Both Computer and Traditional Technology Are Inevitable for Mathematics Teaching:
Revisiting why we use technology

Masami Isoda
isoda@criced.tsukuba.ac.jp
University of Tsukuba
Japan

Abstract: We use technology to improve mathematics education. The first part of this lecture, the logical inconsistencies for introducing technology come from the difference of society, curriculum and technology itself will be mentioned. Because inconsistencies logically existed, we should develop the judicious position of using technology. The judicious users will be teachers and students. The second part, I would illustrate the Japanese approach by focusing on teachers. The later half, I would addressed the importance of judicious using of technology if it is to be a necessity in enhancing students mathematical explorations and developing mental object in order to support their mathematization. The basic theories of mathematics education will be applied to explain this position. To illustrate the importance of both the traditional and computer technology, I would use the perspective drawing and mechanical motions as for the example.

1. Beyond the inconsistencies of using technology in classroom

Technology is a part of necessary tools for knowledge based society. It is true that technology has been pushing globalization and changing society. It is necessary to prepare children, and technology should be introduced into classroom in preparation of the changing society. On the other hands, we should be concerned about some of the logical inconsistencies based on the difference of society, aims of reform and technology itself.

First inconsistency is the mismatch between technology and society of academy and teachers. There is a simplistic motion of introducing technology into classroom because of the existence of some advanced countries in using certain special technology. Even if using technology itself is mathematically interesting, its usage in classroom is meaningful in some countries but not in some other countries. In 1990s, the reform movements of using of innovative technology had influenced the world. The reform of AP-Calculus in USA influenced the other countries. It spreads the use of graphing calculators to limited countries which shared the similar setting and target in their reforms but not with countries having different setting and target. In the case of East Asia, many students have good achievement in their mathematics curriculum without innovative technology. In these countries, teachers are reluctant to use technology even if they well recognize the significance of technology for instance the power of visualization and importance of exploration. In comparison to East Asia, there are many countries in which the mathematics teachers are not well prepared for teaching mathematics. It is not uncommon that mathematics educators who teach elementary mathematics to future prospective teachers do not have good experience in geometrical proof themselves. Judicious technology using (Lynda Ball, Kaye Stacy. 2005) is a general necessary expectation in the teaching content for the knowledge based society when we teach students both on how to use technology and thinking mathematically. On the other hands, if teachers use technology such as Dynamic Geometry Software in their classroom and does not well understand geometrical proof, their exploration with DGS will be limited hence the meaning of judicious using itself is not similar to other countries.

Second inconsistency is the mismatch between curriculum and technology. Introducing new technology into classroom sometimes means the change in the teaching content and aims of education. There is a simplistic motion to use technology as an alternative to paper and pencil approach. When students explore the free fall phenomena using the digitizing system of the motion of graphing calculator, the approximation by the four degree function is better than the quadrilateral function. Here, teachers are teaching students how to explore the phenomena with graphing calculator on its statistical meaning or its mathematical modeling but not teaching them to consider mathematical structure of the free fall phenomena. If we introduce technology as a necessary tool for learning, it changes the content of mathematics teaching itself. If calculus teacher want to use the free fall phenomena as a model of fundamental theorem of calculus, he hopes that pre-calculus teachers teach it as an example of the quadrilateral function. Many teachers who are teaching upper level of mathematics deny teachers who are teaching lower level mathematics, change the content with technology. Upper level mathematics usually uses lower level mathematics as the mental object for the base of constructing upper level mathematics. In some countries, the movement of introducing technology is ongoing with the regression of mathematics teaching content.

Third inconsistency is among innovations. Even if one learned how to use the innovative technology it gets out dated very fast. On the integrity and fair competition of technological innovation, new technology is tentatively new until new products come and governments have to spend too much for every revision.

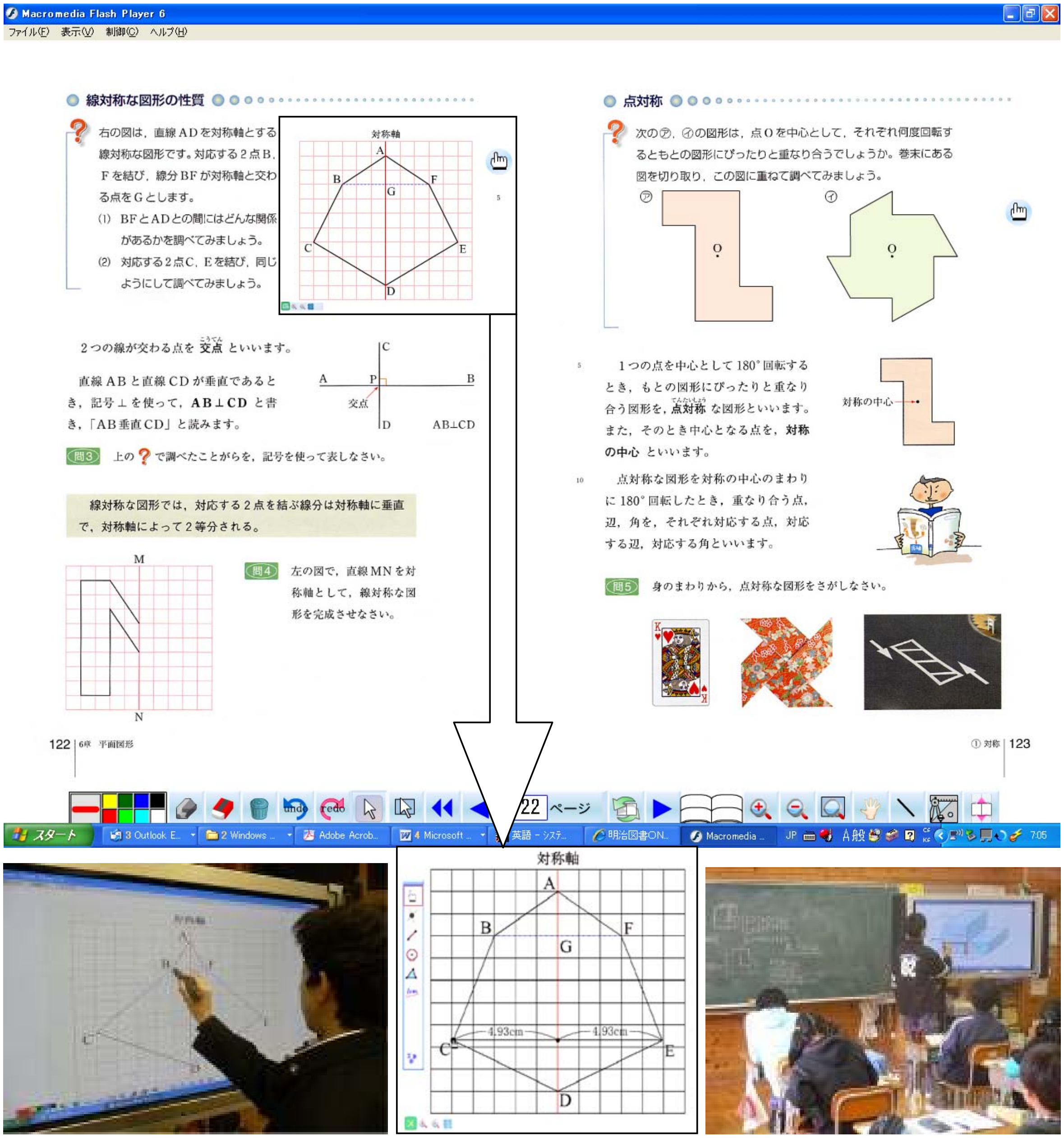


Figure 1. e-textbook and its interactive way of using a part of corkboard.

Against these inconsistencies, there are some approaches. In the case of Japan, for avoiding these inconsistencies, technology is being fixed by both teaching approaches and textbook based on curriculum standard. To introduce the latest technology into education in keeping tracks with the curriculum innovative ideas, e-textbooks with embedded mathematical software on traditional textbook have been developed in Japan. The feature of e-textbook is a part of teacher's guidebook and it almost means free because teachers usually buy teacher's guidebook even if they do not use it. Figure 1 is a sample page taken from the e-textbook which is the same as students textbook but could be projected in the classroom by flash player and demonstrate the way of using software such as DGS and graphing tool embedded in the e-textbook. On the screen in figure 2, parts of the textbook were displayed and used like an interactive chalkboard and integrating its use as a whole classroom activity.

The important feature of this Japanese approach is that it still keeps track with the same curriculum plan of the traditional textbook and Japanese way of teaching. Teachers do not need to change their ways of teaching (Figure 2). Students do not need to learn how to use technology even if teacher uses e-textbook. Teachers are expected to consider judicious technology using but not students because in this case the using of e-textbook is preferred to be used by teachers. This is a weakness but Japanese approach does not aimed in teaching how to use certain technology.

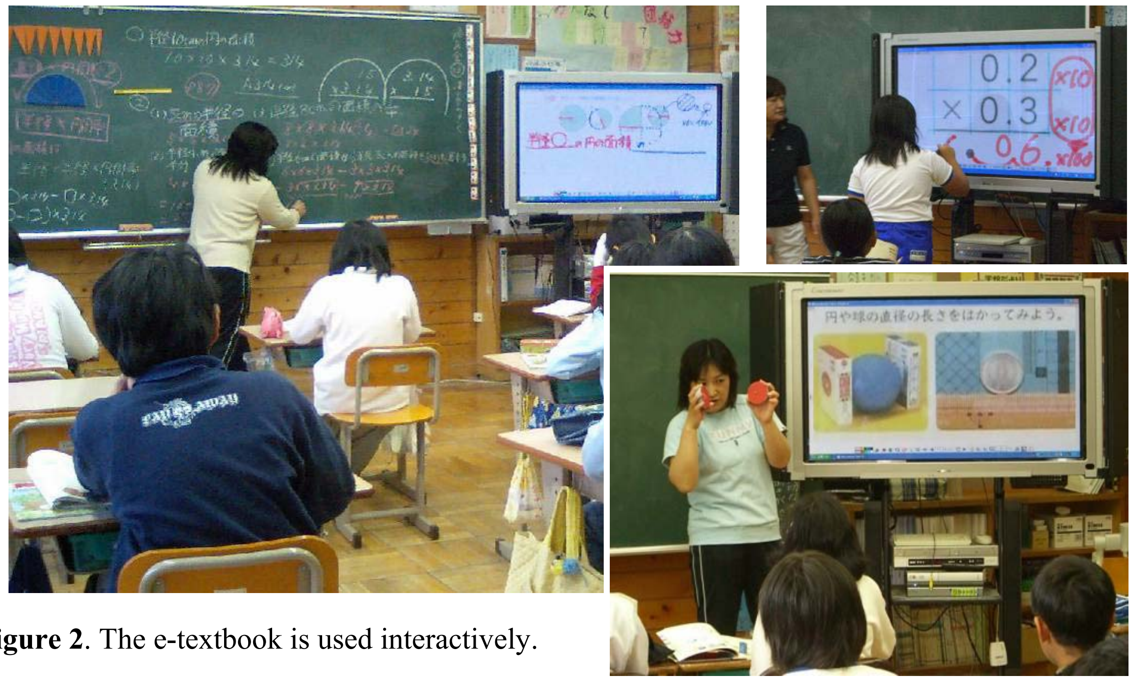


Figure 2. The e-textbook is used interactively.

2. Using Technology for Mathematical Exploration.

Besides those inconsistencies, there is a consistency for developing technology from the view point of mathematics. All technology in mathematics has been developed to enhance the using of mathematics and exploration of mathematics. These kinds of technology are useful and meaningful for students to be able to experience mathematical exploration.

People who use technology in mathematics know that technology is developed to explore mathematics and used for acquiring invariant in mathematics. For example, it is developed for enabling us to

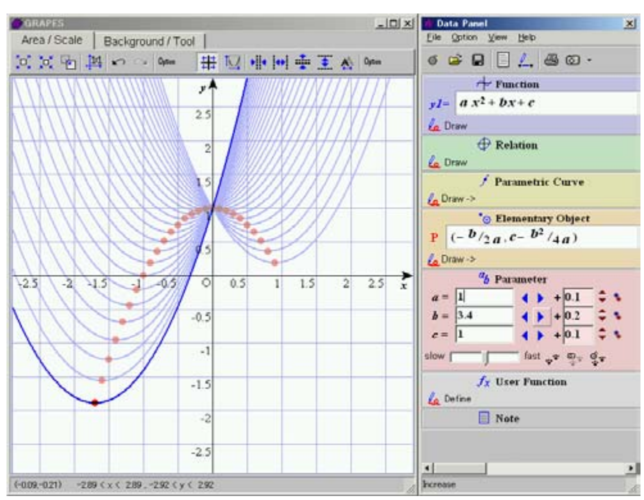


Figure 3. Visualized Invariant on GRAPES

visualize invisible object and increased our possibility of exploring mathematics. Figure 3 is a well known example. The approach in figure 3 is known by the visualization of the graphing tools. Changing parameters are an important approach for exploring invariant of the family of functions. Before graphing tools existed, only limited students can imagine those kinds of invariant. In traditional curriculum, most of students only learned quadratic function with the equation of $y = a(x - \alpha)^2 + \beta$ but not the meaning of parameters a, b and c on the equation of $y = ax^2 + bx + c$, even though they learned the meaning of a and b on $y = ax + b$.

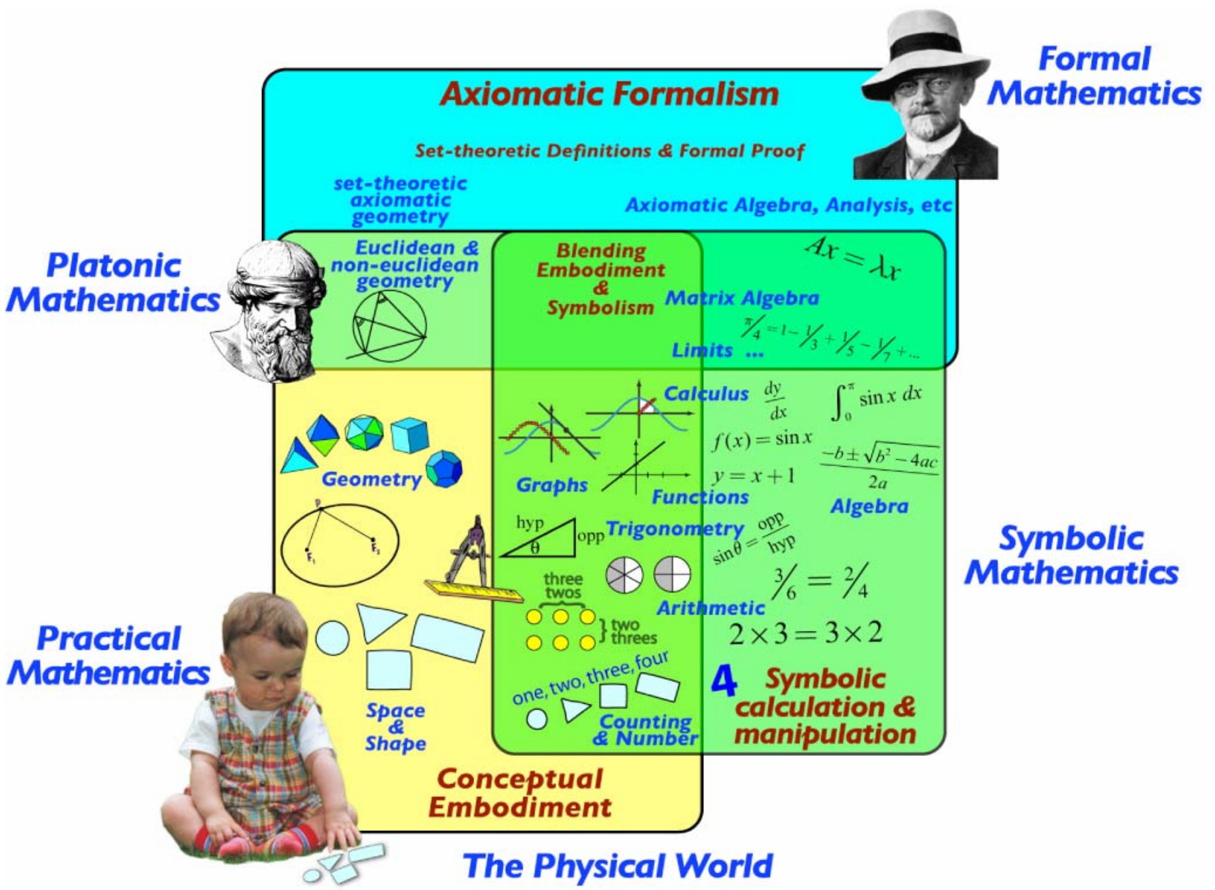


Figure 4. Conceptual Embodiment, Symbolic Calculation & Manipulation, and Axiomatic Formalism (David Tall)

Technology is developed to enable to understand mathematics in another way such as embodiment. Before the graphing tools, it is very difficult to imagine the graphs of { $y = ax^2 + bx + c$; a=1, c=1, and b={…-3,-2,-1,0, 1,2,3,…}}. The graphing tools allow us to manipulate these parameters.

David Tall introduced the three world of mathematics in explaining the simple features of conceptual development in mathematics (Figure 4. see such as Tall & Isoda, to appear). There are many difficulties for conceptual development in mathematics hence we needs long learning processes in each area of mathematics. On the other hands, if we simplify the processes, there are

three major activities in mathematics: Conceptual Embodiment, Symbolic calculation & manipulation, and Axiomatic formalism. We can map technology into his diagram as follows.
CAS (Computer Algebra System): It enables us beyond hand calculation and the embodiment through visualization. There are some researches such as Michele Cerulli who developed CAS for Axiomatic Formation of Algebraic Calculation.
DGS (Dynamic Geometry Software): It enables us the manipulation of geometrical object. Some system includes automatic proof system.
Graphing Tools including calculator: Most cases used the Blending part. Some tools such as CBR enables us the embodiment in physical activity.

When we consider the way of using and developing innovative technology for mathematics, his map is also useful. For example, we can identify the limitation of current technology and the necessary development such as integrating different software and its mathematical difficulty. At the same time, we should also consider the limitation of figure 4. For example, it shows the existence of other worlds such as Physical World but he focused more on mathematical thinking within the three worlds. On his map, there are traditional technologies which showed that he does not only focus on current technology for conceptual development even though he was the earlier developer of graphing tools. We are not teaching mathematics for technology. For teaching mathematics, if it is necessary we prefer some necessary technology.

3. Technology for developing mental object

Judicious using of various tools and ideas is one of the key idea of mathematics education. There is no matheducator who only chose to use one technology. Depending on the aim of mathematics teaching, the meaning of judicious itself will change too. In this chapter we do not want to distinguish old, traditional, new and innovative technology because its preferences depend on the aim of education. For considering of how to develop mental object, we fix mathematics teaching to teach mathematics through Mathematization. Mathematization is a specialized idea of mathematical activities by Freudenthal (1973): organizing reality by mathematical means (methods) is called mathematizing. From the view point of phenomenology, he also enhanced the development of mental object (entity) in mathematics (Freudenthal 1983).

He argued that to teach mathematics as a given is an anti-didactic inversion, and that students should reinvent mathematics as well as mathematicians invented mathematics through organizing reality. Based on his mathematical experience and historian's experiences, he described the learning process through mathematization using some examples of van Hiele levels and emphasized the importance of reflection on experience at a lower level for overcoming the discontinuity between levels. This means that students should experience the mathematization of reality. People are accustomed to use tools in their life with mathematical reasoning and developed mathematics through the reflection of these experiences.

To illustrate the process of mathematization, I would like to use the example from the development of perspective drawing.

Today in art lessons, different perspective drawings using one-point, two-point or three-point perspective drawings are taught as a composition technique for drawing pictures. Some people misunderstand that this is the theory of perspective drawing.

Through the screen windows, painters recognized the way of drawing as mental object and researched the way of using the screen window to draw what they actually saw and tried to translate their geometrical experience onto their drawing board (see Figure 10). Figure 10 is explained by

Figure 11 using Dynamic Geometry Software. During that era, how to draw the depth of pictures was an essential problem. Based on the idea of figure 11, painters could know how to draw (contract) the depth in figure 10. The idea of figure 10 was expanded to the anamorphose through inclining screen windows shown in figure 12 and 13.

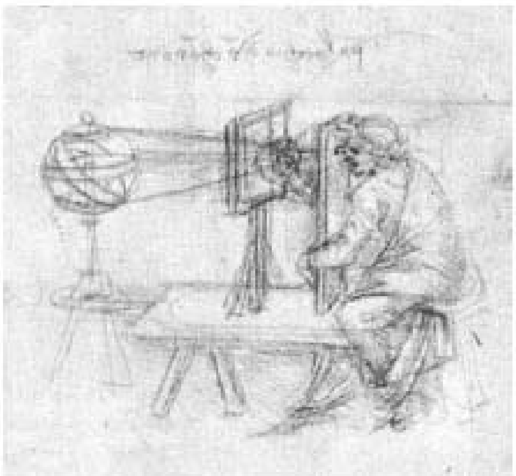


Figure 5. da Vinci

Figure 6. Düler

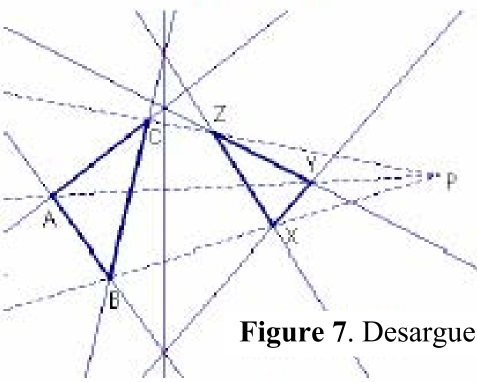


Figure 7. Desargues

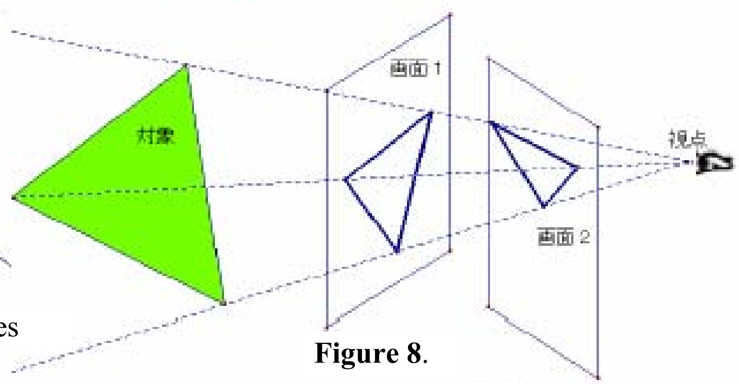


Figure 8.

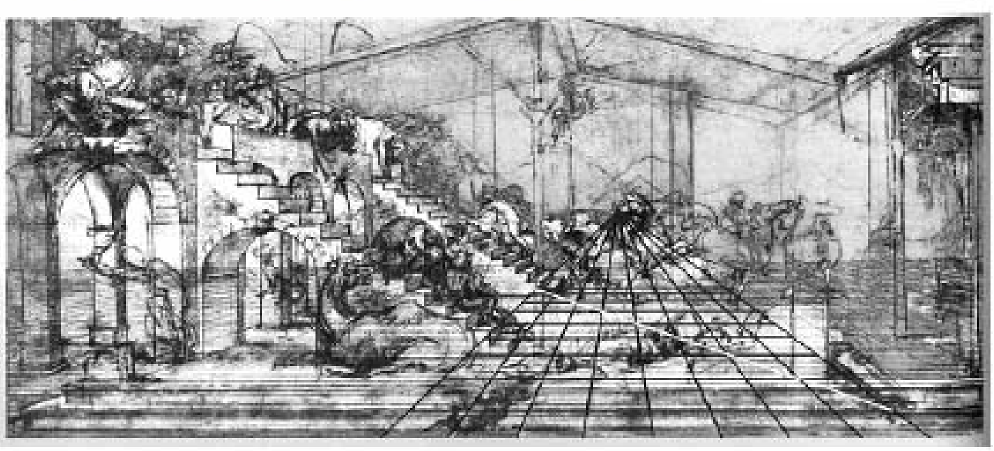


Figure 9. da Vinci

The ways of how to construct the depth on the board using a set of parallel lines and focusing lines develop the mental object for mathematization because there are mathematical invariant three or more lines intersect at one points. Pascal reconsidered the phenomena upset ways and considered it from 'ordonnance' (beam, the set of parallel lines and lines thorough one point). If we use the pencil, it is not easy to write the lines which enable to meet at one point. Because we have experience the difficulty, we feel strange when three lines meet at one point.

The embodiment (Lakoff, Nunez, 2000), explained an even higher mathematical concepts originated from some metaphors derived from bodily motion, is the basic ideas when we think about the role of technology. One cannot understand mathematics appropriately without reliable metaphors. The theory gives a central role of the appropriate metaphors of bodily motion for understanding abstract mathematical ideas and overcoming discontinuity of learning process. If we do not know the difficulty to draw three lines intersect at one point, we can not recognize it as mental object.

In Figure 7, the Theorem of Desargues is explained by Figure 8, which uses the same metaphor as the pictures in Figures 5 and 6. Projective geometry generalized the eye-beam metaphor. The eye-beam also existed in the pictures of Christ in the middle ages but it was not a human eye-beam. It came from heaven and eyes of God. Painters imagined the existence of God and trying to draw the benediction with eye-beam from God or heaven on the drawing boards. Thus, normally, Christ should be larger than apostles because they follow and receive God's eye-beam.

These new tools treated the eye-beam like the eye-beam of a human painter and humanized reality. Through the use of these geometrical tools, it was possible to see reality as a human construction and enable people again to use eye-beam metaphors as well as Euclid did at his Optics. Today, the latest technology, DGS, enables us to construct figure 12 and figure 13. Because we can drag the Eye Point and Screen, we can easily realize the appropriateness of the construction based on the perspective drawing theory. We can embody the perspective drawing theory to the real world which we feel in reality through the dragging the object. Through the embodiment by the DGS, mental object is now the really object could be dragged.

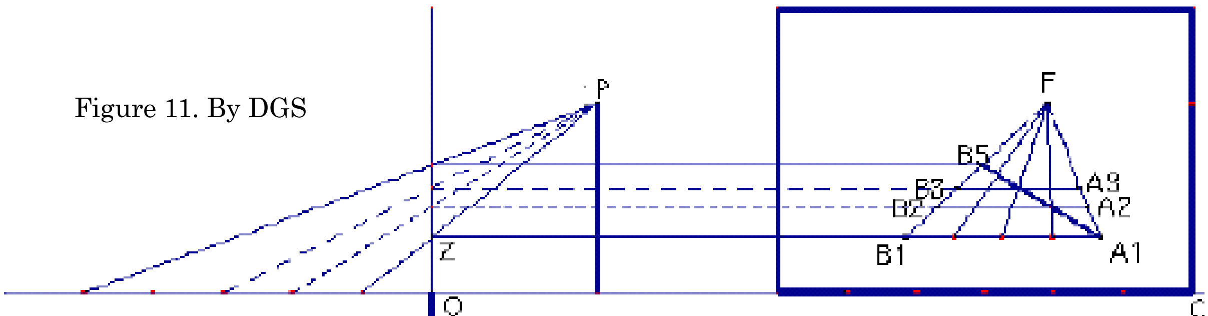
We have to note that this treatment by new technology is not the same recognition as da Vinci had because he did not have DGS. On the other hands, DGS supported to see those processes from the view point of hermeneutics that the understanding of mathematics in a social context includes the developer's own, author's or another's perspective (Janke, 1994). Mathematics is most reliable subject to represent or reinvent by other people and thus understanding mathematics include expecting other people's mind. Historical tools and current technology enable us to imagine or simulate developer's perspective.

Because we know the meaning of Figures 5, 7 and 8, we can imagine the activity in Figures 9 and 10, and imagine the mind of da Vinci and his view of the world. If we understand the Figure 10 as Figure 11, we understand well that there is only one perspective drawing theory even if, in art, it is technically explained with many kinds of composition such as using one-point, two-point or three-point perspective drawings. If we do not know the perspective drawing tools used in Figures 5 and 6, and if we do not imagine that the structure is the same as looking outside through a window, we cannot imagine the process by which mathematics developed. Today's perspective drawing theory in art that counts the number of vanishing points is only one technique for the composition of a given picture. It is not only graphics theory but also does not include the theory of perspective drawing that da Vinci and Dürer developed in their time.

Counting the number of vanishing points of the picture of da Vinci in art classroom in school teaches technique for the composition of picture but lacks interpretation of it as a human endeavor: For interpreting, it is necessary to imagine the author's/painter's wishes at that age in trying to explore with tools. This personal interpretation was not constructed without the tools of Figures 5 and 6.

The example illustrates Judicious using of old and new technology, and both technology supported to construct the important mathematical object on the context. This is the reason why we do not need to distinguish between old and new technology in the context of education such as mathematization. Depending on the time and aim, teacher can chose it in the most meaningful way.

Figure 11. By DGS



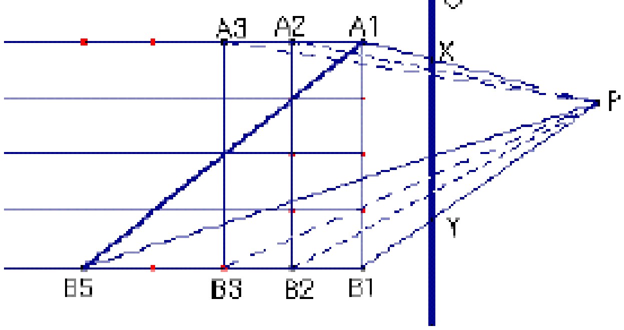


Figure 10. da Vinci

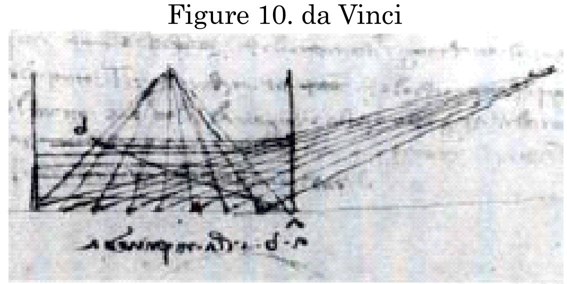


Figure 12

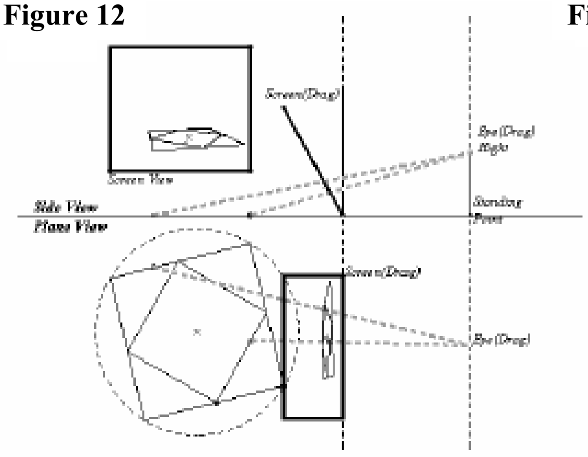
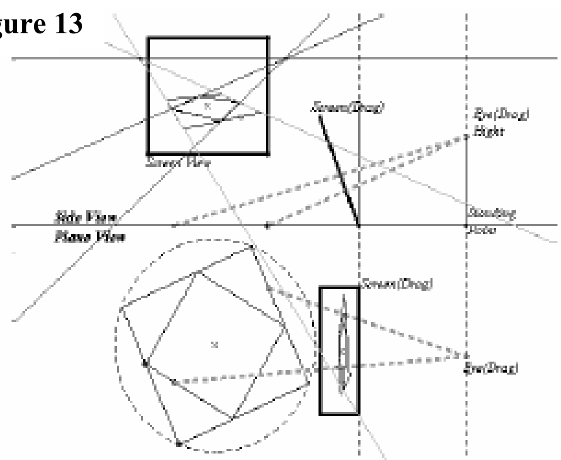


Figure 13



4. The intuition supported by mental object and visualization supported by technology.

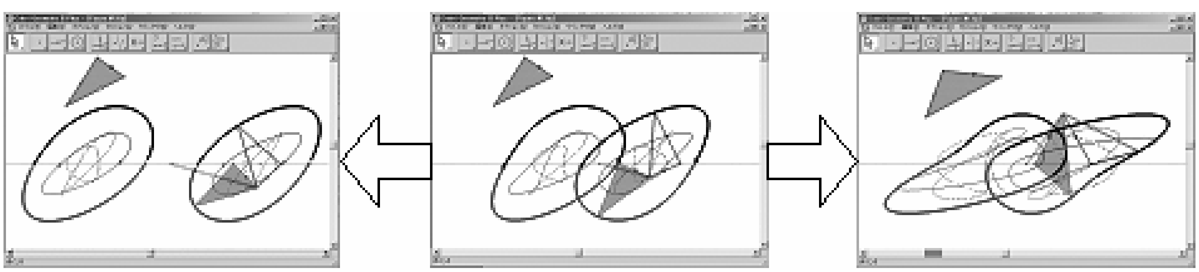
Every tool has certain intuition. It is this reason why we should choose the appropriate technology depending on the aims of education and necessary to develop the ability for judicious using.

The problems of locus on mechanics are good examples to illustrate what mechanical technology is. Technicians need the appropriate ability and knowledge in order to develop mechanics since majority of us do not have such mathematical intuition. Figure 14 was sited from Japanese Secondary School Textbooks in the 1943. In figure 14a, if point C move the slider AB, where point D is fixed and EC=ED=EF, then how does point F move? The guessing and answer could be illustrated cleary by the window mechanism (figure 14b). In case of figure 14c, how about the case of point C (middle) and point D (1/3). On further grade, they are analyzed by algebraic representations (figure 14d).

What is astonishing about this example is that figures 14a – 14d do look different but they are the same in mechanical meaning. To understand these different mechanics as one mechanics you must solve these problems geometrically according to the order of the textbooks. If you could imagine those mechanics are the same, you must have the mental object that is necessary to develop these kinds of mechanical tools. The similar descriptions existed in the textbook by van Schooten in 1646 (Figure 15). His textbook also began from guessing to geometric and finally algebraic reasoning. The intuition which we could acquire from those textbook is supported by the mental object of geometric reasoning. It bridges mechanics and geometry.

On the other hands, even if we recognize mechanics with geometrical mental object, it is very difficult to imagine the following locus (see bellow) because geometry can treat s limited number of curves as for mental object. This is why it is necessary to use technology for visualizing following curves (see bellow).

For technicians who develop mehanics, mechanical and geometric reasoning is important. From the view point of mathemaitzation, the following curves became the mathematical object which will be expressed by equations of functions. If technicians need to control the motion by computer, they must use the equations, too. Through mathematization, they will develop further intuitions to treat the following motions and develop better imagination as mental object without visualization by computer.



5. Conclusion

Logical inconsistencies for introducing technology come from the difference of society, curriculum and technology itself. Because of these inconsistencies, we should develop the judicious position to use technology. Teachers and students should be judicious user. Japanese approach which focused on teachers is one example. In any example, there is limitation. To enhance students' mathematical exploration, the importance of judicious using of technology is mentioned, in order to enhance students' mathematical explorations and developing the mental object that support their mathematization. The basic theories of mathematics education are used to explain this position. To illustrate the importance of both the traditional and computer technology, I used perspective drawing and mechanical motions as the main examples.

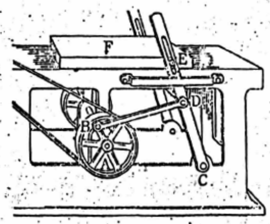
We use technology to improve mathematics education. This is the standpoint in this lecture even though there is some necessity to use mathematics education for technology in some occasions.

1. 軌　跡

§1. 機械ノ運動

スベテ機械ガ運轉シテキルノヲ見ルト,原動力ヲ受ケテ動ク部分ト,目的ノ運動ヲスル部分トノ間ニアル仕掛ガアツテ,初ノ運動ヲ次々ニ傳ヘテ,終ノ運動ヲ起サセルヤウニナツテキル。

問1. 右ノ圖ハ,印刷機ナドニ使ハレル裝置デアル。車輪Aヲ回轉サセルト,上方ノ板Fハドンナ運動ヲスルカ。



問2. 右ノ圖デ,BガAノ周リヲ回轉スルト,Dハドンナ運動ヲスルカ。

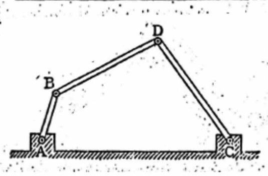
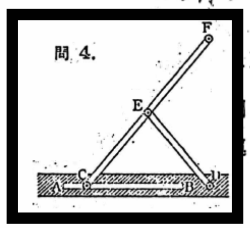
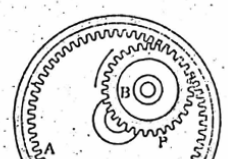


Figure 14a



問5. 右ノ圖デ,歯車Bハ外ノ歯車Aトカミ合ツテコロガル。マタ,Bノ直徑ハチヤ … 半徑ニ等シ … キル。… 上ノ點Pハ, … 動ヲスルカ。



1. 右ノ圖ノヤウナ構造ノ窓ガアル。支ヘノ棒GHハ縦ノ框ノ眞中ニトリツケテアツテ,棒ABヲ突キ出スト,ガ … ラス戸ノ上ノ框CDハ溝ニ沿ツテ下ガルヤウニナツテキル。

コノトキ,下ノ框EFハド … ンナ運動ヲスルカ。

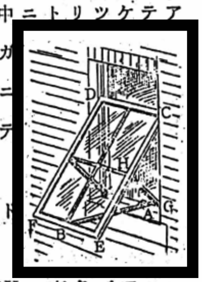
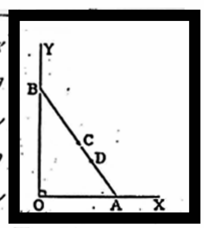


Figure 14b

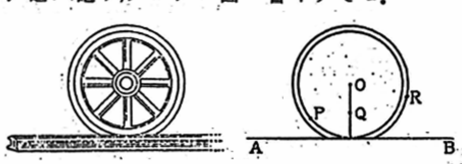
2. 次ノ頁ノ圖デ∠XOYハ … ABハ長サ6cmノ棒デ,ソノ両端

Figure 14c

ノ角ノ邊上ニアツテ動クヤウニナツテキル。コノ棒ヲ動カスト,棒ノ中點Cハドンナ運動ヲスルカ。マタ,棒ノ下カラ$\frac{1}{3}$ノ所ノ點Dハドンナ運動ヲスルカ。ソノ道ヲ圖ニ書イテミヨ。



3. 線路ノ上ヲ車輪ガコロガルトキ,車輪上ノ點ハドンナ運動ヲスルカ。次ノ各ノ點ノ通ル道ヲ,ソレゾレ圖ニ書イテミヨ。



(1) 車軸ノ中心O。

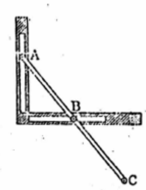
(2) 線路ニ接スル面ノ上ノ點P。

(3) Pヨリモ車軸ノ中心Oニ近イ點Q。

(4) 同ジク遠イ點R。

4. 小サイ圓ガ大キイ圓ノ中ニアツテ,コレニ接シナガラコロガルトキ,ソノ周上ノ點

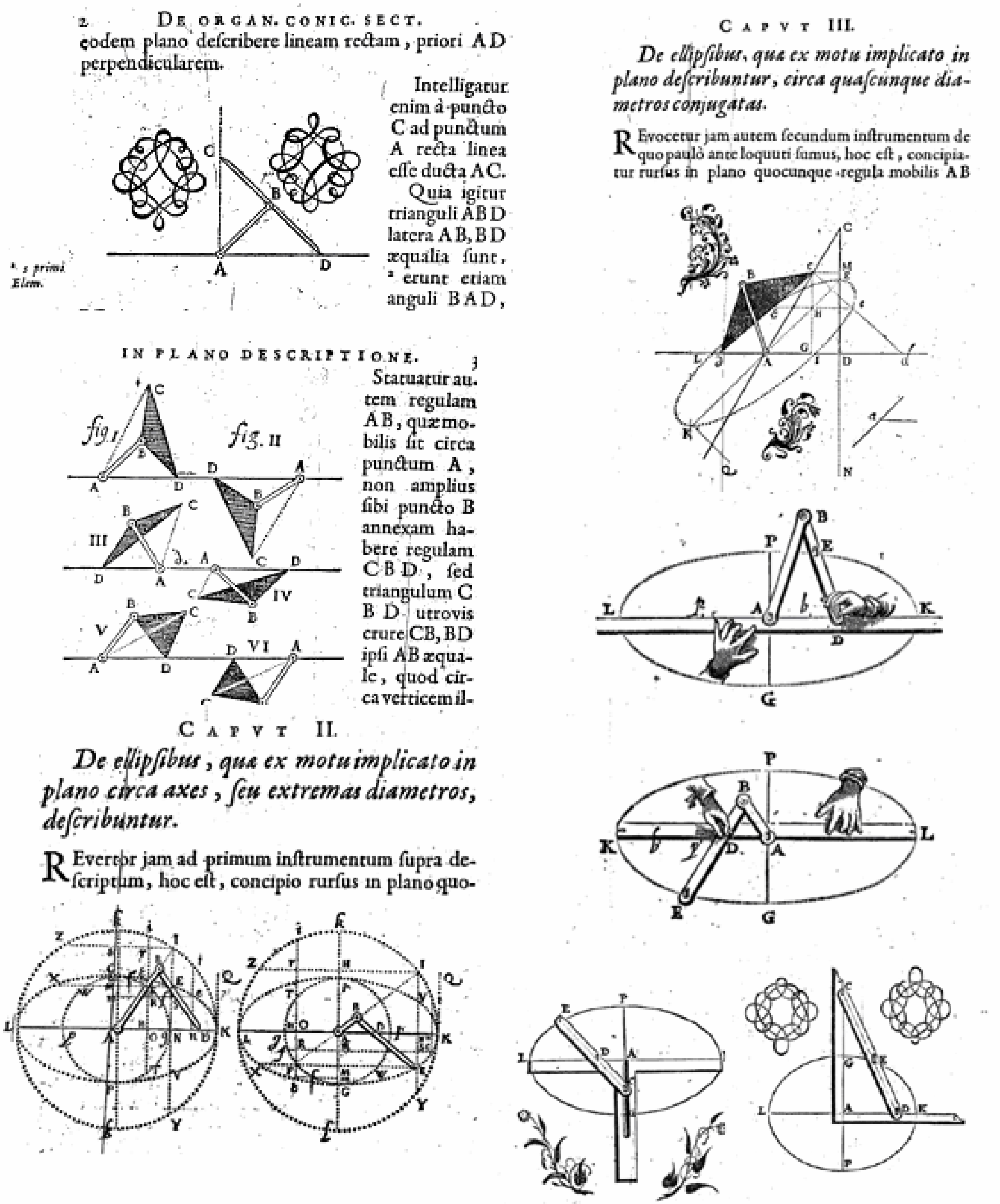
10. 右ノ圖デ,棒ACノ一端Aト中點Bトハ,直角ニ交ハル溝ヲ通路トシテ動ク。Cハドンナ線ヲ書クカ。



11. 座標ガ次ノ不等式ニ

Figure 14d

Figure 14
1943, Japanese Secondary School Mathematics Textbooks
a-c: from guessing to geometrically reasoning
d: after grade, in analytic geometry



2 DE ORGAN. CONIC. SECT.

eodem plano deſcribere lineam rectam, priori AD perpendicularem.

Intelligatur enim à puncto C ad punctum A recta linea eſſe ducta AC. Quia igitur trianguli ABD latera AB, BD æqualia ſunt, * erunt etiam anguli BAD,

* 5 primi Elem.

IN PLANO DESCRIPTIONE. 3

Statuatur autem regulam AB, quæ mobilis ſit circa punctum A, non amplius ſibi puncto B annexam habere regulam CBD, ſed triangulum CBD utrovis crure CB, BD ipſi AB æquale, quod circa verticem il-

CAPUT II.

De ellipſibus, quæ ex motu implicato in plano circa axes, ſeu extremas diametros, deſcribuntur.

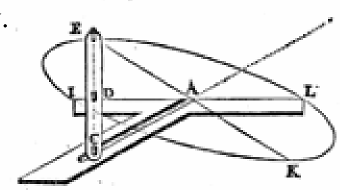
Revertor jam ad primum inſtrumentum ſupra deſcriptum, hoc eſt, concipio rurſus in plano quo-

CAPUT III.

De ellipſibus, quæ ex motu implicato in plano deſcribuntur, circa quaſcunque diametros conjugatas.

Revocetur jam autem ſecundum inſtrumentum de quo paulò ante loquuti ſumus, hoc eſt, concipiatur rurſus in plano quocunque regula mobilis AB

Figure 15 Schooten 1646,
from Guessing to Geometrically and Analytically.



References

[1] Lynda Ball, Kaye Stacey., (2005). Teaching Strategies for Developing Judicious Technology Use., Edited by William J. Masalski., Portia C. Elliott., National Council of Teachers of Mathematics; VA. 3-16.

[2] Bartolini Bussi, Maria G. (2000). Ancient Instruments in the Modern Classroom., Edited by John Fauvel, Jan van Maanen. , History in Mathematics Education. 343-350.

[3] Michele Cerulli. Introducing pupils to algebra as a theory: L'Algebrista as an instrument of semiotic mediation (http://poisson.phc.unipi.it/~cerulli/tesi/)

[4] Hans Freudenthal (1973)., Mathematics as an Educational Task., D. Reidel: Dordrecht.

[5] Hans Freudenthal (1983). Didactical Phenomenology of Mathematical Structures., D. Reidel: Dordrecht.

[6] Masami Isoda (2004). Why We Use Mechanical Tools and Computer Software in Creative Mathematics Education., Proceedings of Aisa-Pacific HPM 2004 Conference. National Taichung Teachers College; Taiwan. 19-30.

[7] Masami Isoda et al. (2006). e-Textbook: Advancing Mathematics Teaching and Learning into the Future., Proceedings of the Eleventh Asian Technology Conference in Mathematics., ATCM:VA. 54-63

[8] Hans Niles Janhke (1994). The Historical Dimension of Mathematical Understanding: Objectifying the Subjective. Proceeedings of International Conference for the Psychology of Mathematics Education 18.(1). 139-156..

[9] George Lakoff, Rafael Nunez., (2000). Where Mathematics Comes From. Basic Books: NY.

[10] Franz van Schooten (1646). De organica conicarum sectionum constructione. Lione, Elsevier,

[11] David Tall, Masami Isoda (to appear)., Long-Term development of Mathematical Thinking and and Lesson study.

[12] James Wertch (1991)., Voices of the Mind. Harvard University Press: Cambridge.

Free Software

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GRAPES (Graphing Tools)
GCL (Dynamic Geometry Software enable to construct on the browser by Flash)
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