this point, a few simple operations are needed to produce the equation of the locus curve,

```
f'(x) // obtain derivative of the polynomial
```

$b + 2x$

```
u := op(solve(b+2*x=0,b)) // solve equation f'(x)=0 in respect to b
```

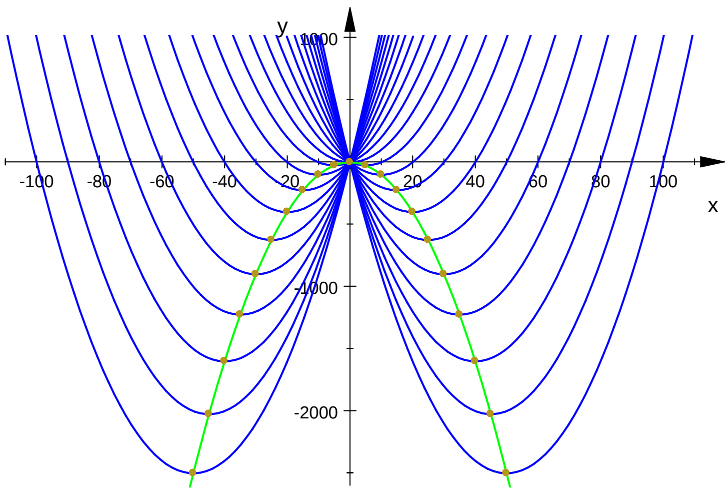
$-2x$

```
locus := subs(f(x), b=u) // substitute the solution for b into f(x) to
                         // obtain the locus curve
```

$-x^2 - 4$

Finally we can plot all the polynomials, vertices and the locus curve together:

```
plot(curves, points, locus, x=-110..110, ViewingBox=[-110..110,-2600..1000])
```



Although typing in commands requires at least some minimal knowledge of the software, the obtained results can be invaluable tools in visualizing many mathematical problems and theorems (example 2 and 3).

As we said before, some of the CAS use command-line instruction. Other CAS are menu-driven, e.g. Scientific Notebook, Scientific Workplace, and Derive. In all these programs, we do not need to type in even a simple command, and all the operations can be obtained by selecting them from menus and toolbars. In such cases, we do not need to memorize commands but we still need to have a reasonably good knowledge of what is included in each menu.

Finally, we have to notice that all the command-line CAS offer some menus with shortcuts to many commands. MuPAD, for example, has a sidebar with menus and toolbars for the most frequently used commands. It is important to note that even the most extensive menu system is not big enough to accommodate all commands and parameters available in a CAS. Therefore, the menu-driven CAS will have always limited functionality.

Example 2 tells us much more than only how the Riemann sums define the approximate value of an integral. It shows us that operations, in this case calculating the Riemann sums, that were usually beyond student abilities, in particular when it came to high school students, can be taken over by CAS and students will still be able to deal with and understand a mathematical concept. A similar situation occurs in example 1. Finally, example 3 shows that CAS can be used as an integrated environment for solving mathematical problems, experimenting with mathematical concepts and visualizing them.

Mathematics education researchers as well as mathematics instructors emphasize the numerous advantages of using CAS in teaching mathematics. There is a huge number of publications analyzing the use of CAS in the classroom and showing its role in transforming mathematics education. Some of the mentioned advantages have a concrete topical value, like creating mathematical graphs illustrating a given topic (see Bowers [4]), some other may lead to serious consequences for the whole teaching process. Let me mention a few of the most important arguments from the second group (compare Heugl [13]).

1. *CAS help in abstracting mathematical problems.* Most mathematical problems start with a concrete problem, then an abstract model is created. By formulating such problems in a CAS language students gain a better understanding of the problem and the algorithms needed to solve the problem (see [13]).

2. *CAS help to increase the value of the knowledge and the degree of interest of students.* Example 2 shows that a student can easily go beyond calculations and concentrate more on the knowledge itself than on tedious transformations needed to produce the Riemann sums.

3. *With CAS we can offer students a more application-oriented, and definitely more interesting mathematics that has a strong meaning.* This, in particular, is important for students who do not posses mathematical skills and do not see the point of learning mathematics.

4. *CAS build skills in translating mathematical problems from a native language into the language of mathematics.* Before students will be able to solve a real life problem

with CAS, they have to translate it into the mathematical model (see Kutzler [17]). We know that such models, developed by students, can be very inaccurate and sometimes even miss some special cases of the problem. While translating such problem into mathematical language and later into CAS commands students will be forced to think about roles of constants, variables and parameters in the problem, their ranges, etc.

5. *Using CAS will result in expanding students' mathematical language and bring new meanings in mathematical reasoning.* For example, in the past mathematical activities of students were frequently limited to transforming mathematical equations, with CAS students will move to the level where they treat equations as objects and apply CAS operations to them. In many situations, a process approach will be replaced by an object approach (see [14]).

6. *While solving problems with CAS there will be a significant shift from calculating to planning and interpreting* (see [8], [13], [7]). Students will be able to spend less time on performing calculations and more time on interpreting the obtained results and applying them in real-life situations.

7. *With CAS students will be able to develop new elements of their thinking technology.* New heuristic strategies will become part of their problem solving skills: experimenting with mathematical concepts, proceeding step by step through calculations, visualizing concepts using various types of graphs, developing models or real-life problems and simulating them with a computer (see [13] [6], [24], [19]).

8. *With CAS students will be able to develop a modular way of thinking and working.* In traditional mathematics, i.e. mathematics without CAS, we did not emphasize the role of modules (see [21], [20]). However, modules have existed in mathematics for a long time. For instance, as soon as we learn how to integrate, integration becomes a kind of black-box tool that we apply whenever we need it without even thinking about what the definition of the integral is. With CAS, we will be able to skip the whole process of integration and use a respective command. Therefore, with CAS students will be able to create more general modules, then modules of modules etc.

9. *CAS will become a medium for creating interactive and dynamic prototypes.* This is a completely new technique in mathematics education. For years, we created mathematical prototypes of given situations. However, we were rarely able to observe how these prototypes change with the change of their parameters. Besides the prototypes we had already been using: word formulas, symbolic prototypes like terms and equations, graphs, and tables; CAS offer us some new prototypes like recursive models or programs (see [13]).

The list of advantages of using CAS is certainly much longer (see the discussion section in Bowers [4]). I did not mention here some disputable arguments (see [22]), like increasing motivation of students, increasing their understanding of mathematics, making students more confident in problem solving, etc. The ongoing discussion on the benefits of using

CAS in mathematics teaching is enormous and almost every conference on technology for mathematics can be a source of papers advocating for the use of CAS and against it (see [1], [3], [4], [12], [14], [17], [18], [24], etc.)

# 3 Who teaches mathematics with CAS and how?

After this brief overview of the benefits that CAS may bring to our mathematics classes, let us come back to the question: *are there countries or places where using CAS for teaching undergraduate mathematics is taken more seriously*? An extensive survey of literature, conference papers and online discussion list shows that there are very few countries where CAS or computer technology in general, got more attention from educational bodies or government institutions. In most other places using technology for teaching is completely ignored, or even prohibited. However, even there, we can find enthusiasts experimenting with technology in their classes. Professor Fred Szabo in one of his letters wrote *technology has produced fabulous tools and is providing us with an environment that allows us to reach our intellectual potential on a grand scale and beyond our wildest dreams* ([23]). I have reasons to believe that many of us feel the same way.

For six years now, I have been working for a university that uses information technology as one of their learning outcomes. The university has a university-wide Maple license and I had my own unlimited MuPAD license. I have been discouraged many times in using any of these programs in my discrete mathematics classes. I guess this is a rather extreme case, but nonetheless some of my colleagues from other universities have had similar experiences.

More positive experiences have been noted by our colleagues from Austria, Victoria in Australia, France, USA and perhaps a few other countries. The Austrian initiatives are worth noting here. According to my observations, no other country has made as much effort to implement CAS in teaching undergraduate mathematics.

CAS in Austria is tightly connected with Derive and a few people pioneering the use of Derive in Austrian mathematics educations. Let me mention them in alphabetic order: Josef Böhm, Bruno Buchberger, Helmut Heugl and Bernard Kutlzer. Professor Buchberger is the author of a famous black-box/white-box principle that we will analyze later in this paper.

In the early nineties school authorities and a group of teachers created the Austrian Center for Didactics in Austria (ACDCA). ACDCA was created *with the intention to attend and support teachers in using revolutionary tools in a meaningful and responsible way unlike to the introduction of pocket calculator several years ago* (see Böhm [3]). In 1991 the Austrian Ministry of Education purchased Derive for all general secondary schools. It is important to note that the ACDCA for many years was, and still is, the main power involved in a very systematic way of introducing CAS in undergraduate mathematics teaching. The word *systematic* used here is very important. Since 1991 we have witnessed five large, country-wide projects (see [3], [25]):

- CAS I 1993-1994 Derive project
- CAS II 1997-1998 TI-92 Project (creation of teaching materials, research on influence of CAS on teaching), 44 schools, 70 classes, 65 teachers, 680 female students, 1570

male students

- CAS III 1999-2000 2nd TI-92 Project (research on influence of CAS in teaching, learning, curriculum and assessment), 94 classes with more than 2000 students
- CAS IV 2001-2002 CAS Project (research on the new culture problems, establishing a Service Center for Teachers), 140 classes with more than 2200 students
- CAS V 2003-2005 Variety of Media Project (together with GeoGebra, Mathe-Online, online learning, teaching in laptop classes)
- CAS VI (in preparation).

Josef Böhm in his paper entitled *What is Happening with CAS in Classrooms? Example Austria* presents a history of the ACDCA; he also analyses the outcomes of the projects initiated by this organization (see [3]). Very extensive information about the ACDCA can also be found on their web site (see [1]).

A number of similar, well-organized, albeit on a much smaller scale, attempts to introduce CAS in teaching mathematics can be observed in literature—Jos Bertemes describes some experiences from Luxemburg (see [2]), Peter Flynn presents the experiences of introducing CAS in Victoria in Australia (see [9]), David Bowers presents some valuable experiences in introducing CAS in a middle school in England and Wales (see [4]).

For a few years now, the MuPAD Education Group has been organizing regular training for German teachers in using MuPAD. The MUMM project (Mathematik-Unterricht mit MuPAD) was first large scale project organized in 2003/2004 by the MuPAD Education group and Ministry of Education of Nordrhein-Westfallen. In this state MuPAD was evaluated by the ministry and recommended to all schools. In the MUMM project participated 280 secondary schools, and about 50 teachers training sessions took place. One of the outcomes of this action is a collection of about 100 MuPAD notebooks that demonstrate selected aspects of high school mathematics. After three years the collection contains about 500 notebooks and is growing rapidly. In Germany each federal state has so called *Kulturhoheit*, which simply means that each state can make their own school laws independent from the German Government. Therefore, in each state situation of CAS is different. A very positive example is state Baden-Württemberg—a precursor of using CAS in Germany. They take teaching mathematics with technology very seriously.

For the last six years, the author of this paper has been organizing regular MuPAD conferences at the Nicholas Copernicus University (about 60-70 participants each year). These conferences, sponsored by the university and SciFace Software, have been mostly targeting mathematics teachers, and their general purpose is to provide training on using MuPAD in the classroom as well as to prepare some printed and electronic materials for mathematics teachers.

In each case, we can observe one important thing — all the actions like those mentioned above are very important as they build an interest of teachers, students and education ministers regarding the use technology in teaching and, in particular, in teaching mathematics.

However, the successful implementation of CAS in teaching mathematics requires much more than official conferences, research projects, etc. We have to consider a few other parties that are or will be involved in this process. These are teachers, examination boards, and students. A deep analysis of this aspect in relation to Austrian environment we can find in Wazir's article (see [24]). In this paper, I will add also some experiences from Europe and the Middle East.

### 3.1 Teachers' point of view on teaching mathematics with CAS

The first and the most important observation is that we are still far away from a good methodology for using CAS in mathematics classes. While talking to Kai Ghers (see [11]) I discovered that for example German teachers still do not have a clear strategy on how to use MuPAD notebooks provided to them by the MuPAD Education team. According to him, *some of the teachers used notebooks to demonstrate certain topics by themselves, without letting the students work in these notebooks on their own. Later on, when students learned enough of MuPAD, the teacher took students to the computer lab and each student could work through the topics demonstrated in notebooks. ... Some other teachers were not using MuPAD in the school at all, but they provided the notebooks to the students so they could experiment with them at home. Finally, some other teachers were using only graphical features of MuPAD to demonstrate certain topics, e.g. solids of revolution, planes and lines in 3D, etc. without letting the students use MuPAD at all.*

I observed a completely different situation in Poland. Polish teachers, at least some of them, are keener to involve students in mathematical explorations with MuPAD and some of them are very open to use any technology in the class in order to build the students' interest in learning mathematics. This way they can motivate students to develop a kind of ownership of the learned mathematics. Similar aspects are also mentioned by other authors (see Bowers [4]).

In general, teachers hesitate to integrate CAS into their teaching practice (see Wazir [24]) and there are many reasons for this. Any technology, including CAS, requires learning it first, developing strategies how to use it, and developing new teaching materials. CAS is a new teaching aid and very few teachers learned how to use it during their studies. Therefore, teachers are often afraid that students, especially some of the computer-addicted students, will come to know the computer program better than they do. In fact, in most of the cases where CAS is used at school, teachers are one step behind their students. Another serious reason is the lack of mathematics textbooks using CAS as a teaching tool. Most the teachers prefer to stick to a textbook and follow it tightly in the classroom. They will not try anything new unless they are not forced to. We have to remember that in many countries and schools, teachers are heavily overloaded with the amount of classes and assessments. They simply may not have time to work out a strategy and methodology to use CAS in their classes. Finally, some teachers simply hate computers and/or prefer pure mathematics.

The textbook issue seems to be different at the university level. Here we can find a few textbooks that claim to use Maple or Mathematica to a particular mathematics course. However, many of these books contain in reality a very traditional course material sprinkled with a few CAS commands. Usually, in the first few chapters of such a book we can find more technology-based material, while the last chapters completely ignore the technology and the course goes on like it went 20 or more years ago.

Finally, it is important to mention that, according to my knowledge (July 2007), there is no country-wide high school curriculum in which CAS is officially accepted as a tool for teaching mathematics. There are some exceptions allowing the use of calculators, though often with some restrictions to calculator type. In such situations many teachers are afraid that their time for introducing CAS in mathematics classes will be wasted if CAS will be banned by the ministry of education or examination boards.

## 3.2 The students' view on using CAS in mathematics classes

In literature there are quite mixed points of view on students' appreciation of CAS. Some authors claim that introducing CAS in their mathematics classes developed more interest of students in mathematics, in particular mathematics with computer. Others say that their students did not like technology in mathematics classes at all. According to my personal experience, liking or disliking technology by students depends on their individual skills and attitudes. Intelligent students, who usually do not have problems with mathematics, like the technology component in the class and they quickly learn how to use it to expand their possibilities. There is no doubt that such students, at least some of them, with CAS can go far beyond of what they could do using traditional paper-and-pencil approach.

For students, who are struggling with mathematical concepts, the introduction of CAS is just another burden that will take their time. In such cases, we have to very carefully design the strategies and methodology of using CAS in teaching.

## 3.3 Examination boards and CAS in mathematics classes

It is well known that in most countries, the requirements of the examination boards have a major influence on what we teach and how we teach. This refers to both international examination boards like IB and local national-level school examination boards. The issue of what will be allowed on the exam, in reality, legalizes the tools used in the classroom. Therefore, why teach mathematics with CAS, if CAS are prohibited in all international exams and also prohibited in many countries in their countrywide exams? There is always a concern that students who learned to solve problems with CAS will fail on a paper-and-pencil exam. Although in literature there are opinions that students using CAS are still able to solve problems using traditional methods, some authors claim that their CAS students performed on the exam even better than non-CAS students, most educators have rather mixed feelings about this outcome. A serious step towards using CAS in exams was taken recently in Austria. In this country all final exams have two parts—one to be solved manually, and one to be solved using technology. Some critical comments in literature show that this solution is also not perfect.

A major question is how long we will have to wait for a more favorable look of examination boards on CAS? We have to accept that they have their rational arguments also — not enough computers in schools, lack or examination rooms equipped with computers and CAS, etc. However, even in some wealthy countries where such problems are marginal, the decision to introduce CAS in mathematics exams are not easily pushed through. There is one issue that should not be neglected — it was mentioned by a few authors that for many examination boards, the preparation of exam questions with a CAS component was a major difficulty and the preparation of such questions took much more time than standard exam questions.

I believe that if we at the university level start demanding from our candidates a better knowledge of technology for mathematics, then we will be able to change the point of view of the examination boards. This process may take many years.

# 4 Black-box/white-box principle

Until now we have discussed some social and psychological issues. Let us investigate one of the reasons why some teachers or university instructors do not wish to use CAS in teaching and why some other are in favor of CAS in the classroom. This discussion started when the first CAS were invented and it is still present in various conferences and seminars.

The major question asked frequently is — what will my students learn if such a tool does everything for them? Will they learn how to solve equations, differentiate functions or integrate if any CAS will do it for them in a result of a single command? At the same time some other educators ask — why do students still have to learn this *boring stuff* if computers can do it faster and better? We will leave the second question without an answer and concentrate for a while on the first one.

In 1990 Professor Buchberger formulated his famous black-box/white-box principle (see [6]). The major point of it is when and how we should use CAS in mathematics classes. According to Buchberger, while learning a given topic there is a moment when the topic is *trivialized*. For example, after learning how to integrate functions of one variable we are coming to the stage when we know the concept well, or reasonably well. This is the moment when we no longer need to think about how to integrate functions and we move on to topics where we can apply integration to solve another problems. We consider this to be the moment when integration is trivialized.

According to the black-box/white-box principle, we should not use CAS as a black-box in mathematics classes if the studied concept is new to students and is not trivialiazed, and we should use respective black-box commands as soon as the concept has been thoroughly studied and trivialized (like in the example 2). For example, while teaching how to solve systems of linear equations, we should go first through the phase where students learn the principles of solving such systems, theorems, algorithms and the theorems they're based on, and calculate by hand a number of examples. During this phase, we could use CAS in the form of a white-box to simplify some calculations or show how algorithms work (like in the example 3), but without using the command to solve a system of linear equations. After this phase, when we are sure that the concept was learned properly, we can introduce the command `linsolve` (MuPAD) and allow students to use it to solve systems of linear equations in further applications. In practice, the white-box and black-box phases occur at the same time in respect to different topics. The table on the next page presents an example of such a situation from a course of linear algebra:

| Topic | Black-box phase for | White-box phase for |
|---|---|---|
| Determinants | arithmetical operations | determinants |
| Inverse matrices | arithmetical operations<br>calculation of determinants | finding inverse matrices |
| Solving systems of linear eq. | arithmetical operations<br>calculation of determinants<br>finding inverse matrices | solving systems of lin. eq. |
| Eigenspace | arithmetical operations<br>calculation of determinants<br>finding inverse matrices<br>solving systems of lin. eq. | eigenspaces |
| ... | ... | ... |

The black-box/white-box principle was discussed in a number of papers. In literature, we can also find some modified versions of it. The principle was consequently applied in all Derive materials produced during CAS projects in Austria. The Austrian experience shows how the black-box/white-box principle allow us to produce more structured, and definitely more interesting teaching materials encouraging students to experiments with mathematical concepts on a computer.

# 5 MuPAD for the classroom project

Having been involved in MuPAD-related matters for many years, I am confident that MuPAD can be a very useful software for teaching mathematics. Therefore, in the nearest future I am going to start, with a group of mathematics teachers and university professors, a research project called *MuPAD for the classroom*. At the moment, there is no institution formally supporting this project or attached to it, and there are no official participants of the project. The project is completely open and everybody interested in teaching mathematics classes with MuPAD can join the project as long as they want to share their works with the wider community. There is also no budget for the project. However, I expect that some educational institutions may support it. There will be certainly support from SciFace GmBH in the form of web space for the project and MuPAD licenses for the participants.

The objectives of the *MuPAD for the classroom* project will be research on implementing MuPAD in teaching undergraduate mathematics classes, the development of methodology of teaching with MuPAD, the development of strategies for using MuPAD in the classroom, and the development MuPAD notebooks and printed materials for many courses in mathematics. Such materials should be constructed using the black-box/white-box principle as well as the relevant Gagne's events of instruction (see discussion on Gagne's events of instruction in [10] or[15]).

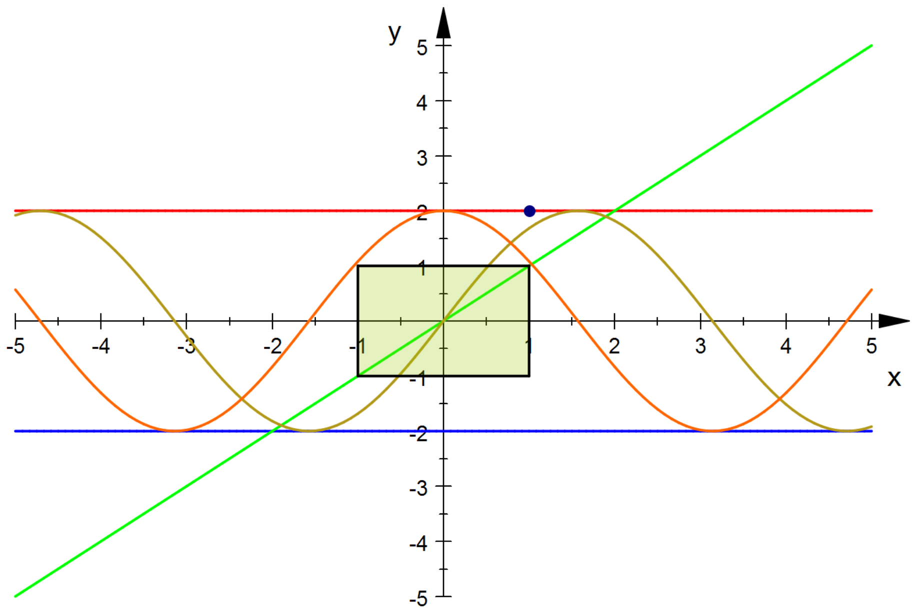
Recently, MuPAD has gone through many changes that are important from a didactic point of view. Here I wish to mention two of them: the intelligent plotting procedure, so-called eplot, and a new, very flexible interface.

The intelligent plotting procedure was developed recently and makes producing MuPAD graphs less formal than with the standard plotting tools. The procedure can plot almost everything what we drop into it. Here is an example showing how it works.

While using the standard plotting concept in MuPAD, we have to declare each object to be plotted with its parameters. Such a concept can be acceptable for an university student in some advanced courses (see the code in example 4). However, it may be too difficult for a high school student.

**Example 4** *Standard plotting commands against the intelligent plot procedure*

```
f1 := plot::Function2d(-2):
f2 := plot::Function2d(2):
f3 := plot::Function2d(x):
f4 := plot::Function2d(2*sin(x)):
f5 := plot::Function2d(2*cos(x)):
p1 := plot::Point2d([1,2]):
poly1 := plot::Polygon2d(
   [[-1,-1],[-1,1],[1,1],[1,-1]], Filled=TRUE, Closed=TRUE
):
plot(f1, f2, f3, f4, f5, p1, poly1)
```




We can obtain the same result, or almost the same, using the smart plot concept:

```
plot(-2, 2, x, 2*sin(x), 2*cos(x), [1,2],
  {[[-1,-1],[-1,1], [1,1], [1,-1], [-1,-1]], Filled}
);
```

Another important thing that was introduced with MuPAD 4.0, is the new concept of a flexible interface. In fact, the interface in MuPAD, including its toolbar, can be modified for the needs of any specific mathematics course. The figure 1 shows two of the many possible versions of the toolbar.

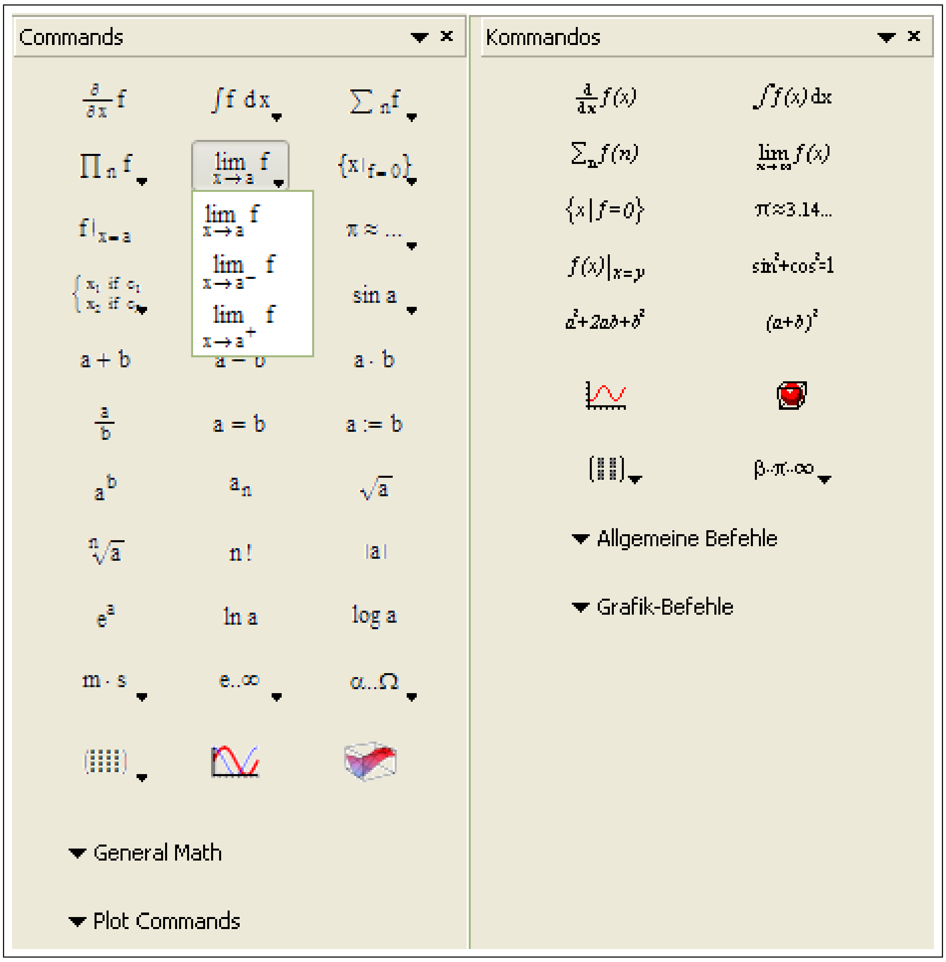



Figure 1: *Examples of two different toobars in MuPAD*

The new concept of MuPAD's interface is based on XML files, and it is a significant step towards a software interface that is user friendly and at the same time very flexible. The need for such interfaces has been discussed in many publications (see Kutzler [18] and Buchberger [6]).

There have also been some other developments in MuPAD addressing its usability in mathematics teaching.

# 6 Conclusions

There is no doubt that CAS can bring a number of improvements in teaching mathematics. However, before CAS will be widely used we have to change many things, and many developments have to be done. Here are some of them:

1. We have to develop methodologies for teaching mathematics with CAS and strategies for their introduction in our classrooms.
2. We have to produce teaching materials (textbooks, notebooks, online materials),

that will incorporate CAS into mathematics content according to good educational practices and principles.

3. We have to organize workshops, seminars and training sessions for current mathematics teachers as well as introduce CAS into teaching programs for mathematics teachers at the university and teachers' colleges.

4. We need appropriate changes in the official mathematics curriculum as well as changes in examination policies allowing CAS to be used on exams.

5. We need a lot of research on many aspects related to using CAS in mathematics teaching, in particular research on the influence of CAS on students' attitudes and the learning of mathematics, psychology of learning with CAS, etc.

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