# Spreadsheets in Mathematics: Accessibility, Creativity, and Fun

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**Abstract:** A spreadsheet, such as *Microsoft Excel*, provides educators with a creative tool for the study and teaching of mathematics, mathematical modeling, and mathematical visualization. It enables students to gain mathematical insights into a diverse range of interesting and significant applications in an engaging setting while they simultaneously acquire practical skills in using the principal mathematical tool of the workplace. This paper illustrates novel ways to use this powerful and accessible tool and its outstanding graphic features creatively in teaching a surprising number of mathematically oriented topics. Illustrations come from such disciplines as the physical and social sciences, statistics, mathematics, computer sciences, and the arts. The paper describes how *Excel*'s graphics can create eye-catching animated graphic displays and inject more fun into the study of mathematics.

## 1. Introduction

The computer spreadsheet is nearly 30 years old. During its lifetime, applications of this creative software have diversified and spread widely from the original uses in financial fields into all areas of human endeavor. Today it is the principal mathematical tool of the workplace. It is readily available on virtually every computer, with the most common version being *Microsoft Excel*. It also increasingly appears in the teaching and communication of mathematics at all levels. In this paper, we provide some visual glimpses into a broad range of applications of spreadsheets in mathematics education.

The spreadsheet offers many advantages for learning mathematical concepts. It is an easily accessible and creative tool whose basic operations are familiar to most students. Educators can use spreadsheets so that the actual process of creating a spreadsheet model itself teaches and reinforces mathematical concepts. In addition, the spreadsheet design often allows students to study successfully numerous topics ordinarily considered as being too difficult for them. Its use also provides students with valuable practical experience in working with a tool that they will use in their future jobs. Finally, the spreadsheet helps teachers and students to find the study of mathematics to be a fun experience.

Our approach to the use of spreadsheets contains an emphasis on designing effective graphics to promote the development of visualization skills. To do this, we present illustrative examples that describe creative ways to use spreadsheets to produce interactive animated graphics. Our examples come from a wide range of applications and disciplines. Each of these examples incorporates the use of mathematics.

We have designed this paper to give a visual overview of our approach. We create our examples using *Microsoft Excel 2003*. Anyone interested in the technical details of spreadsheet operation, how to create scroll bars and simple macros for animated graphics, and ways to use the Solver and Data Table tools, can consult [8] or download a paper and *Excel* application files from [2].

A spreadsheet lends itself to a variety of educational uses. First, it provides educators with a natural way to implement mathematical algorithms and models and to create interactive graphs for use in student assignments and activities. In the latter case, it provides a way for students to work in groups on more substantial projects. Second, it enables teachers to prepare original and effective classroom demonstrations to illustrate mathematical ideas. In addition, it allows teachers to create visual models for most textbook topics, including those in algebra, calculus, statistics, numerical analysis, and linear algebra. Third, it can provide an avenue for the professional development of educators, opening opportunities for them to give professional presentations of new approaches to teaching and research. This applies not only to mathematics, but also to virtually any other discipline. Finally, it is an excellent avenue for teaching continuing education courses, and for communicating with the public and with colleagues from other disciplines. The author has used spreadsheets successfully in presenting mathematics to wide variety of adult learners in both developing and technologically advanced nations. Additional discussions of teaching uses of spreadsheets may be found in [1], [3] and on the Web [9], [10].

## 2. Mathematical Modeling

Originally, the business community was the principal user of spreadsheets, employing them to create interactive financial mathematical models that they interrogated in a "What if …?" manner. Over the ensuing years, many additional mathematical applications for spreadsheets arose, causing their usage to expand into the design of mathematical models from diverse fields. As a result, today we use spreadsheets in many classrooms to study a wide and complex range of interesting mathematical models. Many topics that ordinarily we might regard as too advanced are now accessible to our students. The spreadsheet provides students with a tool that encourages creativity, and enables them to have fun while investigating significant mathematical ideas.

As our first example, in Figure 2.1 we present an ecological model that involves the study of the interaction of two competing species of animals. We base it on a more traditional presentation found in pages 41-45 of [7]. Here we consider two species of squirrels, gray and red, whose populations grow exponentially if there were no competition between them. However, since there is an interaction effect that reduces population growth, this also is included in the model.

For our spreadsheet descriptions, we use the arrow notation of [8]. In Figures 2.2-2.3, we first enter the population sizes and growth rates, and then form a table that gives the sizes of the populations in successive time periods. After reproducing the initial populations in the top row of our table, we compute the number of new grey species as the product of their population and the growth rate. We indicate that the growth rate is an absolute reference by placing a pin in the source cell. We next compute the amount of grey population reduction that occurs due to competition between the species by assuming that it is proportional to the number of possible interactions between individuals (the product of the populations). We assume that studies have approximated the value of the parameter for the reduction rate. We then find the next period's population of the grey species in the following row. We treat the red species similarly. We then simply copy the formulas.

| | | | | | | |
|---|---|---|---|---|---|---|
| | Initial | New | Interact | | | Period |
| | Population | Growth | Effect | | N | 20 |
| Grey | 145 | 0.24 | 0.003 | | Grey | 41.3 |
| Red | 79 | 0.3 | 0.002 | | Red | 204.9 |
| | | | | | | |
| Period | Grey | Red | New G | Loss R | New G | Loss G |
| 1 | 145.0 | 79.0 | 34.8 | 34.4 | 23.7 | 22.9 |
| 2 | 145.4 | 79.8 | 34.9 | 34.8 | 23.9 | 23.2 |
| 3 | 145.5 | 80.5 | 34.9 | 35.2 | 24.2 | 23.4 |

**Figure 2.1.** Species Competition Output

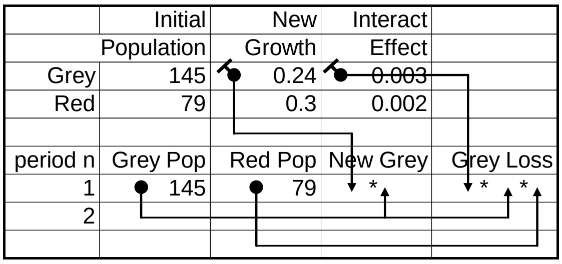



**Figure 2.2.** Species Formulas I

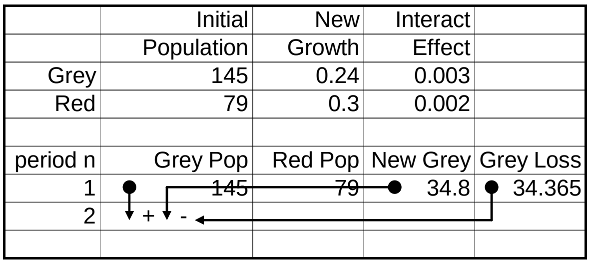



**Figure 2.3.** Species Formulas II

In order to produce an image to help us in visualizing the results, we create the *xy*-graph in Figure 2.4. It shows the population sizes over a sequence of periods. We see that with our assumptions the red species eventually drives out the grey one. Students can also discover that there are initial population values that will result in equilibrium, but one that is extremely unstable. The slightest changes cause the extinction of one of the species. Our second graph in Figure 2.5 gives us a different way to visualize the results. The randomly placed markers represent the numbers of each species in a given period. We further enhance our model by inserting scroll bars (see [8], [2]), and move their sliders to see the effects of varying the number of periods or the model's parameters. In the second graph, the original mixture of red and grey gradually becomes entirely red.

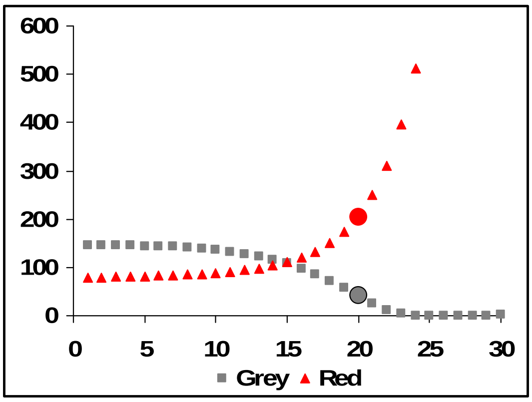



**Figure 2.4.** Species Graph I

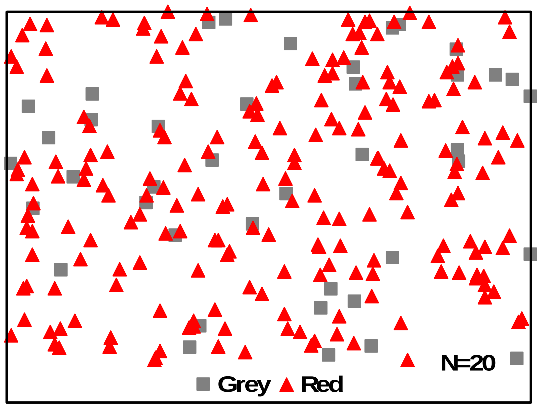



**Figure 2.5.** Species Graph II

Similar models and visualization techniques appear in [8], with investigations of such topics as population growth, resource harvesting, epidemics, predator-prey interactions, genetics, medicine dosage, financial analyses, projectiles, heat flow, apportionment, and planetary motion. Other possible topics include pollution, cooling, drug testing, and many more. These topics provide interesting group projects, allowing students to design creative implementations and graphics.

## 3. Graphing Functions

Spreadsheets are also effective tools for creating animated graphs of functions. Here we graph the polar equation $r = \cos(t/c)$. In the model of Figures 3.1-3.2, the parameter, *c*, is set to 2. The left column counts degrees. We convert these to radians using *Excel*'s built-in RADIANS function in the second column, enter the formula for $r$ in the third one, and compute the values of $x$ and $y$ in the next two by $x = r\cos t$, $y = r\sin t$.

| | | | | | | |
|---|---|---|---|---|---|---|
| c | 2 | | N | 4 | | |
| step | 1 | | | | | |
| n | t | r | x | y | X | Y |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0.017 | 1 | 1 | 0.017 | 1 | 0.017 |
| 2 | 0.035 | 1 | 0.999 | 0.035 | 0.999 | 0.035 |
| 3 | 0.052 | 1 | 0.998 | 0.052 | 0.998 | 0.052 |
| 4 | 0.07 | 0.999 | 0.997 | 0.07 | 0.997 | 0.07 |
| 5 | 0.087 | 0.999 | 0.995 | 0.087 | 0.997 | 0.07 |

**Figure 3.1**. Graphing Output

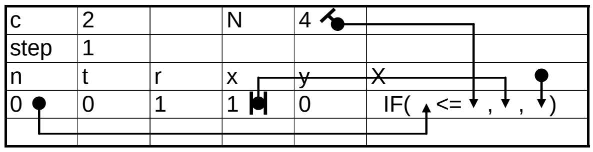



**Figure 3.2**. Graph Formulas

We use the 4th and 5th columns to create the complete *xy*-graph in Figure 3.3. However, it is easy to animate the tracing of the curve to appear much as we would draw it manually in class. We generate only the first $N$ points of the curve by employing the IF…THEN…ELSE structure in the two rightmost columns. These expressions reproduce the $(x, y)$ values when $n \le N$, and otherwise copy the value from the cell above. We then link a scroll bar to $N$. As we move it, we see the curve being traced out. Many additional graphing applications appear in [4] and [8].

Students can use mathematical concepts to have fun in enhancing the model. In Figure 3.4, we have created a simple image of a butterfly, and then scaled, translated, and rotated it so that it moves at the trace point and remains tangent to the path. Using a few more mathematical ideas and spreadsheet techniques, students can color in the wings, cause them to flap periodically, draw parametric curves in the shape of flowers along the curve, and create a simple macro to continually update $N$, thereby creating a movie of the butterfly moving from flower to flower.

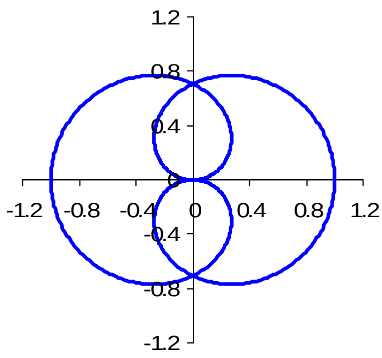



**Figure 3.3.** Static Graph

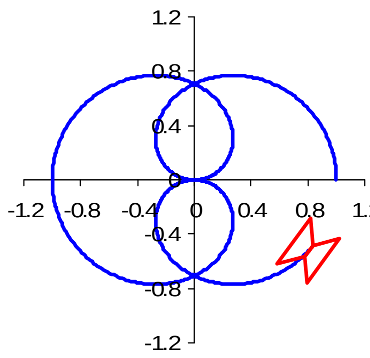


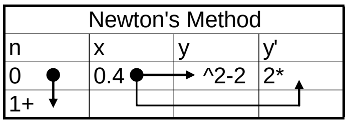

**Figure 3.4.** Animated Trace Graph

## 4. Numerical Algorithms

We also implement the algorithms of numerical analysis effectively in a spreadsheet, including those for finding zeroes, numerical integration, solving differential equations, fitting curves, solving linear systems, and computing eigenvalues. We can also create animated versions of virtually every static diagram that appears in a text. These creations provide students with new ways to visualize the effects of changes in parameters, to observe possible divergence, and to explore other aspects.

Teachers and students can also create animated demonstrations of algorithms. In Figures 4.1-4.3, we implement Newton's Method in a table format to find a zero of $f(x) = x^2 - 2$. We first enter 0.4 as an initial estimate. We enter formulas to compute $f(x)$ and $f'(x)$ in the cells to the right. In the second row, we obtain the next approximation of the zero by $x - f(x)/f'(x)$. We copy these expressions down their columns. We can observe the rate of convergence, experiment with the initial estimate, and change functions. By designing other animated graphics as in [8], we can better illustrate functions in which the algorithm does not converge or is sensitive to the initial estimate. Such graphics generally attract more student attention than do traditional techniques.

| Newton's Method | | | |
|---|---|---|---|
| n | x | y | y' |
| 0 | 0.4 | -1.84 | 0.8 |
| 1 | 2.7 | 5.29 | 5.4 |
| 2 | 1.72 | 0.96 | 3.441 |
| 3 | 1.441 | 0.078 | 2.883 |
| 4 | 1.414 | 7E-04 | 2.829 |
| 5 | 1.414 | 7E-08 | 2.828 |
| 6 | 1.414 | 0 | 2.828 |



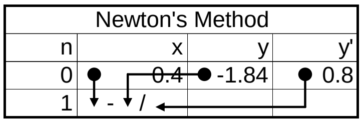
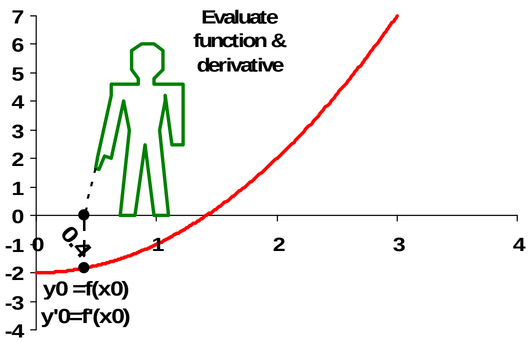




**Figure 4.1.** Newton **Figure 4.2.** Formulas I **Figure 4.3.** Formulas II

Additionally, teachers can design demonstrations that lead students through an algorithm in a step-by-step manner. Two steps in an animated teaching construction that we update by clicking on a button appear in Figures 4.4-4.5.



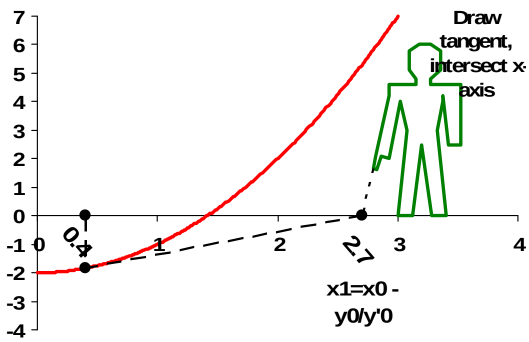




**Figure 4.4.** Instructional Graphic I **Figure 4.5.** Instructional Graphic II

## 5. Linear Algebra and Vectors

*Excel* provides built-in matrix functions for multiplication, inversion, and determinates (see [8], [2]). We can use these and other standard features of *Excel* to create interactive animated displays for a great range of ideas from the linear algebra of $\Re^2$, including such topics as the grids for various bases, eigenvalues, linear programming, pivoting, and the visualizations of linear transformations. We employed some of these techniques in creating the butterfly graph above and the reflection transformation shown in Figure 5.1.

In Figure 5.2, we use the ideas of vectors in illustrating pursuit problems, enhanced by forming *xy*-images of airplanes. We first parameterize the path of one airplane, and then use vectors and a rotation matrix to cause another airplane to pursue the first as it moves directly toward the first in small discrete increments. We set velocities of the planes as parameters, and use a scroll bar to vary time, causing the planes to trace out curves. We observe the resulting path of the pursuit plane and when, or if, it overtakes the first. Students can also discover and investigate other pursuit strategies.

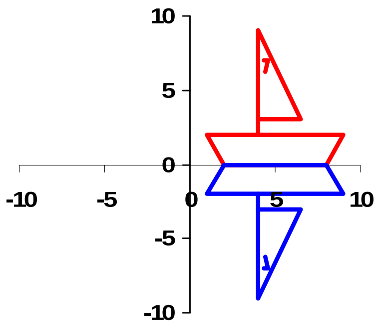



**Figure 5.1.** Matrices (reflection)

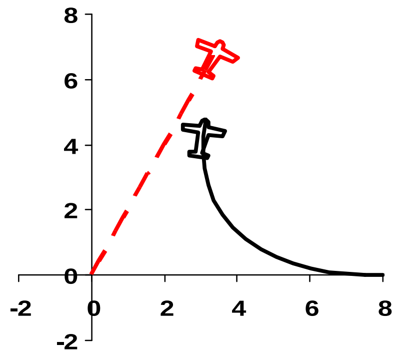



**Figure 5.2.** Matrices (rotation)

## 6. Computer Science

We can study a broad range of computer science concepts naturally on a spreadsheet. In addition to using the spreadsheet's built-in database features, students can discover creative ways to study such data structures topics as algorithms for sorting, searching, and stacks with this widely used computing tool. The act of creating an implementation enhances a student's understanding of the concepts. We also can create different ways to visualize traditional computer science topics. In Figures 6.1-6.2, we show two steps in the iterative solution of the classical Towers of Hanoi problem [5], while Figures 6.3-6.5 illustrate the continuous morphing of a butterfly into a rabbit.

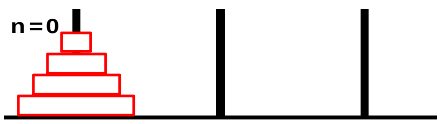



**Figure 6.1.** Towers Hanoi Graphic I

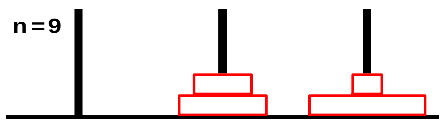



**Figure 6.2.** Towers Hanoi Graphic II

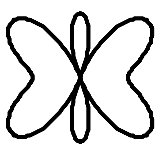


**Figure 6.3.** Morphing I

**Figure 6.4.** Morphing II

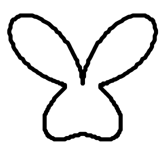


**Figure 6.5.** Morphing III

## 7. Art and Culture

Another fascinating source for interest, fun, and enjoyment comes from examining the arts and cultures of different societies. For example, symmetry, geometry, and other mathematical ideas often appear in the design of national flags and traditional art. A few spreadsheet creations of these appear in Figures 7.1-7.4. A challenge to students is to animate the flags to simulate waving in the breeze. Doing this requires both inventiveness and mathematical insights. Other sources of cultural projects come from traditional arts and crafts, such as the design of quilts, needlepoint, string-and-nail art, bilums (net bags) of Papua New Guinea, and kilims (rugs) of Turkey. Using a spreadsheet to design these cultural images provides many students who otherwise may not be attracted to mathematics with an attractive pathway to encounter the subject through its geometrical aspects by the creative application of a popular and accessible technological tool.

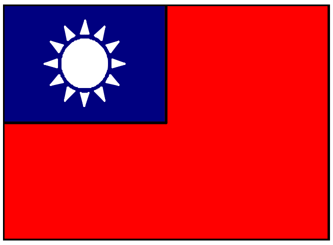


**Figure 7.1.** Flag Design I

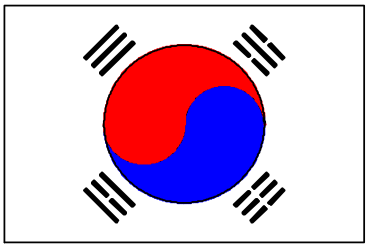


**Figure 7.2.** Flag Design II

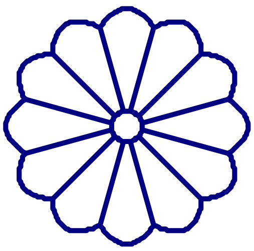


**Figure 7.3.** Cultural Design

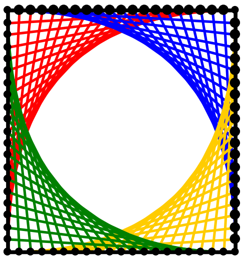


**Figure 7.4.** Nail and String Art

## 8. Statistics

Despite shortcomings in some of its statistical features, *Excel* is a valuable tool for learning the fundamental concepts of statistics. It invariably supplies us with several levels at which to approach a topic. Thus, we can pursue a concept such as the standard deviation either directly from definitions or by using built-in functions. In either case, we can supplement our models with effective original graphic visualizations.

One inventive way to display data in a map, as in Figure 8.1, is by using a bubble graph, where the geographical border is composed of tiny circles, and the circles for cities are proportional to their populations. This also is an excellent way for students to create attractive scatter diagrams for topics of particular interest to them. One of the author's earliest statistical memories is of a magazine's graphic that showed the relative strengths of U.S. university football teams via scaled footballs in a map.

Although we enter the coordinates of this map manually, the Web provides some data sets that supply (*x*,*y*) coordinates of many cartographic boundaries, and freeware such as *PlotDigitizer* give us a more convenient way to generate more easily coordinates to digitize a wide variety of images.

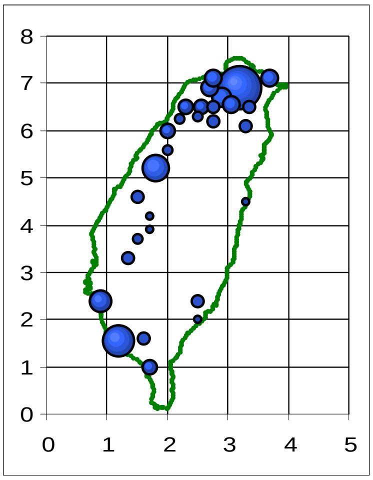



**Figure 8.1**. Scatter Display via Bubble Graph

Another versatile spreadsheet tool is the Solver. In Figures 8.2-8.4 we first employ the RANDOM function to generate 100 (*x*,*y*) points over a chosen range, and use the CORRELATION function to observe that the correlation coefficient, *r*, is nearly 0. We then convert the values into constants and use the Solver to set the correlation coefficient to a chosen value ($r = 0.8$) by having the Solver change *x* and *y* values. The spreadsheet makes the adjustments and produces a graph of a distribution giving the desired value of *r*. If we use 1 or –1 for *r*, then we will obtain a straight line.

| | | |
|---|---|---|
| r | 0.019 | |
| | | |
| n | x | y |
| 1 | -0.06 | 0.185 |
| 2 | 0.468 | -0.68 |
| 3 | -0.52 | -0.05 |
| 4 | -0.11 | 0.872 |

**Figure 8.2.** Random Data

| | | |
|---|---|---|
| r | 0.8 | |
| | | |
| n | x | y |
| 1 | 0.032 | 0.16 |
| 2 | 0.129 | -0.43 |
| 3 | -0.53 | -0.29 |
| 4 | 0.316 | 0.805 |

**Figure 8.3.** Correlation Data

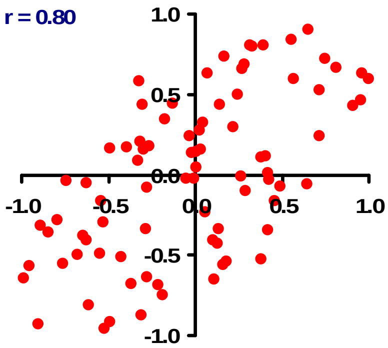



**Figure 8.4.** Correlation Graph

## 9. Historical Topics

During the author's university studies and his early years of teaching, the principal computational tool was the slide rule. Most students today are completely unfamiliar with its operation and its use of logarithms. This and other historical ideas provide wonderful projects for making classes interesting and fun. In the display below, we use scroll bars to move the center rule and the crosshair. The display of Figure 9.1 also provides the underlying numbers to better show the ideas involved. Other historical topics for student projects include the abacus, Galileo's sector compass, Napier's bones, and the Galton board of statistics.

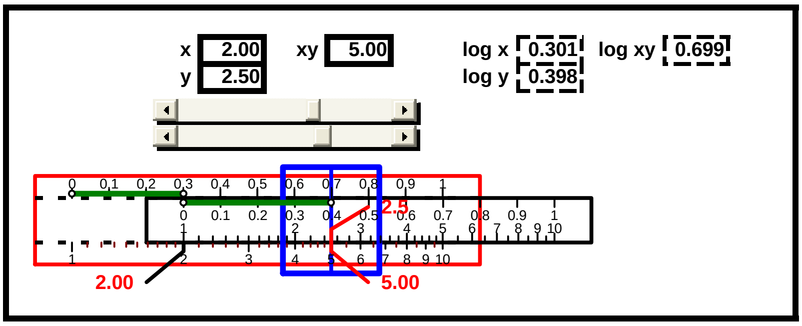



**Figure 9.1.** Animated Slide Rule Display

## 10. Creative Fun

Finally, students are generally quite adept at finding interesting and fun things to use in illustrating mathematical concepts. We show two examples in Figures 10.1-10.2. One of these is a carnival ride, called a Ferris wheel in the U.S. We create it using our earlier graphic techniques. We then use a rotation matrix to rotate the wheel through *k* degrees, and build a macro containing a loop to iterate the process many times, causing the wheel to rotate. The larger the value of *k*, the faster the wheel moves. Using a negative value for *k* reverses the direction of rotation.

Another interesting project is creating a traditional analog clock by using mathematics to insure that the second, minute, and hour hands advance appropriately. We then link the time to the computer's clock. Students can also draw a building in which to display the clock, or modify the design for a "backward" moving clock, such as the historical Josefov clock in Prague.

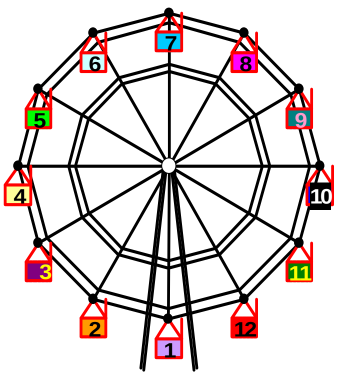



**Figure 10.1.** Animated Recreations

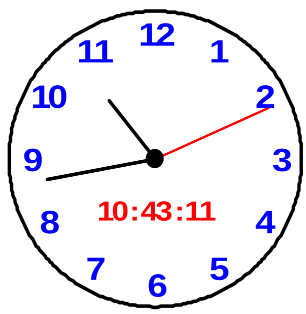



**Figure 10.2.** Animated Clock

Another creative application comes from psychology, where optical illusions help in the analysis of visual perception. Students create designs using a spreadsheet, and manipulate then through sliders, spinners, and buttons. In Figure 10.3, the left and right red segments appear not to be in line. However, when we click a button to expose connecting line segment in Figure 10.4, we see that, in fact, they are. In Figure 10.5, we use a scroll bar to move the center arrow to try to divide the line into two equal parts. We seldom will be correct. Once again, we can see the correct location after pushing a button to display the correct location. In many illusions, color plays a vital role, and the spreadsheet provides us with a good tool for this. Consult [5] to see a vast array of examples of optical illusions to implement.

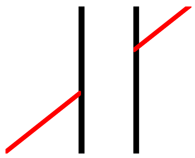


**Figure 10.3.** Illusion Ia

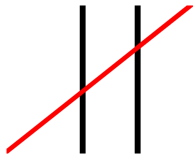


**Figure 10.4.** Illusion Ib

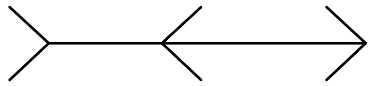


**Figure 10.5.** Optical Illusion II

**Note:** While we have created our images using *Excel 2003*, the models created in this version run on well on *Excel 2007*, which contains almost all of the features of the earlier version. However, some of these may be more difficult to find (sliders are on the Developer ribbon under Insert; graphing is on the Insert ribbon), and unfortunately the drag-and-drop technique for graphing has been eliminated, although we can accomplish the same thing with a copy and paste special option. F

## References


[1] Arganbright, Deane. "Using Spreadsheets with Mathematically Gifted Students". *Journal of the Korea Society of Mathematical Education*, Series D: Research in Mathematical Education, Vol. 10, No. 1, March 2006, 33-47.

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