

# The Use and Influence of Technology in Mathematics Education

*Ma. Louise Antonette N. De Las Peñas*

Department of Mathematics  
Ateneo de Manila University, Philippines  
[mlp@mathsci.math.admu.edu.ph](mailto:mlp@mathsci.math.admu.edu.ph)

*Wei-Chi Yang*

Department of Mathematics and Statistics  
Radford University, VA, USA  
[wyang@radford.edu](mailto:wyang@radford.edu)

**Abstract.** The use of various types of technologies in the classroom and examinations is growing rapidly and is strongly influencing teaching and learning practices. In this paper, we will look at particular situations on how various technologies such as numerically capable calculators, graphics calculators, and technological tools that are CAS enabled or have CAS with Dynamic Geometry, impact students' learning. We also discuss briefly the educational opportunities that are made available by the emergence of graphics calculators with capabilities of handling electronic learning activities, such as *Casio's Class Pad* (see [1]) and *Casio's 9860* graphics calculator.

## 1. Introduction

The advent of different types of technologies inevitably influences the teaching and the learning of mathematics. A challenge to mathematics educators is how to use these technologies to improve their competencies and increase their efficiencies as teachers and make the mathematical lives of students better (see [2]). The presence of new technological tools is paving way for educators to rethink the mathematics curriculum and to develop instructional and learning materials requiring more in-depth analysis and with more emphasis on real world problems (see [3]). The focus of examination questions is changing depending on the particular type of technology allowed during examinations (see [4] and [5]). One indisputable fact is that the impact of technology varies when technological tools vary.

What strategies can we incorporate in our teaching when a particular type of technology is present? What would be the focus of our examinations? In this paper, we present several situations how a particular type of technology strongly influences students' learning.

## 2. The Impact of Various Technologies

### 2.1 Calculators with Numeric Capability

The emergence of calculators with numeric capabilities redefines the role of scientific calculators in instruction and examinations. We highlight the mathematical opportunities that are made accessible to students with these calculators as follows:

1. The solutions to equations can be obtained.
2. The Tabular and List capabilities of the calculator allow the students to analyze and study the properties of functions.

Illustration: Consider the function  $g(x) = 0.25x^4 - 2x^3 + 1.5x^2 + 21x + 15$ . The images below illustrate the calculation of the critical value of  $g(x)$  using the equation solver of the calculator. The solution to  $x^3 - 6x^2 + 3x + 21 = 0$  ( $g'(x) = 0$ ) is given by  $x \approx -1.49$ .

```
1: aN+bnY=Cn
2: aN+bnY+CnZ=dn
3: aX2+bX+c=0
4: aX3+bX2+cX+d=0
```

Figure 1

```
1  b  c  d
-6  3  21
```

Figure 2

```
X1=
-1.486416764
```

Figure 3

The next set of images gives the table of values of the first derivative before and after the critical value  $x = -1.49$ . The sign of the first derivative is positive at  $-1.48$  (after  $-1.49$ ) and negative at  $-1.5$  (before  $-1.49$ ). These explain some properties of  $g$ :  $g$  is increasing on  $(-1.49, \infty)$ , decreasing on  $(-\infty, -1.49)$  and  $g$  has a local maximum near  $x = -1.49$ .

```
f(X)=6X2+3X+21
```

Figure 4

X	F(X)
-1.49	-0.375
-1.48	0.1758

-1.5

Figure 5

3. Statistical calculations can be performed with such a calculator.

The use of real data can help students learn to value and appreciate mathematics. The students learn the concept of mathematical modeling using a variety of regression models.

Illustration: The following screen shots show the regression models available in a scientific calculator, including determining the regression coefficient  $r$ , and calculating interpolation and extrapolation values using the functional model obtained.

```
1: 1-VAR  2: A+BX
3: 1+cX2 4: 1n X
5: e^X   6: A*B^X
7: A*X^B 8: 1/X
```

Figure 6

```
1: A      2: B
3: r      4: x
5: s
```

Figure 7

```
r
0.9980324381
```

Figure 8

4. Matrix calculations can be carried out using the calculator.

Students can use matrices to solve systems of linear equations. In addition, students can explore elementary numerical linear algebra problems, which involve iterations of matrix multiplications,  $A^n$  (where  $n$  is a positive integer) including Markov Chain types of problems.

Illustration: The following screen shots show the calculation of  $A^3$  using a scientific calculator

where  $A = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.6 \\ 0.1 & 0.5 & 0.2 \end{bmatrix}$ . (The second screen shot initializes the value of  $A^3$  to 0; the result is

given in the third screen shot after pressing the execute key )

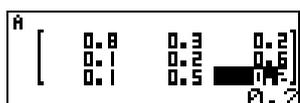


Figure 9



Figure 10

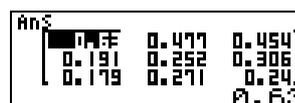


Figure 11

Remarks: We give some consequences when this type of technology is used as follows (see [4]):

- Students would rely on numerical values to “guess” graphs of functions. For instance, students can use the tabular and numerical capabilities offered by the calculator to sketch graphs of functions.
- Since students have access to solving equations, students can solve zeros of polynomial and other elementary functions, which are important and helpful in solving real world problems.

## 2.2 Graphics Calculators and other graphing utilities (but with no CAS):

1. Visualization as a technique of teaching mathematics has become important using graphics calculators and technological tools with graphing capabilities. Visualization allows students to relate algebraic and graphical representations.

Illustration: The following is studying how the parameters  $A$  and  $B$  affect the graph of the function  $y = |x - A| + |x - B|$ : First we let the parameter  $A$  vary from  $-3$  to  $3$  as shown in Figure 12 and we fix the parameter  $B$  to be  $1$  as shown in Figure 13. The sequence of animation is captured in Figures 14 to 20 below.



Figure 12



Figure 13

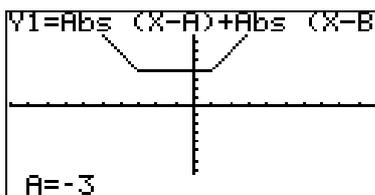


Figure 14

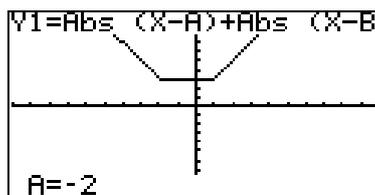


Figure 15

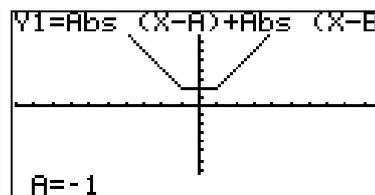


Figure 16

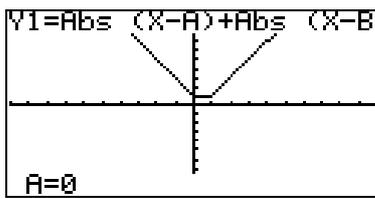


Figure 17

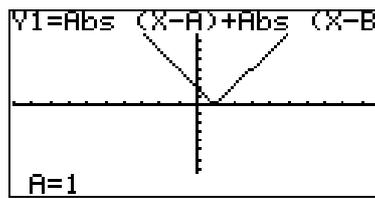


Figure 18

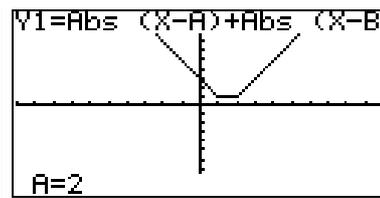


Figure 19

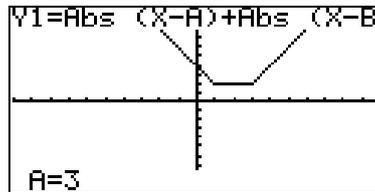


Figure 20

Consider Figures 14 to 16 which illustrate the respective graphs of  $y = |x+3| + |x-1|$ ,  $y = |x+2| + |x-1|$ , and  $y = |x+1| + |x-1|$ . We make the following observations: the horizontal line segment in Figure 14 is from  $x = -3$  to  $x = 1$ . This suggests what students have to do algebraically if they wish to graph  $y = |x+3| + |x-1|$ . The function is defined as a piecewise function on the given intervals:  $x < -3$ ,  $-3 \leq x \leq 1$  or  $x > 1$ . Similarly, the horizontal line segment in Figure 15 is from  $x = -2$  to  $x = 1$ , so that the piecewise function is defined for the intervals  $x < -2$ ,  $-2 \leq x \leq 1$  or  $x > 1$ . Finally, the horizontal line segment in Figure 16 is from  $x = -1$  to  $x = 1$ , which gives rise to the following intervals:  $x < -1$ ,  $-1 \leq x \leq 1$ , or  $x > 1$ . Similar results can be obtained from Figures 17 to 20.

Visualization allows the graphical interpretation of analytical concepts and processes.

Illustration 1: There are many concepts on algebraic inequalities that are difficult for students to comprehend, so that a graphical approach allows for a visualization of the problem. Consider for example, the problem of finding the value of the definite integral  $\int_a^b |x-A| + |x-B|$ , where  $a < -A$  and  $b > B$ . It is easier to sketch the graph of  $y = |x-A| + |x-B|$  as we have discussed previously, and find the area under (or above)  $y = |x-A| + |x-B|$  in the interval  $[a, b]$ .

Illustration 2: Consider for instance the graph supporting the calculation  $\lim_{x \rightarrow 3^+} \frac{x-1}{x-3} = +\infty$ :

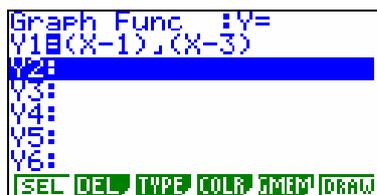


Figure 21

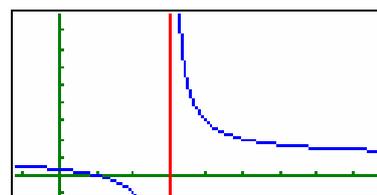


Figure 22

Using graphical approach can also help students understand  $\lim_{x \rightarrow 3^-} \frac{x-1}{x-3} = -\infty$ . We note that using both graphical and tabular approaches help students observe the behavior of the graph of a function.

**Illustration 3:** The Mean Value Theorem is visualized from the graph in this example. For the function  $f(x) = x^3 + x^2 - x$  on the interval  $[-2,1]$ , we use the trace capability of the calculator to highlight the fact that  $f'(x) \approx 1$  when  $x \approx -1.2$  and  $x \approx 0.5$ .

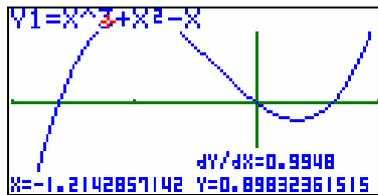


Figure 23

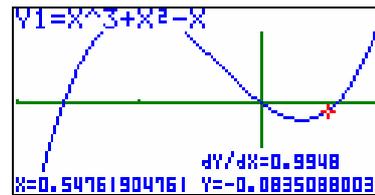


Figure 24

2. The spreadsheet capability of the calculator allows students to generate charts and bar graphs to help students in data representation, interpretation and calculation of quantitative information.

**Illustration:** The left screen shot illustrates data in the spreadsheet editor where column B denotes frequencies. The relative frequencies are calculated in column C, with corresponding cumulative frequencies in column D. The right screen shot gives the graph of the probability distribution.

TUSE	A	B	C	D
1	0	120	0.0436	0.0436
2	1	970	0.3527	0.3963
3	2	730	0.2654	0.6618
4	3	410	0.149	0.8109
5	4	270	0.0981	0.909

Figure 25

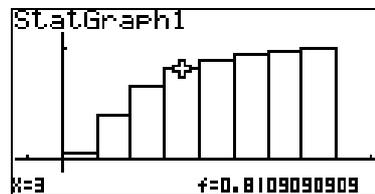


Figure 26

**Remarks:** Some implications when this technological tool is used are described as follows: (also see [4]):

- In analyzing certain mathematical problems, the graphing capabilities of the utility will have to be supplemented with rigorous calculations. For example, graphics calculators can provide the graphs of a function  $f(x)$  and its derivatives, however manual calculations have to be performed to obtain  $f'(x)$  and  $f''(x)$ .
- Students may not be able to solve problems symbolically. For instance, consider the following problem: Let  $f(x) = \sin(x) - x^2 + 5$ , find a horizontal line ( $y = c$ ) such that the line cuts the area of  $y = f(x) \geq 0$  into equal halves.

### 2.3 CAS enabled technological tools

Using technological tools equipped with CAS, algebraic and symbolic manipulation is trivialized, so that students can experiment on interesting theorems and outcomes and “what-if?” situations.

**Illustration:** The following exercises are difficult or time consuming to solve without CAS:

Factor  $x^n + 1$  for  $n = 1, 2, \dots, 150$ . (a) Determine if the coefficients of the polynomial factors are 1 for all exponents  $n$ . (b) Explore and conjecture that the first exponent which leads to integer coefficients  $> 1$  is  $n = 105$ .

The Figures 27-29 are screen shots from ClassPad (see [1]), we can obtain the similar results with any devices that are CAS enabled.

```
factor(x^105+1)
(x+1)·(x^2-x+1)·(x^4-x^3+x^2
```

Figure 27

```
factor(x^105+1)
x^41-x^40+x^39+x^36-x^35+x
```

Figure 28

```
factor(x^105+1)
-x^8+2·x^7-x^6+x^5+x^2-x+1)
```

Figure 29

## 2.4 CAS with Dynamic Geometry

The use of dynamic geometry simultaneously with CAS allows the students to explore the geometrical situation, to make conjectures, and to test and verify the answers symbolically and numerically. The link between geometric, numeric, algebraic as well as graphical representations is established and can be shown.

Illustration 1. Consider for example the following problem: Determine the measure of the vertex angle,  $FED = 2x$  for which the given triangle, with sides FE and ED of length 1, will attain a maximum area. By animating the isosceles triangle FED, students can explore and give conjectures. Referring to the screen shots given below: When the triangle is animated, a table is generated showing how the areas of the triangle are changing as the angle varies. The table suggests that we get the maximum area of  $0.499 \approx 0.5$  when  $2x \approx 90.4$  or  $x \approx 45.2$  degrees or  $0.79$  radians.

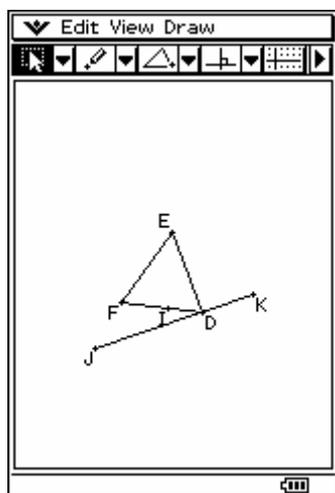


Figure 30

vertex angle-2x	Area
136.9437263	0.341361
157.8251771	0.188721
161.3960988	0.1595071
110.9912973	0.466815
77.17689082	0.4875311
62.16500426	0.442150
57.8923217	0.4235281
59.20367956	0.429499
63.49187114	0.447437
69.38987657	0.468000
76.12478968	0.485411
83.22978102	0.496513
90.40974499	0.4999871

Figure 31

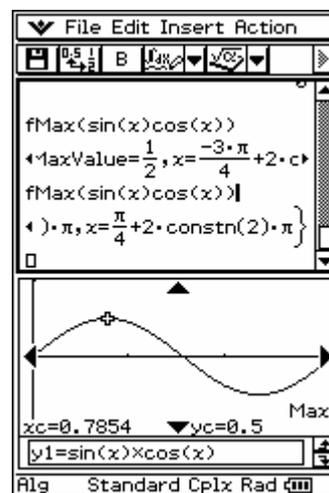


Figure 32

Note that the area  $A$  of the triangle in terms of  $x$  is given by  $A = (\sin x)(\cos x)$ . The rightmost (upper) screen (top half of Figure 32) gives the exact calculation: the maximum area is  $\frac{1}{2}$ , which occurs when  $x = \frac{\pi}{4}$ , considering the domain of the given problem. A graphical interpretation is

shown in the rightmost (bottom) screen (bottom half of Figure 32) and is helpful. It can be verified that the maximum value of the function occurs at approximately 0.79 radians.

**Remarks:**

- Because of the dynamic geometry and animation capabilities of the technological tool, many traditional problems can be modified to more challenging problems. In the mean time, some challenging problems in the past become more accessible and easily understood by the students.
- The “CAS-CAT research project” is a trial project conducted by the Victorian Curriculum and Assessment Authority (VCAA), the state government agency in Australia responsible for senior secondary school programs and end-of-school assessment; in conjunction with a research team from the University of Melbourne. Under this project, students are allowed to use CAS enabled calculators in examinations. Many examples and situations may be obtained from [5] in this respect.

**3. Graphics Calculators and e-Activities**

The recent development of Casio of the *ClassPad* and the 9860 graphics calculator provides new opportunities for learning mathematics. These handheld devices offer students small electronic activities, *e-Activities*, which link the calculator’s capabilities for different learning purposes. The e-Activity can be considered as a pre-designed lesson that connects various mathematical ideas through different applications. The e-Activity designed on the *ClassPad* for instance, links graphical, numeric, symbolic and geometric applications, while the e-Activity written on the 9860 device connects capabilities of a graphics calculator.

Illustration 1: Consider the following extracts from the e-Activity *Integration* designed for the *ClassPad* which are concerned with teaching the concept of the definite integral from various perspectives. The student is given the opportunity to explore aspects of the mathematical concept simultaneously from symbolical, numerical and geometrical points of view in order to enrich his conceptual understanding.

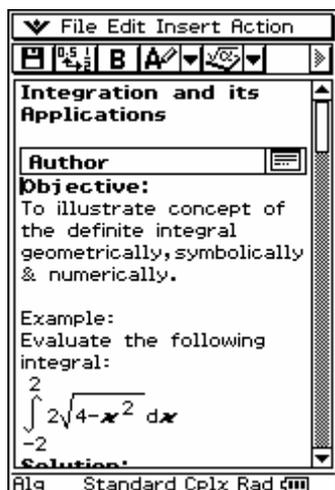


Figure 33

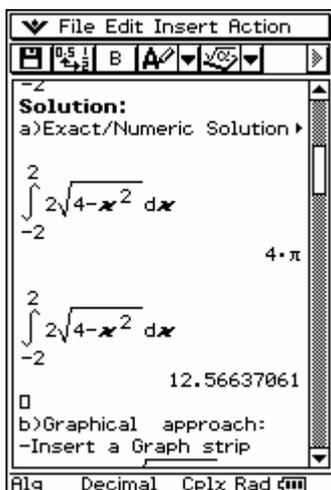


Figure 34

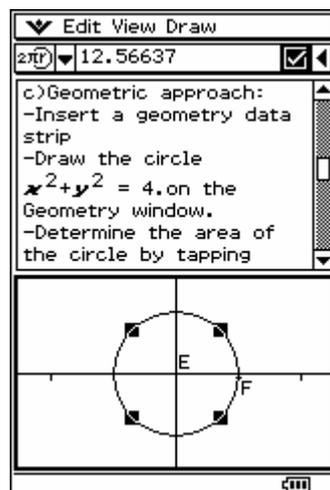


Figure 35

**Illustration 2:** The following screen shots are from the e-Activity *Trigonometric Identities* written for the 9860 graphics calculator which show the accessibility of both the graph and tabular functionalities within the e-Activity. This environment helps the student make potential mathematical discoveries through some controlled interaction between the graphical and the tabular approach.

In the first screen shot the graph strip is highlighted, which leads the student to the graph menu and the graphs, as suggested by the last two screen shots.



Figure 36

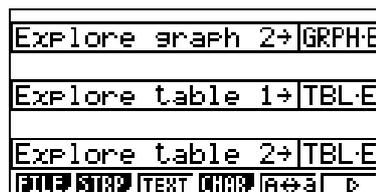


Figure 37

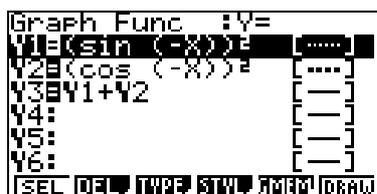


Figure 38

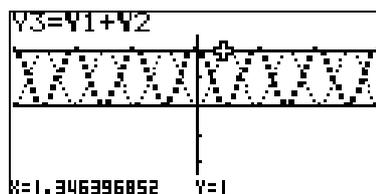


Figure 39

The next example shows a powerful feature of the *ClassPad* through the e-Activity, the capability to make connections between different areas of mathematics such as linear algebra and transformation geometry.

**Illustration 3:** The product of the matrices  $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix}$  can be interpreted geometrically as

the application of a  $90^\circ$  rotation given by the transformation matrix  $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  to the triangle EFG

with vertices (1,1), (2,3) and (4,2). The resulting matrix  $\begin{bmatrix} -1 & -3 & -2 \\ 1 & 2 & 4 \end{bmatrix}$  represents the vertices (-1,1), (-3,2) and (-2,4) of the resulting triangle.

The calculation link within the e-Activity allows the student to “drag” the product of the matrices to the original triangle to obtain the resulting triangle, as seen in the second and third screen shots below. The last screen shot suggests that the resulting transformation is a  $90^\circ$  rotation

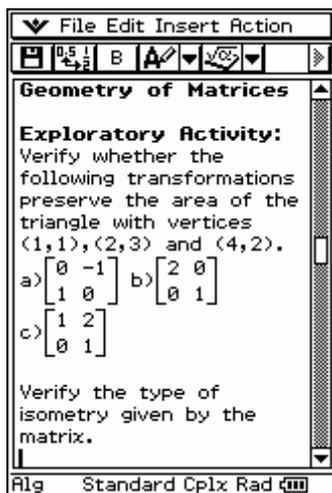


Figure 40

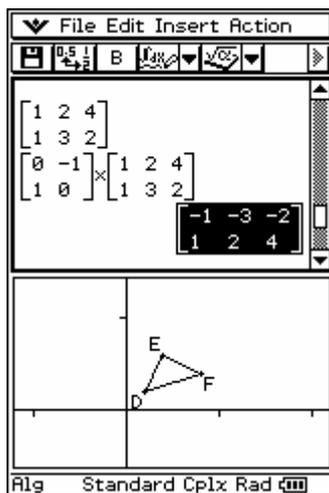


Figure 41

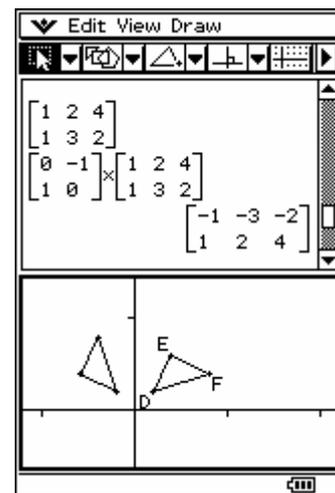


Figure 42

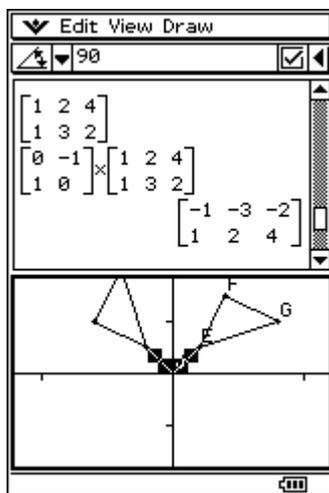


Figure 43

From the illustration above, we find the benefits of integrating dynamic geometry within a technological tool. We may extend this example to understand the geometric interpretation of eigenvalues and eigenvectors.

As we have seen from the above examples, electronic activities available in calculators are effective tools for teaching and learning. However, several issues have been raised by mathematics educators regarding the suitability of such calculators in examinations. Concerns, for instance, regarding allowing calculators with the capabilities of handling both text lines and mathematical computations in examinations have to be addressed.

#### 4. Conclusion

From the examples and situations discussed in this paper, we can conclude that the use of various types of technologies offer different implications in teaching mathematics and assessment. The emergence of new technologies gives rise to new opportunities for learning mathematics. In addition, most examples we mentioned in this paper can be explored with an appropriate handheld technological tool. Due to limitations of individual hardware devices, there are plenty exercises that

one may not be able solve with a specific hand-held technology. Therefore, it is necessary for teachers to introduce timely by using a software program such as Maple, Mathematica and etc. to allow students to explore more in-depth problems. This also allows teachers to assign more challenging questions which students are not constrained by time to solve. Depending on the type of technological tool available in a learning environment, mathematics curriculum will have to be redefined and the focus of examinations adjusted accordingly.

## References

- [1] ClassPad, a product by CASIO Computer Co., Ltd. (<http://classpad.net/>)
- [2] De Las Penas, M.L. & Yang, W.C. (2004) "*Inspiring Students to Study and Learn Mathematics Using Technology*", Proceedings of ATCM'04, ISBN 0-9763064-0-9.
- [3] Kissane, Barry. (2004) "*The Design and Use of Eactivities for Learning Mathematics*", Proceedings of ATCM'04, ISBN 0-9763064-0-9.
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- [5] Stacey, Kaye, "*Using Computer Algebra Systems in Secondary School Mathematics: Issues of Curriculum, Assessment and Teaching*", Proceedings of ATCM '03, (<http://epatcm.any2any.net/EP/2003/2003S172/fullpaper.pdf>)